HERACLITEAN TIME PROPOSAL REVISITED

Orfeu Bertolami*

CERN, Theory Division
CH-1211 Geneva 23
Switzerland

ABSTRACT

Difficulties in the interpretation of the wave function of the Universe in canonical quantum gravity suggest that the use of dynamical variables to play the rôle of time is not quite consistent. A formulation of canonical quantum gravity in which time is an extrinsic variable has been previously studied with the problem of being compatible, at classical level, to General Relativity with a non-vanishing unspecified cosmological constant. We argue that last this problem can be circumvented by introducing a non-dynamical scalar field which allows for a relaxation mechanism for the cosmological term.

CERN-TH.7147/94

April 1994

* On leave of absence from Instituto Superior Técnico, Departamento de Física, Av. Rovisco Pais, 1096 Lisboa Codex, Portugal.

The issue of "time" is intimately related with a host of key questions in quantum gravity. These include interpretational problems such as the ones of probability and its conservation, causality, unitarity and the emergence of the classical features from the primordial inherently quantum world. In canonical quantum gravity, as repeatedly discussed in the literature, the proposals advanced so far to tackle the problem of time (see refs. [1] for thorough discussions), are not completely satisfactory as consistency in the full quantum theory sense is difficult to achieve. The most discussed proposals, such as for instance, the WKB interpretation [2] and the conditional probability interpretation [3] illustrate quite well the situation. Indeed, the former approach is valid only in the restricted domain of the semiclassical approximation of the quantum theory, while the latter requires one to favour in an arbitrary way a specific time variable among all possible dynamical variables. The main difficulties in these approaches can be ultimately attributed to the hyperbolic nature of the Wheeler-DeWitt equation.

Another interesting proposal to the abovementioned problem is the so-called Heraclitean time proposal [4]. In this approach, the time flow is viewed as a dialectics, such that a suitable variable allows for the setting of conditions for the remaining variables to evolve. In this proposal, the time parameter seems to be essentially non-dynamical since it is shown, in the context of ordinary Schrödinger quantum mechanics, that no dynamical variable can correlate monotonically with the Schrödinger time parameter provided the Hamiltonian of the system is bounded from below. Although this is not the situation encountered in canonical quantum gravity, the Heraclitean approach leads, at least at the minisuperspace level, to satisfactory interpretative features [4].

From the technical point of view, the extrinsic or Heraclitean time proposal of ref. [4] is built in analogy with a parametrized form of quantum mechanics in the Schrödinger picture through fixing the determinant of the metric and therefore the lapse function in the ADM formalism. This fixing implies that the Hamiltonian constraint of canonical quantum gravity can be only partially recovered via the closure condition of the constraint algebra. The classical Hamiltonian is then fixed to the value of an arbitrary unspecified non-vanishing cosmological constant which emerges as an integration constant. This strategy

gives origin to the so-called Modified or Unimodular Theories of Gravity which have been recently studied both in connection with the question of time in quantum gravity [4] and also with the problem of the cosmological constant itself [5]. These theories are reminiscent of early considerations by Einstein [6] concerning the validity of the field equations of General Relativity to matter. As pointed out in ref. [7], the fixing of the determinant of the metric and other combination of dynamical variables can be more easily implemented in a covariant fashion by introducing into the action one or, more generally, a set of constraints, $C^a = 0$ ($a = 1, 2, \dots, n$), with their associated Lagrange multipliers, λ_a . In our approach this set will comprise a single constraint to fix the determinant of the metric.

Clearly, the Heraclitean approach is only feasible in the presence of a non-vanishing cosmological constant, being therefore incompatible with the main trend of the cosmological observations which are consistent with a vanishingly small cosmological constant. The aim of this paper is to address this last difficulty by introducing a non-dynamical scalar field, i.e. a scalar field that has no kinetic term, that allows for a type of relaxation mechanism for the cosmological term. This renders the Heraclitean time proposal at least consistent with observation. The introduction of this additional scalar field can be justified as scalar fields are present in the gravitational multiplet of various unification schemes such as for instance in some supergravity models and string theories. The non-dynamical feature of this field can be found in for instance, singularity-free cosmological models where it is responsible for the curvature assuming a limiting value [8].

Of course, likewise in the original Heraclitean time proposal, the question of relating the cosmological time of this approach with the standard time which in General Relativity arises from spacetime foliations, remains as stressed in ref. [9], an open issue that we shall not attempt to address here.

Let us now turn to the description of our model. We consider first the Lagrangian formulation and the following action in the manifold M :

$$S = \int_M d^4x \sqrt{-g} \left\{ \frac{\varphi}{16\pi} \left[-R + \frac{\lambda C}{\varphi \sqrt{-g}} + \frac{V(\varphi)}{\varphi} \right] + L_m(\chi) \right\}, \quad (1)$$

where φ is a non-dynamical scalar field endowed with a potential $V(\varphi)$ and $L_m(\chi)$ denotes the Lagrangian density of generic matter fields, χ .

The constraint C , which is obtained extremizing action (1) with respect to λ , is chosen to depend only on the square root of the determinant of the metric, $C = C(\sqrt{-g})$, which is fixed to a certain value, as for instance

$$C \equiv \sqrt{-g} \cdot \sqrt{-\eta} = 0, \quad (2)$$

where η is the determinant of the Minkowski metric.

Extremizing action (1) with respect to the metric and to the scalar field gives respectively:

$$G_{\mu\nu} = \frac{1}{\varphi} \left[\varphi_{;\mu\nu} - g_{\mu\nu} \square \varphi \right] + \frac{\lambda}{2} \frac{\partial C}{\partial \sqrt{-g}} g_{\mu\nu} + \frac{8\pi}{\varphi} T_{\mu\nu} + \frac{1}{2\varphi} V(\varphi) g_{\mu\nu}, \quad (3)$$

$$-R + \frac{\partial V(\varphi)}{\partial \varphi} = 0, \quad (4)$$

where $G_{\mu\nu}$ is the Einstein tensor and $T_{\mu\nu}$ the energy - momentum tensor associated with L_m .

From the Bianchi identity $D^\mu G_{\mu\nu} = 0$, it follows that the most general form of the right-hand side of eq. (3) is given by:

$$X_{\mu\nu} = A(\varphi) g_{\mu\nu} + B(\varphi) T_{\mu\nu} + C(\varphi) g_{\mu\nu}, \quad (5)$$

where $A(\varphi)$, $B(\varphi)$ and $C(\varphi)$ are tensor functions of φ as specified in (3).

Taking the trace of (5) and comparing with (3) and (4), yields:

$$-\frac{\partial V}{\partial \varphi} = 4A(\varphi) + B(\varphi)T + C(\varphi) \quad (6)$$

where $T = T^\lambda{}_\lambda$ and $C(\varphi) = C(\varphi)^\lambda{}_\lambda$.

Clearly, eq. (4) fixes the scalar curvature allowing in this way for the consistency condition (6). The same procedure can be basically repeated in the Hamiltonian formalism as we discuss next.

We consider the ADM formalism and add to action (1) the following boundary

term:

$$S_B = \frac{1}{8\pi} \int_{\partial M} d^3x \sqrt{h} K [1 + \varphi] , \quad (7)$$

where $h = \det(h_{ij})$, $h_{ij} = {}^{(3)}g_{ij}$ ($i, j = 1, 2, 3$) is the three-dimensional Riemannian metric, $K = h^{ij} K_{ij}$ and K_{ij} is the extrinsic curvature of the three-dimensional manifold Σ , the

boundary of M (∂M). In the canonical formalism, the scalar curvature is expressed through the three-dimensional curvature ${}^{(3)}R$, its metric h_{ij} , the extrinsic curvature K_{ij} , the lapse function N and the shift vector N^i (see e.g. ref. [10])

$$\begin{aligned} \sqrt{-g} R &= N \sqrt{h} \left[{}^{(3)}R + K_{ij} K^{ij} - K^2 \right] - 2 \left[\frac{\partial}{\partial t} (\sqrt{h} K) + \right. \\ &\quad \left. \frac{\partial}{\partial x^i} (\sqrt{h} \{ K N^i - h^{ij} \sqrt{h} N_{;j} \}) \right] , \end{aligned} \quad (8)$$

from which, after integration by parts yields

$$\begin{aligned} \sqrt{-g} \varphi R &= N \sqrt{h} \varphi \left[{}^{(3)}R + K_{ij} K^{ij} - K^2 \right] - 2 \left[\frac{\partial}{\partial t} (\sqrt{h} \varphi K) - \sqrt{h} F(\varphi) K + \right. \\ &\quad \left. \frac{\partial}{\partial x^i} (\sqrt{h} \varphi [K N^i - h^{ij} \sqrt{h} N_{;j}]) + \frac{\partial \varphi}{\partial x^i} h^{ij} N_{;j} \right] , \end{aligned} \quad (9)$$

where $F(\varphi) \equiv \varphi + \frac{\partial \varphi}{\partial x^i} N^i$ and the overdot denotes derivative with respect to time.

We consider Σ to be a compact three-dimensional manifold. It then follows that

the third term inside the second square bracket in (9) vanishes upon integration. The first

term in there drops out on account of (7) while the last one can be eliminated by further integrating by parts. Notice that in (9), the term $\sqrt{h} \dot{\varphi} K$, which mixes $\dot{\varphi}$ and h_{ij} can be absorbed by the field redefinition, $\Phi = \varphi h$ [11], however since that yields that Φ turns out to be a dynamical field, we shall prefer instead to pursue another approach.

Hence we obtain for action (1):

$$S = \frac{1}{16\pi} \int_M dt d^3x N \sqrt{h} \left\{ -\varphi \left({}^{(3)}R - K_{ij} K^{ij} + K^2 \right) - 2 \frac{F(\varphi)}{N} K + \frac{\lambda C}{N \sqrt{h}} + 16\pi L_m + V(\varphi) \right\} . \quad (10)$$

Using the definition of the extrinsic curvature K_{ij} in terms of the rate of change of the three-dimensional metric

$$h_{ij} = -2 N K_{ij} + 2 D_{(i} N_{j)} , \quad (11)$$

$D_{(i} N_{j)}$ being the symmetrized covariant derivative on Σ , allows one to obtain the canonical conjugate variables:

$$\pi_{ij} = \frac{\delta S}{\delta h^{ij}} = \sqrt{h} \varphi (K_{ij} - h_{ij} K) + \frac{F(\varphi)}{N} \sqrt{h} h_{ij} , \quad (a)$$

$$\pi_{N_i} = \frac{\delta S}{\delta N^i} = 0 , \quad (b)$$

(12-b) meaning that N^i has vanishing momentum which is related to the invariance of the theory under three-dimensional diffeomorphisms.

The Hamiltonian associated to action (10) can be then written as

$$H = \int_M dt d^3x \left\{ N H_0 + N^i H_i \right\} \equiv \int dt H , \quad (13)$$

where

$$H_0 \equiv \sqrt{\hbar} \left[\phi H_G + \frac{\lambda C}{N\sqrt{\hbar}} + 16\pi H_m + V(\phi) \right], \quad (14)$$

and in terms of the canonical conjugate momenta π_{ij} :

$$H_G \equiv \left[- ({}^3R + \hbar^{-1} \phi^{-2} (\pi_{ij} \pi^{ij} - \frac{1}{2} \pi^2 + \frac{3}{2N^2} F(\phi)^2 \hbar) \right], \quad (15)$$

and furthermore

$$H^i \equiv -D_j \left[\frac{\pi^{ij}}{\phi \sqrt{\hbar}} \right] + H_m^i, \quad (16)$$

H_m being the Hamiltonian of matter fields and $H_m^i = 16\pi T_0^i$ where T_0^i are the (0, i) components of the energy-momentum tensor for matter.

Notice, that, on account of the fixing (2), the lapse function now evolves

$$N = \hbar^{-1/2}, \quad (17)$$

preventing one to obtain the Hamiltonian constraint in the usual way. The momentum constraints remain, however, unchanged:

$$H^i = 0. \quad (18)$$

The Hamiltonian constraint can still be nearly recovered from the condition that the dynamical evolution satisfies the momentum constraints (18). This comes about as the

Poisson bracket of the Hamiltonian H with the integrated momentum constraints on Σ vanishes [4]:

$$\left\{ H; \int_{\Sigma} d^3x \xi^i H_i \right\} = 0, \quad (19)$$

where ξ^i is an arbitrary vector field on Σ .

Conditions (18) together with (19) imply that

$$\int_{\Sigma} d^3x \xi^i D_i [NH_0] = 0. \quad (20)$$

$\therefore$ solution of (20) is given of course by $NH_0 = \text{Constant}$, where this integration constant is essentially the cosmological constant [4]. Eq. (20) admits however, another solution, namely:

$$H_0 = \sqrt{\hbar} \Lambda(\phi), \quad (21)$$

where $\Lambda(\phi)$ stands for an arbitrary spatially constant function of ϕ which replaces the spatial integration constant of the original Heracleean proposal. Naturally, this amounts for ϕ being spatially constant itself and hence this solution is consistent only with a rather special class of metrics.

Clearly solution (21) is at the heart of our proposal, as the function $\Lambda(\phi)$ can be set to guarantee that the "cosmological constant" vanishes for $\sqrt{\hbar} \gg L_p^3$ (L_p being the Planck length), allowing for a non-vanishing value for $\sqrt{\hbar} \leq L_p^3$.

Notice that we also have to consider the additional constraint associated to the variation of the Hamiltonian with respect to the scalar field, which in the gauge $N_i = 0$ is given by:

$$H_G + \frac{\partial V(\phi)}{\partial \phi} = 0. \quad (22)$$

Condition (22) allows for consistency between constraints (18) and (21) and implies in a relation analogous to (6), previously obtained in the Lagrangian formalism:

$$-\phi \frac{\partial V(\phi)}{\partial \phi} + 16\pi H_m + V(\phi) = \Lambda(\phi). \quad (23)$$

The canonical quantization of the model proceeds as usual. The state vector is a functional of the dynamical variables on Σ , i.e. $\psi = \psi [\tau = \varphi(t)^{1/2}, h_{ij}, \chi]$. The momentum constraints (18) are imposed by requiring the state vector to satisfy

$$\int_{\Sigma} d^3x (D_i \xi_j) \frac{\delta \psi}{\delta h_{ij}} = 0, \quad (24)$$

which imply that ψ depends only on three-geometries.

From (20) we get the condition

$$\int_{\Sigma} d^3x \xi^i D_i \left[h^{-1/2} H_0 \right] \psi = 0, \quad (25)$$

where now ψ can evolve via the Schrödinger equation

$$i \frac{\partial \psi}{\partial \tau} = H \psi = \int_{\Sigma} d^3x h^{-1/2} H_0 \psi, \quad (26)$$

as the right-hand side of (21) is non-vanishing. Hence, the introduction of the scalar field allows the state vector to have a nontrivial time dependence.

Notice that some solutions of (26) are eigenstates of the $\Lambda(\varphi)$, in the very way they were eigenstates of the cosmological constant in ref. [4]:

$$H \psi_F = \Lambda(\varphi) \psi_F, \quad (27)$$

meaning that a general solution of the Schrödinger problem (26) can be obtained via integration over the values of $\Lambda(\varphi)$ with an appropriate measure:

$$\psi(\tau) = \int d\Lambda(\varphi) \mu(\Lambda(\varphi)) \exp \left[-i \int_{\Sigma} d^3x \Lambda(\varphi) \right] \psi_F, \quad (28)$$

where ψ_F satisfies eq. (27).

The setting we propose operates then in the following way: (i) choose $V(\varphi)$ and $\Lambda(\varphi)$ such that initially, say for $\varphi^{1/2} < L_P^{-1}$ and $\sqrt{h} < L_P^3$, $V(\varphi)$, $\frac{\partial V(\varphi)}{\partial \varphi}$ and $\Lambda(\varphi)$ are non-

vanishing; (ii) solve eq. (21) to get the evolution of the three-geometry, (iii) consistency equations (22) and (23) provide then the next value of $\tau = \varphi^{1/2}$ compatible with the evolution of the dynamical variables.

The choice of $\Lambda(\varphi)$ must of course allow for $\varphi^{1/2} \rightarrow L_P^{-1}$ and $\sqrt{h} \gg L_P^3$ that $\Lambda(\varphi) \rightarrow 0$ as to avoid conflict with the vanishing of the cosmological term on large scales. Once this condition is met our proposal leads to a description that at minisuperspace level, is essentially similar to the one of ref. [4].

Thus we have seen that the introduction of a non-dynamical scalar field allows, within the framework of canonical quantum gravity, to obtain an Heraclitean time proposal, which at classical level, is consistent with observation. Naturally, the introduction of a scalar field and, in general, any field to allow for the setting of an extrinsic time variable brings some degree of arbitrariness into the quantum gravity problem. Although that may be regarded as an undesirable feature of our model, that has at least the virtue of illustrating in a concrete fashion the problems one faces in attempting to solve some of the interpretational difficulties of quantizing gravity in the canonical framework adopting the Heraclitean proposal and making it consistent with observation.

REFERENCES

- [1] C.J. Isham, "Conceptual and Geometrical Problems in Quantum Gravity" 1991 Schadrning Winter School, Imperial College Preprint /TP/90-9/14; "Canonical Quantum Gravity and the Problem of Time" 1992 NATO Advanced Study Institute "Recent Problems in Mathematical Physics", Imperial College Preprint /TP/91-92/25; K.V. Kuchar, "Time and the Interpretations of Quantum Gravity" in the Proceedings of the 4th Canadian Conference on General Relativity and Relativistic Astrophysics, 1992.
- [2] A. Vilenkin, Phys. Rev. **D39** (1988) 1116.
- [3] J.B. Hartle and S.W. Hawking, Phys. Rev. **D28** (1983) 2960.
- [4] W. Unruh, Phys. Rev. **D40** (1989) 1048;
W. Unruh and R.M. Wald, Phys. Rev. **D40** (1989) 2598.
- [5] S. Weinberg, Rev. Mod. Phys. **61** (1989) 1.
- [6] A. Einstein, Sitz. Berl. Preuss. Akad. Wiss. (1919).
- [7] A.N. Petrov, Mod. Phys. Lett. **A6** (1991) 2107.
- [8] V. Mukhanov and R. Brandenberger, Phys. Rev. Lett. **68** (1992) 1969.
- [9] K.V. Kuchar, Phys. Rev. **D43** (1991) 227.
- [10] R.M. Wald in "General Relativity" (University of Chicago Press, 1984).
- [11] C. Kiefer, Phys. Lett. **B225** (1989) 227.