Leptonic flavor violations
in the presence of an extra $Z$

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Abstract

Gauged extensions $[SU(2)_L \otimes U(1)_Y \otimes U(1)_X]$ of the standard $SU(2)_L \otimes U(1)_Y$ model obtained without extending the fermion content of the model are studied. Models that are possible when $U(1)_X$ is identified with some combination of the family lepton numbers are systematically classified. Most of these contain flavor violations in the leptonic sector. These flavor violations are correlated to the mixing of the $U(1)_X$ gauge boson $Z'$ with the ordinary $Z$ in the models considered here. Detailed phenomenological implications of a typical model are discussed. Constraints on the $Z'$ mass and the $Z-Z'$ mixing following from (i) the observations at LEP, (ii) rare processes like $Z \to e\tau$ and (iii) flavor violating $\tau$ decays are presented. It is found that the constraints coming from the LEP allows the rare processes like $\tau \to eee$ at the level of the present limits on its branching ratio. Thus the future search could either improve on the existing LEP limits or would find such flavor violating decays.

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1 Introduction

Lepton number $L_i$ for each family $i = e, \mu, \tau$ are globally conserved in the standard $SU(2)_L \otimes U(1)_Y$ model. Within the minimal $SU(2)_L \otimes U(1)_Y$ theory, these symmetries have to be global since attempt to gauge them would introduce anomalies and would spoil the renormalizability of the theory. Nevertheless, it is possible to gauge some linear combinations of $L_i$. Specifically, it was shown in ref. [1] that only three linear combinations of $L_i$ namely $X = L_e - L_\mu, L_e - L_\tau$ and $L_\mu - L_\tau$ are gaugable and only one can be gauged along with $SU(2)_L \otimes U(1)_Y$ at a time. Thus the maximal permissible gauge group with the minimal content of fermions and higgs field is $SU(2)_L \otimes U(1)_Y \otimes U(1)_X$. One could easily enlarge the gauge symmetry by adding more fermions as happens for example in the left-right symmetric or $E_6$ models [2]. But it is possible to enlarge the available choices of $U(1)_X$ by adding more Higgs doublets transforming non-trivially under it. The point is that the requirement of anomaly cancellations allow for more than the above mentioned three possibilities for $X$. In the absence of Higgs doublets which are non-trivial under $U(1)_X$, these additional choices of $U(1)_X$ are physically indistinguishable from the above three choices. Otherwise, they are inequivalent and could lead to different predictions. $U(1)_X$ groups of this type fall in the category of horizontal symmetries characterised by enlargement of the gauge and Higgs sector. Many examples of such gauge symmetries have been proposed [3] and studied. Crucial tests of such symmetries are flavor violations associated with these symmetries. Most study of these flavor violation were done before the results of the $e^+e^-$ collider became available. These results can provide additional constraints on such theory. We wish to study here simplest of such horizontal symmetries. We shall study various choices of $U(1)_X$ under the assumption that (a) the fermion sector of the standard model is not extended, (b) $X$ is some linear combination of lepton family numbers, (c) the Higgs sector of the SM is enlarged by adding one or more Higgs doublets transforming non-trivially under the gauge group.

It is possible to systematically classify all the allowed choices of $U(1)_X$ in the presence of an enlarged Higgs sector. We do such a classification. Different $U(1)_X$ groups studied here differ from groups of ref. [1] in three important ways. The $U(1)_X$ current in the present case is non-vectorial. Secondly, the $U(1)_X$ current due to its horizontal nature leads to flavor violations in the leptonic sector. Thirdly, the neutral gauge boson $Z'$ associated with $U(1)_X$ necessarily mix with $Z$ in these models. The last two properties put significant constraints on parameters of the model. We systematically work them out. It follows from the analysis presented here that the observable flavor violations, e.g. in $\tau \rightarrow eee$ decay are possible within the models in spite of the severe constraints imposed by the LEP observations.

We shall discuss in the next section all possible choices of $SU(2)_L \otimes U(1)_Y \otimes U(1)_X$ and general structure of the current coupled to $Z$ and $Z'$. Then we discuss a specific model in section 3. Section 4 contains a discussion on constraints on parameters taking the model of section 3 as an illustration. A summary is contained in section 5.

2 Possible Extensions

We shall confine ourselves to the minimal fermionic content as in the standard model but would consider a general gauge group $SU(2)_L \otimes U(1)_Y \otimes U(1)_X$, where $X$ is taken to be a
linear combination of three lepton numbers. In general $X$ need not act vectorially on the weak interaction basis $e'_{i,R}(i = 1, 2, 3)$ although the $X$ assignments of members of a given $SU(2)_L$ doublet have to be identical. For notational convenience let us write $X$-charges in terms of diagonal matrices in the generation space.

$$X_{L,R} = \text{diag}(\alpha_1, \alpha_2, \alpha_3)_{L,R}$$

$X_L$ determine the $U(1)_X$ assignment of the leptonic doublet while $X_R$ that of the charged right-handed leptons. The possible choices of $\alpha_{iL}$ and $\alpha_{iR}$ are restricted due to anomaly cancellation which require:

$$\sum_i \alpha_{iL} = \sum_i \alpha_{iR} = 0$$
$$\sum_i \alpha_{iL}^2 = \sum_i \alpha_{iR}^2$$
$$2 \sum_i \alpha_{iL}^3 = \sum_i \alpha_{iR}^3$$

(1)

These constraints can be satisfied by taking any two of $\alpha_{iL}$ and $\alpha_{iR}$ to be $\pm 1$ and the third to be zero. This can be done in variety of ways but a particularly simple choice results when one takes $X_L = X_R$. In this case the allowed $X$ is restricted [1] either to $L_e - L_\mu$, $L_e - L_\tau$ or $L_\tau - L_\mu$. The current coupled to $U(1)_X$ boson $Z'$ is vectorial when expressed in the weak basis in this case. Since the initial choice of basis is arbitrary one could always redefine the right-handed fields $e'_{iR}$ to obtain the choice $X_L = X_R$. But the structure of physical current coupled to mass eigenstates of fermions depends upon the choice of Higgs fields. In the event of only one Higgs doublet neutral under $U(1)_X$, the charged leptonic mass matrix is diagonal and the physical current coupled to $Z'$ is vectorial. When one introduces more Higgs fields transforming non-trivially under $U(1)_X$, the $U(1)_X$ no-longer remains vectorial. The possible choices of $X \equiv X_L = X_R$ are severely limited due to the anomaly constraints, eq.(1). In particular only three choices are possible which respectively correspond to

$$L_\mu - L_\tau \quad X = \text{diag}(0, 1, -1)$$
$$L_e - L_\tau \quad X = \text{diag}(1, 0, -1)$$
$$L_e - L_\mu \quad X = \text{diag}(1, -1, 0)$$

(2)

The structure of the current associated with the new $Z'$ can be written as,

$$L_{Z'} = \frac{g'}{\cos \theta} (\tilde{e}'_{L} X \gamma_{\mu} e'_{L} + \tilde{e}'_{R} X \gamma_{\mu} e'_{R}) Z'^\mu$$

(3)

where $e'_{iL,R}$ are column vector in generation space and $\theta$ is the weak mixing angle introduced here purely for notational convenience. The coupling of the physical (i.e. mass eigenstate) fermions to $Z'$ depend upon the structure of the mass matrix $M_i$ for the charged leptons. This is dictated by the charge matrix $Q$ whose $(i,j)_{th}$ element correspond to the $X$-charge of bilinear $\tilde{e}'_{iL} e'_{jR}$. For example, we have in case of $L_e - L_\tau$

$$Q = \begin{pmatrix} 0 & -1 & -2 \\ 1 & 0 & -1 \\ 2 & 1 & 0 \end{pmatrix}$$

(4)

The possible structures of mass matrices follow from that of $Q$. In particular, a Higgs field with charge $-Q_{ij}$ would contribute to the $(i,j)^{th}$ element of the mass matrix $M_i$. Note
that the two different fields contribute to the \((M_i)_{ij}\) and \((M_i)_{ji}\). Hence \(M_i\) is necessarily non-hermitian except when it is diagonal with only one Higg’s doublet carrying zero charge under \(U(1)_X\). In this case, the weak basis \(e'_{L,R}\) coincide with the mass basis \(e_{L,R}\) and \(Z'\) couples to a vector current corresponding to \(X\). When one introduces one or more additional doublets transforming non-trivially under \(U(1)_X\) then \(M_i\) is necessarily non-hermitian and can be diagonalized by a biunitary transformation:

\[
U_L M_i U_R^\dagger = \text{diag.}(m_e, m_{\mu}, m_{\tau}) \tag{5}
\]

\[
e_{L,R} = U_L e'_{L,R} \tag{6}
\]

\(\mathcal{L}'_Z\) then assumes the following form in terms of the mass eigenstates:

\[
\mathcal{L}'_Z = \frac{g}{\cos \theta} (\kappa_{Lij} \bar{e}_i \gamma_{\mu} e_j L + \kappa_{Rij} \bar{e}_i \gamma_{\mu} e_j R) Z'^\mu \tag{7}
\]

where

\[
\kappa_a \equiv U_a X U_a^\dagger \quad a = L, R. \tag{8}
\]

Eq.\( (7) \) represents the general form of the \(Z'\) interactions in all the \(SU(2)_L \otimes U(1)_Y \otimes U(1)_X\) models under study. Different models are specified by the choice of \(X\) and the Higgs fields which determine \(M_i\) and hence \(U_L, R\). The following two important properties are enforced by the structure of \(X\):

(i) The current coupled to \(Z'\) is non-vectorial except in a specified case \(U_L = U_R = I\). This follows since \(M_i\) is necessarily non-hermitian when it is not diagonal as already discussed. Hence \(U_L \neq U_R\). Moreover, for \(U_L \neq U_R\), \(U_L X U_L^\dagger\) and \(U_R X U_R^\dagger\) cannot be identical\(^1\) leading to a non-vector current.

(ii) The current coupled to \(Z'\) would violate leptonic flavor, i.e. \(\kappa_{aij}\) are non-zero for \(i \neq j\), if \(M_i\) is not diagonal. In this case, \(U_L\) and/or \(U_R\) are different from unity. To see this, consider \(X \equiv L_e - L_.\). Because of the form of \(X\) given in eq.\((2)\), it is easy to see that \(U_L X U_L^\dagger\) (\(U_R X U_R^\dagger\)) will have non-zero off diagonal couplings unless mixing between \(e'_{L}(e'_{R})\) and \(\tau'_{L}(\tau'_{R})\) is forbidden. Since such couplings would invariably occur in models with extended Higgs structure, one expects the flavor changing \(Z'\) couplings in these cases. This occurrence of the flavor changing current is a well-known phenomena [3] which arises when fermions of the same charge and helicity transform differently under a gauge group; \(U(1)_X\) in the present case.

Since the structure of the \(Z'\) current is fixed by \(X\) and \(M_i\), it is easy to classify all models that are possible within the present scheme. One has basically three types of models. (i) Model with only one Higgs doublet neutral under \(U(1)_X\). In these, eq.\((4)\) require \(M_i\) to be diagonal. Hence one has vector currents and no flavor violation. There are three models in this category studied in ref. [1]. (ii) Models with two Higgs carrying \(U(1)_X\) charge 0, and \(\pm 1\) or \(\pm 2\). In this case only one non-diagonal entry is possible in \(M_i\) (see eq.\((10)\)). In these types of models, one of the lepton remains unmixed while the other two mix with some mixing angles \(\theta_{L,R}\).

\(^1\)To prove this explicitly, we write \(U_R = U_L V\), \(V\) being a unitary matrix different from \(I\). Then \(U_L X U_L^\dagger\) = \(U_R X U_R^\dagger\) only if \(V X = X V\). This is not possible because of the restricted structure of \(X\).
(iii) Third category of the models follow when one introduces two or more additional Higgs fields carrying $U(1)_X$ charge ±1 or ±2. These represent general class of models with mixing among all three generations.

We shall study in the next section detailed phenomenology of a model in category (ii).

3 $SU(2)_L \otimes U(1)_Y \otimes U(1)_{L_e - L_\tau}$ Model:

We consider $SU(2)_L \otimes U(1)_Y \otimes U(1)_X$ model containing standard fermions, two Higgs doublets $\phi_{1,2}$ and an $SU(2)_L \otimes U(1)_Y$ singlet $\eta$. $X$ is chosen to be $L_e - L_\tau$ charge from eq.(2). $X$ charges of $\phi_1$ and $\phi_2$ are chosen to be 0 and +2 respectively. The field $\eta$ is assumed to carry some non-zero charge under $U(1)_X$ and it is solely introduced to provide a different mass scale characteristic of the $U(1)_X$ breaking.

The quark sector of the model remains the same as in the SM model while lepton couplings to the neutral Higgs fields are given by the following:

$$-\mathcal{L}_Y = h_{ii} \tilde{e}_i^c L_i^c \phi_1^0 + h_{12} \tilde{e}_1^c L_2^c \phi_2^0$$

$$\equiv \frac{m_i}{\langle \phi_1 \rangle} \tilde{e}_i^c L_i^c \phi_1^0 + \frac{\delta}{\langle \phi_2 \rangle} \tilde{e}_2^c L_2^c \phi_2^0 + h.c$$

This leads to the following mass matrix $M_i$:

$$M_i = \begin{pmatrix} m_1 & 0 & \delta \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}$$

Let $U_{L,R}$ diagonalize $M_i$, i.e.

$$U_L M_i U_R^\dagger = \text{diag}(m_e, m_\mu, m_\tau)$$

where

$$m_\mu^2 = m_2^2$$

$$m_e^2 = \frac{1}{2} \left\{ m_1^2 + m_3^2 + \delta^2 + \left[ (m_1^2 - m_3^2)^2 + 2\delta^2 (m_1^2 + m_3^2) + \delta^4 \right]^\frac{1}{2} \right\}$$

$$m_\tau^2 = \frac{1}{2} \left\{ m_1^2 + m_3^2 + \delta^2 - \left[ (m_1^2 - m_3^2)^2 + 2\delta^2 (m_1^2 + m_3^2) + \delta^4 \right]^\frac{1}{2} \right\}$$

$$U_{L,R} = \begin{bmatrix} \cos \theta_{L,R} & 0 & \sin \theta_{L,R} \\ 0 & 1 & 0 \\ -\sin \theta_{L,R} & 0 & \cos \theta_{L,R} \end{bmatrix}$$

The mixing angles $\theta_{L,R}$ are given by:

$$\sin 2\theta_L = -\frac{2\delta m_3}{m_\tau^2 - m_e^2} \quad \sin 2\theta_R = -\frac{2\delta m_1}{m_\tau^2 - m_e^2}$$

5
As we will soon see, the $\theta_{L,R}$ are constrained to be quite small. It is therefore appropriate to work in the approximation $\delta < m_1, m_3$. In this limit,
\[
\sin 2\theta_R \approx -\frac{2\delta m_e}{m_2^2} \quad \sin 2\theta_L \approx -\frac{2\delta}{m_\tau} \quad (12)
\]

The parameters $\kappa_{aij}$ ($a = L, R$) determining the couplings of $Z'$ to leptons through eq.(8) are explicitly given in the present case by
\[
\begin{align*}
\kappa_{a11} &= \cos 2\theta_a = -\kappa_{a33} \\
\kappa_{a13} &= -\sin 2\theta_a \\
\kappa_{a2i} &= 0 \quad i = 1, 2, 3 \quad (13)
\end{align*}
\]

Since one of the doublets carry non-zero $U(1)_X$ charge, the $Z'$ will mix with the conventional $Z$ boson to produce two mass eigenstates $Z_{1,2}$.
\[
\begin{align*}
Z &= \cos \phi \ 2_1 + \sin \phi \ 2_2 \\
Z' &= -\sin \phi \ 2_1 + \cos \phi \ 2_2 \quad (14)
\end{align*}
\]

The couplings of the neutral gauge boson $Z_{1,2}$ to the leptons are now given by
\[
L_Z = \frac{g}{\cos \theta} \left\{ \sum_{m=1,2} F_{Lmij} \bar{\ell}_i \gamma_\mu \ell_j Z_m^\mu + L \leftrightarrow R \right\} \quad (15)
\]

where
\[
\begin{align*}
F_{L11j} &= \cos \phi \left(-\frac{1}{2} + \sin^2 \theta\right) \delta_{ij} - \sin \phi \frac{g'}{g} \kappa_{Lij} \\
F_{R11j} &= \cos \phi \sin^2 \theta \delta_{ij} - \sin \phi \frac{g'}{g} \kappa_{Rij} \\
F_{L2ij} &= \sin \phi \left(-\frac{1}{2} + \sin^2 \theta\right) \delta_{ij} + \cos \phi \frac{g'}{g} \kappa_{Lij} \\
F_{R2ij} &= \sin \phi \sin^2 \theta \delta_{ij} + \cos \phi \frac{g'}{g} \kappa_{Rij}
\end{align*}
\]

As would be expected, eqs.(10) and (13) show that the muon number is exactly conserved in the model. This is a consequence of the fact that both the $Z'$ interactions as well as the mass matrix, eq.(10), respect this symmetry. When $\delta << m_\tau$, the flavor violations and departure from vectorial symmetry are very small. Moreover, these departures are more suppressed in the right-handed sector compared to the left-handed sector.

The generalization to other models in this category is obvious. One could construct another model with additional Higgs carrying $L_e - L_\tau$ charge $-2$ instead of $+2$. In this case $(M_i)_{31}$ will be non-zero instead of $(M_i)_{13}$ as in eq.(10). All the couplings of this model are then obtained by interchange of $\theta_L \leftrightarrow \theta_R$ in eq.(13). In addition to these two models with $L_e - L_\tau$ symmetry, one could construct pair of models each with symmetry $L_e - L_\mu$ and $L_\mu - L_\tau$. These are respectively characterized by an unbroken $L_\tau$ and $L_e$. 

6
In addition to the flavor violations induced by $Z'$, there exists other flavor violations associated with the Higgs fields. These arise in a well-known [4] manner whenever the fermions with the same charge obtain their masses from two different Higgses as in eq.(10). Using eqns.(10-11) it follows that

$$-\mathcal{L}_{FCNC} = \delta \left( \frac{\phi_1^0}{\phi_1^0 < \phi_1^0 >} - \frac{\phi_2^0}{\phi_2^0 < \phi_2^0 >} \right) \cos \theta_L \sin \theta_R \tau_R + \cos \theta_R \sin \theta_L \tau_L e_R + h.c.$$  

It follows from eq.(12) that these flavor violations are of $O \left( \delta^2 / m_\tau < \phi_1^0 > \right)$ and hence would be suppressed in the limit $\delta << m_\tau$ compared to $Z'$ induced flavor violations unless the associated Higgs is much lighter than $Z'$. We shall therefore concentrate on the $Z'$ induced flavor violations in the next section.

We close this section with a brief mention of the $Z-Z'$ mixing in these models and quote well known formulae [4] to be used later on. The neutral gauge boson mass matrix $M_0^2$ in the $Z-Z'$ basis is given by

$$M_0^2 = \begin{pmatrix} M_2^2 & \delta M^2 \\ \delta M^2 & M_2^Z \end{pmatrix}$$  

where

$$M_2^2 = \frac{1}{4} g^2 [\langle \phi_1^2 \rangle + \langle \phi_2^2 \rangle]; \quad M_2^Z = g'/2 [\langle \phi_2^2 \rangle + \langle \eta \rangle^2]$$

$$\frac{\delta M^2}{M_2^Z} = 4(g'/g) \sin^2 \beta \quad \text{where} \quad \sin^2 \beta = \frac{\langle \phi_2^2 \rangle}{\langle \phi_1^2 \rangle + \langle \phi_2^2 \rangle}$$  

(16)

The mixing angle $\phi$ appearing in eq.(14) is then given by

$$\tan^2 \phi = \frac{M_2^2 - M_1^2}{M_2^Z - M_2^2}$$  

(17)

In addition, one has

$$M_1^2 \cos^2 \phi + M_2^2 \sin^2 \phi = \frac{M_W^2}{\cos^2 \theta} \quad \text{and} \quad \sin \phi \cos \phi = \frac{\delta M^2}{M_2^Z - M_1^2}$$  

(18)

$\theta(M_W)$ being the Weinberg angle ($W$-mass) at the tree level.

4 **Phenomenology of $SU(2)_L \otimes U(1)_Y \otimes U(1)_{Le-L\tau}$**

We shall now explore the phenomenological consequences of the $SU(2)_L \times U(1)_Y \times U(1)_X$ models. The extra $Z$-boson associated with $U(1)_X$ change the phenomenology of the SM in two ways. The extra $Z'$ contribute to the known processes induced by the $Z$ boson. In addition, in the present case, $Z'$ induce new flavor violating processes. The detailed phenomenology will depend upon the model. We shall take the model presented in the last section as an illustrative example and work out consequences within that model.

In the absence of additional Higgs, the $Z'$ induced flavor violation disappears. Moreover the $Z'$ does not mix with the ordinary $Z$. In this case $Z'$ makes its effect felt by
contributing to known processes like $e^+e^- \rightarrow \mu^+\mu^-$ scattering. The detailed restrictions on the relevant parameters by LEP results have been worked out in ref. [1] for this case. These restrictions continue to hold in the present case. But additionally one gets more stringent restrictions due to flavor violations and $Z-Z'$ mixing. We shall concentrate on these in the following.

The phenomenology of models with extra $Z$ boson is extensively discussed in the literature [4, 5]. The present class of models have characteristic differences arising due to the fact that $Z'$ couples only to leptons. In other models, an important restriction on the $Z'$ mass arises from the direct experimental observations at the hadronic colliders. These restrictions though model dependent strongly constrain the $Z'$ mass. For example in the left-right symmetric model [6], the search in $p\bar{p}$ collisions imply $|7| M_{Z_{L,R}} > 310$ GeV. Similar restrictions are not applicable here since $Z'$ couples only to leptons. Its production at the hadronic colliders arise only through mixing with the ordinary $Z$ and is therefore highly suppressed. The $Z'$ mass as well as its mixing with $Z$ is constrained in the present case by (a) the observations at LEP and (b) the observed limits on the leptonic flavor violations. We discuss them in turn.

4.1 Constraints from the LEP data

We closely follow the analysis of ref. [4] in deriving constraints on the relevant parameters from observations at LEP. These constraints have been derived in two different ways. The observations of the ratio $M_W/M_1$ and the $Z$-mass $M_1$, at CDF and LEP respectively, constrain the $\rho$ parameter and lead to restrictions on $M_2$ and $\tan \phi$. Other method is to use the fact that the extra $Z$ induce changes in observables like width to fermions, peak cross section in $e^+e^-$ collisions etc. One could then make a detailed fit to the LEP data and derive constraints on $M_2$ and $\phi$.

The mixing between $Z$ and $Z'$ change the tree level relation between the $W$ and the $Z$ mass. Specifically,

$$\frac{M_W^2}{\rho M_1^2} = \cos^2 \theta$$

$\theta$ being the tree level weak mixing angle. The parameter $\rho_M$ can be read off from the mixing matrix between $Z$ and $Z'$ (see eq.(18)):

$$\rho_M = \frac{1 + \tan^2 \phi M_2^2}{1 + \tan^2 \phi}$$  (19)

One could eliminate $\cos^2 \theta$ in favor of $G_F$, $\alpha$ and $M_1$ to obtain

$$\frac{M_W^2}{\rho M_1^2} = \left( \frac{1}{2} + \sqrt{\frac{1}{4} - \frac{\mu^2}{\rho M_1^2}} \right)$$  (20)

where

$$\mu = \sqrt{\frac{\pi \alpha}{\sqrt{2} G_F}} = (37.280 \text{GeV})$$
These restrictions are valid at the tree level. Since the extra $Z$ induced effects are comparable to the radiative corrections in the standard model, one must incorporate the later. This has been done in ref. [4], assuming that the radiative corrections induced by $Z_2$ are negligible. The radiative corrections of SM are included using the improved Born approximation which changes eq.(20) to the following:

$$ M_W^2 \rho_M^2 = \left( \frac{1}{2} + \frac{1}{4} - \frac{\mu^2}{\rho M_t^2(1-\Delta \alpha)} \right) $$

(21)

where the $\rho$ parameter is now

$$ \rho = \frac{\rho_M}{1-\Delta \rho_T} $$

with

$$ \Delta \rho_T \simeq 3 \frac{G_F m_t^2}{8\pi^2 \sqrt{2}} \quad \text{and} \quad \Delta \alpha = 0.0602 + \frac{40 \alpha}{9 \pi} \ln \frac{M_1(\text{GeV})}{92} \pm 0.0009 $$

The CDF result on $\frac{M_W}{M_1} = 0.779$ together with the LEP result on $^2$ the $Z$-mass $M_1$ can be used to obtain $\rho = 1.005 \pm 0.003$ in eq.(21). This implies at $1\sigma$

$$ \Delta \rho_M = \rho_M - 1 \leq 0.008 - 0.003 \left( \frac{m_t(\text{GeV})}{100} \right)^2 $$

(22)

In addition to this restriction, $\Delta \rho_M$ can also be constrained [4, 5] by the other observables at LEP. Specifically, the presence of $Z'$ would change the three leptonic widths $\Gamma_{\epsilon_{\mu\tau}}$ as well as the hadronic width $\Gamma_h$ of the $Z_1$. These changes can be parameterized [4] in terms of $\Delta \rho_M$ and mixing angle $\phi$:

$$ d\Gamma_i = A_i \Delta \rho_M + B_i \phi $$

(23)

In our case

$$ A_i = 4 N_c \rho f \left[ (T_{3L_i} - \sin^2 \theta_f Q_i)^2 + T_{3L_i}^2 + \frac{4 \sin^2 \theta_f \cos^2 \theta_f}{\cos 2\theta_f} Q_i (T_{3L_i} - \sin^2 \theta_f Q_i) \right] $$

$$ B_i = 8 N_c \rho f \left[ (T_{3L_i} - \sin^2 \theta_f Q_i) g'_{V_i} - T_{3L_i} g'_{A_i} \right] $$

where

$$ g'_{V_i} = \frac{g'}{g} (\kappa_{Lii} + \kappa_{Rii}); \quad g'_{A_i} = \frac{g'}{g} (\kappa_{Lii} - \kappa_{Rii}); $$

$$ \rho f \equiv \frac{\rho}{\rho_M}; \quad \sin^2 \theta_f = \frac{1}{2} - \frac{1}{4} - \frac{\mu^2}{\rho f M_t^2(1-\Delta \alpha)} $$

$N_c = 3(1 + \frac{\alpha S}{\pi})$ for quarks and 1 for leptons. Fermionic width $\Gamma_i$ of $Z_1$ have been extracted from the LEP data in a model independent way. We use the values derived in ref. [8] to
constrain $\Delta \rho_M$ and $\phi$. Specifically,

\[
\begin{align*}
\Gamma_e &= 82.6 \pm 0.7\text{MeV} \\
\Gamma_\mu &= 83.6 \pm 1.1\text{MeV} \\
\Gamma_\tau &= 83.1 \pm 1.2\text{MeV}
\end{align*}
\]

We use these values and determine the best values for $\Delta \rho_M$ and $\phi$ appearing in eq.(23) through a least square fit. This gives (for $m_t = 150\text{GeV}$) at 1$\sigma$:

\[
\Delta \rho_M = -0.0018 \pm 0.004 \quad \phi = 0.0094 \pm 0.012
\] (24)

The value of $\Delta \rho_M$ as determined by eq.(24) is less stringent than following from eq.(22) derived on the basis of the CDF result on $\frac{M_\mu}{M_1}$. We shall therefore use the values given by eq.(22) for $\Delta \rho_M$ in the next section to constrain the parameters of the model.

### 4.2 Constraints from the rare processes

As already discussed, the model of the last section contains flavor violations involving $\tau$ and $e$. The muon number is exactly conserved in the model. As a consequence one expects the following rare processes to occur in the model:

- $Z_{1,2} \rightarrow e\tau$
- $\tau \rightarrow eee$
- $\tau \rightarrow e\mu\mu$

The branching ratios for these processes can be easily worked out and are given by:

\[
\begin{align*}
\frac{\Gamma(\tau \rightarrow e\mu\mu)}{\Gamma(\tau \rightarrow \nu_\tau \nu_\mu \nu_e)} &= 16M_1^4 \left\{ (g_{LL}^e)^2 + (g_{RR}^e)^2 + \frac{1}{2} \left[ (g_{LR}^e)^2 + (g_{RL}^e)^2 \right] \right\} \\
\frac{\Gamma(\tau \rightarrow e\mu\mu)}{\Gamma(\tau \rightarrow \nu_\tau \nu_\mu \nu_e)} &= 4M_1^4 \left\{ (g_{LL}^\mu)^2 + (g_{RR}^\mu)^2 + (g_{LR}^\mu)^2 + (g_{RL}^\mu)^2 \right\} \\
\Gamma(Z \rightarrow \tau e) &= \frac{G_F M_1^2}{3\sqrt{2}\pi} \left\{ (F_{11}^{\tau e})^2 + (F_{21}^{\tau e})^2 \right\}
\end{align*}
\]

where

\[
\begin{align*}
g_{LL}^m &= \frac{F_{11}^{\tau e} F_{11}^{\mu e}}{M_1^2} + \frac{F_{12}^{\tau e} F_{12}^{\mu e}}{M_2^2} \\
g_{LR}^m &= \frac{F_{11}^{\tau e} F_{11}^{\mu e}}{M_1^2} + \frac{F_{12}^{\tau e} F_{12}^{\mu e}}{M_2^2}
\end{align*}
\]

$m = e, \mu$. $g_{RR}$ and $g_{RL}$ are obtained by $L \leftrightarrow R$ interchange in above equation. The difference in the rates for the $\tau \rightarrow eee$ and $\tau \rightarrow e\mu\mu$ arise due to both the $s$ and $t$ channel $Z_{1,2}$ exchanges contributing to the former. In addition to constraints from the LEP discussed earlier the rare decays also provide important constraints on the model. The specific constraints are [7] given by the following:

\[
\begin{align*}
Br(Z \rightarrow e^+\mu^-) &< 2.4 \times 10^{-5} \\
Br(Z \rightarrow e^+\tau^-) &< 3.4 \times 10^{-5} \\
Br(Z \rightarrow \mu^+\tau^-) &< 4.8 \times 10^{-5} \\
Br(\tau \rightarrow eee) &< 2.7 \times 10^{-5} \\
Br(\tau \rightarrow e\mu\mu) &< 2.7 \times 10^{-5} \\
Br(\tau \rightarrow \mu\mu) &< 1.7 \times 10^{-5}
\end{align*}
\]
The basic parameters of models are mixing angles $\theta_{L,R}$, $Z_2$ mass $M_2$, $Z-Z'$ mixing angle $\phi$ and the $U(1)_X$ gauge coupling $g'$. Both the $Z-Z'$ mixing and the flavor violation arise in the model from the presence of the additional doublet $\phi_2$. Thus both are related to the parameter $\tan \beta = \langle \phi_2 \rangle / \langle \phi_1 \rangle$. Relation between $\phi$ and $\beta$ follows from eq.(16) and (18)

$$\sin \phi \sim 4C \left( \frac{M_1}{M_2} \right)^2 \sin^2 \beta$$

(25)

where

$$C \approx \frac{g'}{g} \left( 1 - \frac{M_2^2}{M_1^2} \right)^{-1} \sim O(1)$$

The $\theta_{L,R}$ also goes to zero when $\beta \to 0$. If one assumes that the flavor violating Yukawa coupling $h_{13}$ in eq.(10) is of the same order as flavor conserving one (namely $h_{33}$) then $\delta \approx m_\tau \tan \beta$ and hence from eq.(3)

$$\sin 2\theta_L \approx -2\tan \beta$$

(26)

The existing limits on the $Br(\tau \to eee)$ as well as $Z \to e\tau$ imply restrictions on the parameters $\beta$ and $M_2$. These are displayed in fig.1 assuming $h_{13} = h_{33}$. Analogous constraints also follow from the process $\tau \to e\mu\mu$. This process is comparatively suppressed in the present case and hence imply much weaker constraints. This is not displayed in the figure for simplicity. The same parameters are also constrained by $\Delta \rho_M$ and $\phi$ (see eqs.(19) and (25)).

It follows that the strongest constraints on the parameters are implied by the rare decay $\tau \to eee$. Hence the process $\tau \to eee$ is allowed by the LEP data to occur at a rate consistent with the present experimental precision. Improvement in the limits for this process would either imply more stringent restrictions on $\beta$ and $M_2$ or one should be able to see this decay in future. Fig.1 was based on the assumption of equal Yukawa couplings, $h_{13} = h_{33}$, in eq.(10). For comparison we also display in fig.2 limits on $\beta$ and $M_2$ in case of $h_{13} = 10^{-2} h_{33}$. Reduction in the value of $h_{13}$ strongly suppresses the flavor violating couplings of $\tau$. $\Delta \rho_M$ and $\phi$ remain unchanged. As a result, now the LEP data imply stronger restrictions on $\tan \beta$ and $M_2$. In this case, the LEP observations already rule out possibility of seeing flavor violation in future experiments which are expected to provide improved limits on $\tau \to eee$.

It is clear from fig.1 and 2 that as long as $M_2 < O(TeV)$, $\tan \beta$ is restricted to be $< O(0.1 - 0.5)$. Hence the vacuum expectation value of the field $\phi_2$ responsible for flavor violations is strongly constrained in the model. Likewise, low values of $M_2$ (e.g. 400GeV) are possible only if $\tan \beta$ is chosen small (0.03 in case of $h_{13} = h_{33}$, and 0.3 in case of $h_{13} = 10^{-2} h_{33}$).

Although we restricted ourselves to the $L_e - L_\tau$ model, the analogous constraints would follow in models with $X = L_e - L_\mu$ or $L_\mu - L_\tau$. In particular, one would expect very severe constraint if $L_e - L_\mu$ is gauged since $\mu \to eee$ is much severely constrained experimentally.

5 Summary

We have studied in this paper a specific class of extended gauge models of the form $SU(2)_L \otimes U(1)_Y \otimes U(1)_X$. All these extensions are characterized by the fact that it is
possible to gauge $U(1)_X$ without extending the fermionic sector of the standard model. Thus models studied here are the simplest gauge extensions of the SM. These models are prototype of more general horizontal symmetries [3]. We have concentrated here (a) on a systematic classification of $U(1)_X$ models and (b) on deriving constraints on parameters of a prototype model using the LEP results. In specific case of $U(1)_X$ coupling to leptons, we have categorized all possible choice of $U(1)_X$. In general $U(1)_X$ provide important restrictions on the mixing matrices. Moreover, they also give rise to interesting flavor violations thus providing window into the existence of such symmetry. The mixing of the $U(1)_X$ gauge boson $Z'$ with the ordinary $Z$ is correlated in these models to the flavor violation. In fact both these features originate from the existence of a Higgs doublet carrying non-zero $U(1)_X$ charge. As a result the observations at LEP could indirectly provide important constraints on flavor violations. Detailed study presented here shows that under reasonable assumptions on relevant Yukawa couplings, the LEP observations do allow sizeable flavor violations and it is possible to obtain rate for $\tau \rightarrow eee$ near its present experimental limit. In contrast, the lepton flavor violating decays of $Z$ are considerably suppressed in these models.

We mainly studied models in which $U(1)_X$ acts only on leptons. Models with $U(1)_X$ acting on quarks [9] or both can be analogously studied. A systematic study of these horizontal models and restrictions on flavor violations in these models in the light of LEP observations would be interesting in its own right.

References


Figure Caption

Figure 1: The allowed region in the $M_2$-tan $\beta$ plane implied by various constraints: Curve (A) is a contour for $Br(\tau \to eee) = 2.7 \times 10^{-5}$; (B) for $Br(Z \to e\tau) = 3.4 \times 10^{-5}$; (C) for $\Delta \rho_M = 0.00125$; and (D) for $\phi = 0.021$. These curves are for $h_{13} = h_{33}$ (see text). Region to the left of the curves is allowed.

Figure 2: Same as figure 1 except that $h_{13} = 10^{-2}h_{33}$. 

14