The Electroweak Phase Transition in Extended Models *

J. R. Espinosa †
Instituto de Estructura de la Materia, CSIC
Serrano 123, 28006-Madrid, Spain

Abstract

We study the possibility of relaxing the cosmological bound on the Higgs mass coming from the requirement of non-erasure of the baryon asymmetry by sphalerons. After reviewing the Standard Model case we obtain this bound in two extensions of it: 1) The Standard Model with an additional gauge singlet, and 2) The Minimal Supersymmetric Standard Model. Taking fully into account all experimental constraints and thermal screening effects we found that the situation can be slightly improved with respect to the Standard Model but only in case 1) a non negligible region in parameter space exists where the baryon washout is avoided and the experimental bounds evaded.

†Supported by a grant of Comunidad de Madrid, Spain.
Introduction

The attractive possibility that the observed baryon asymmetry of the universe was created at the electroweak phase transition has deserved a lot of of attention in the last years [1]. It was realized [2] that all the conditions formulated by Sakharov [3] (for generating the baryon asymmetry) can be met in the Standard Model (SM). In particular, the sphaleron transitions were recognized as unsuppressed at high temperatures and so playing a central role in the mechanisms generating the amount of $\Delta B$ we finally observe today.

These sphaleron processes, if unsuppressed after the electroweak phase transition, will drive the previously created baryon asymmetry to zero, and so its rate $\Gamma$ at that time [2, 4] ($\Gamma \sim exp(-E_{sph}/T)$) should be less than the Hubble expansion rate $H$ to go out of equilibrium. Due to the exponential sensitivity of $\Gamma$ to the ratio $E_{sph}(T)/T$, imposing $\Gamma < H$ gives the model independent condition [5]:

$$\frac{E_{sph}(T_c)}{T_c} \geq 45.$$  \hspace{1cm} (1)

After relating the sphaleron energy [6] $E_{sph}$ to the vacuum expectation value of the Higgs field at the critical temperature $T_c$ ($E_{sph}(T_c) \sim \langle \phi(T_c) \rangle / g$) the central relation Eq. (1) translates into

$$\frac{\langle \phi(T_c) \rangle}{T_c} \gtrsim 1,$$ \hspace{1cm} (2)

which tells that the transition should be strongly enough first order so as to avoid the erasure of the baryon asymmetry.

When the dependence of $\langle \phi(T_c) \rangle$ on the parameters of the Higgs potential is taken into account, condition (2) gives an upper bound on the mass of the Higgs boson (in the Standard Model where this calculation was originally performed and also in more general cases). The study of this bound is the central aim of the work here resumed.

All along this paper we will say that a phase transition is strongly first order if condition (2) is satisfied and weakly first order (or even second order) if it is not.

The Standard Model

The effective potential at non zero temperature is the central tool in studying the electroweak phase transition. In one-loop approximation the temperature-dependent potential describes a first order phase transition between the symmetric phase (at high $T$) and the $SU(2) \times U(1)$ breaking one. Using a high temperature expansion the form of the potential is:

$$V(\phi, T) = D(T^2 - T_0^2)\phi^2 - ET\phi^3 + \frac{1}{4}\lambda(T)\phi^4,$$ \hspace{1cm} (3)

where $D, T_0, E$ and $\lambda(T)$ are easily calculable.
Approximating the critical temperature \( T_c \) as the temperature \( T_D \) at which two degenerate minima coexist (\( T_c \) will be somewhere in the narrow range between \( T_D \) and the temperature \( T_0 \) at which the minimum at \( \phi = 0 \) is destabilized) one gets

\[
\langle \phi(T_c) \rangle = \frac{2E}{\lambda(T_c)}.
\]

We see from this formula that the strength of the transition is governed by \( E \) (which gives the cubic term in the potential). Now, \( E \) comes from bosonic loops only (from \( n = 0 \) Matsubara frequencies). Each bosonic degree of freedom, with field dependent mass \( M_i(\phi) \) contributes to the potential a term

\[
\Delta_i V = -\frac{T}{12\pi} \left[ M_i^2(\phi) \right]^{3/2}.
\]

In the SM, the main contribution is the one coming from gauge boson loops:

\[
\Delta V = -\frac{T}{12\pi} \left[ 6 \left( \frac{1}{4} g^2 \phi^2 \right)^{3/2} + 3 \left( \frac{1}{4} (g^2 + g'^2) \phi^2 \right)^{3/2} \right].
\]

Using the value of \( E \) obtained from Eq. (6) and relating \( \lambda \) to the Higgs mass one gets the bound [5, 7]

\[
M_h \lesssim (45 - 50) \text{ GeV},
\]

region which is already ruled out by LEP.

This bound is obtained within the one-loop approximation to the effective potential and as is well known this will only be reliable for \( \phi > T \) (the expansion parameter is \( g^2 T^2 / M^2(\phi) \)). For small \( \phi \) the perturbative expansion is plagued with infrared divergences. To cure this, one has to reorganize the loop expansion resumming the most important infrared divergent diagrams [8] (the so called daisy diagrams). The effect of this daisy resummation is to shift the mass of the \( n = 0 \) modes contributing to the cubic term by a thermal mass (Debye screening effect). In the SM this thermal mass is of order \( g^2 T^2 \) to leading order for the longitudinal gauge bosons, and zero (to this order) for the transverse ones. The effect of this thermal mass is to shield the cubic term of the longitudinal modes while leaving the transverse modes unaffected and so to effectively reduce \( E \) by a factor \( 2/3 \). The improved bound on the Higgs mass is then even lower [9]:

\[
M_h \lesssim \sqrt{2/3(45 - 50)} \text{ GeV} \sim (35 - 40) \text{ GeV}.
\]

There exist further refinements of this calculation in the literature. If the screening of the transverse modes at subleading order \( (g^4 T^2) \) is taken into account [10] the strength of the transition is further reduced. On the other hand, some non-perturbative effects may give a lower value for the critical temperature [11] translating into a larger ratio \( \langle \phi(T_c) \rangle / T_c \). Also there are some two-loop (resummed) calculations [12] that give larger \( \langle \phi(T_c) \rangle \).
Even with these uncertainties the SM has another problems to be able to generate the observed baryon asymmetry. In fact the CP violation effects are by far too small to give the correct amount of $\Delta B$ and some recent attempts [13] to solve this problem have been shown [14] to be flawed. So it seems necessary to move to some extensions of the SM to explain the generation of the baryon asymmetry at the electroweak phase transition.

From what we have learned by studying the Standard Model case we can work out an strategy [15] for the extensions of it to give a strong first order phase transition. A look at Eq. (4) shows that we need a large cubic term in our potential. So, adding more bosonic degrees of freedom may help for only bosons contribute to this cubic term. The screened mass of such bosons will have the general form

$$M^2(\phi) = M^2 + \lambda \phi^2 + \Pi(T),$$

(9)

where $M$ is an invariant mass, $\lambda$ is a generic coupling of the new bosons to the Higgs (here $\phi$ stands generically for the fields driving the transition) and $\Pi$ is the thermal mass. For this to give a non-negligible contribution to the cubic term, small values of $M$ and $\Pi$ and large values of $\lambda$ are required. Usually, there will exist an experimental lower bound for $M$ so that one cannot take it to be arbitrarily small. Concerning the coupling $\lambda$ the requirement of perturbativity up to some large scale will set an upper bound on it. In a given model one has to take all this conditions into account when looking for a region in the parameter space where the phase transition is strong enough.

Another way of increasing the strength of the transition has been considered in the literature [15, 16, 17], namely the possibility of decreasing the value of the coupling in the denominator of Eq. (4) due to one loop corrections (even at zero temperature). But usually for this mechanism to give a sizeable effect large couplings and/or many bosonic degrees of freedom are needed.

Before moving to the discussion of extended models some words about the validity of the perturbative calculations are needed. Roughly speaking, one can trust the (resummed) perturbative calculations in the SM if $m_h^2 \lesssim m_W^2$ (or equivalently $\lambda/g^2 \lesssim 1$). When trying to improve the "sphaleron bound" $m_h \lesssim m_{h,\text{crit}}$, one can face cases where $m_{h,\text{crit}} \sim m_W$ rising some doubts about the reliability of the calculation of $m_{h,\text{crit}}$. In the cases we are going to consider the relevant parameter will no longer be $\lambda/g^2$. In fact $g^2$ will be substituted by $\zeta^2$ in the SM + singlet case and by $h_3^2$ in the MSSM. As we will be interested in large values of these couplings, for a given Higgs mass the expansion parameter will be smaller than in the SM, and the perturbative calculation more reliable.

The Standard Model with a gauge Singlet

This is the simplest extension of the Standard Model that can in principle improve the situation concerning the electroweak baryogenesis problem, as already noted in Ref. [15].
The lagrangian of the model is defined as:
\[
\mathcal{L} = \mathcal{L}_{SM} + \partial^\mu S^* \partial_\mu S - M^2 S^* S - \lambda_S (S^* S)^2 - 2\zeta^2 S^* S H^* H,
\]
where $H$ is the SM doublet with $\langle H \rangle = \phi/\sqrt{2}$, $\phi$ is the classical field, and $M^2, \lambda_S, \zeta^2 \geq 0$, to guarantee that $\langle S \rangle = 0$ at all temperatures.

The temperature dependent effective potential in the one loop approximation was studied in Ref. [15]. The daisy improvement was performed in Refs. [18, 16]. The potential is a function of the classical field $\phi$ as in the SM, but with a new contribution coming from the $S$ bosonic loops of the form (imposing renormalization conditions preserving the tree level value of the vev $v$)
\[
\Delta V_S = g_S \left\{ \frac{m_S^2(\phi) T^2}{24} - \frac{M_S^3(\phi) T}{12\pi} - \frac{m^4_S(\phi)}{64\pi^2} \left[ \log \frac{m_S^2(v)}{c_B T^2} - 2\frac{m_S^2(v)}{m_S^2(\phi)} \right] \right\},
\]
where $g_S = 2$ is the number of degrees of freedom of the (complex) singlet. $m_S(\phi)$ is the field dependent mass of $S$ and $M_S(\phi)$ its Debye screened mass. They are given by
\[
\begin{align*}
    m_S^2(\phi) &= M^2 + \zeta^2 \phi^2, \\
    M_S^2 &= m_S^2(\phi) + \Pi_S(T).
\end{align*}
\]
Here $\Pi_S$ is, to leading order
\[
\Pi_S(T) = \frac{\lambda_S + \zeta^2}{3} T^2.
\]
The thermal polarizations for the rest of SM particles can receive extra contributions coming from $S$ loops (see Refs. [18, 16]).

To understand the numerical results presented at the end of this section it is convenient to study analytically the effective potential assuming as will be the case that the bosonic contribution is dominated by the singlet (and this is the reason for the relevant expansion parameter to be $\lambda/\zeta^2$ rather than $\lambda/g^2$). The high temperature expansion of the effective potential takes then the form
\[
V(\phi) = A(T)\phi^2 + B(T)\phi^4 + C(T) \left( \phi^2 + K^2(T) \right)^{3/2},
\]
where
\[
C(T) = -\frac{\zeta^2 T}{6\pi},
\]
\[
K^2(T) = \frac{(\sqrt{2} + \lambda_S) T^2 + 3M^2}{3\zeta^2}.
\]
From these expressions and the discussion at the end of the previous section is clear that the best case for the phase transition to be strongly first order is to have low values of $M$ and $\lambda_S$ and large values of $\zeta^2$. This is what was found numerically in Ref. [18].

4
For large values of $M$ the transition is weaker and eventually, when $M \gg T$, the singlet decouples and the SM result is recovered. The dependence on $\zeta^2$ is the most interesting one. Imposing the condition that all the couplings in the theory remain perturbative up to some high scale $\Lambda$ an upper bound for $\zeta^2$ at the electroweak scale is derived (as a function of course of the scale $\Lambda$ and the unknown value of the top yukawa coupling $h_t$). This renormalization group study was also performed in Ref. [18].

Translating the requirement of a strong phase transition ($E_{\text{sph}}(T_c)/T_c > 45$ or $\langle \phi(T_c) \rangle \gtrsim 1.3$ with our definition of $T_c$) into a bound on the Higgs mass $M^2_H$, one finds that in this model one can evade the stringent bound of the SM and go to larger values of $M^2_H$. The numerical value of this critical mass depends on how large the coupling $\zeta^2$ is, or equivalently, on the mass of the top and the scale of new physics $\Lambda$. Assuming that this scale is not lower than $10^6$ GeV the value of $M^2_H$ can be of order 80 GeV (see Fig. 1) which is higher than the experimental bound. Larger values of this critical mass can only be obtained at the price of introducing new physics at lower scales.

The Minimal Supersymmetric Standard Model

The Minimal Supersymmetric Standard Model (MSSM) is the physically most motivated and phenomenologically most acceptable among the extensions of the SM. Concerning the generation of the baryon asymmetry the MSSM allows for extra CP-violating phases besides the Kobayashi-Maskawa one, which could help in generating the observed baryon asymmetry [19]. On the other hand the MSSM has a lot of new degrees of freedom and the nature of the phase transition could be significantly modified with respect to the SM.

The main tool for our study is the one-loop, daisy-improved (for previous studies in the one-loop approximation see Refs. [7, 20]) finite-temperature effective potential of the MSSM, $V_{\text{eff}}(\phi, T)$. Now the potential is actually a function of two fields: $\phi_1 \equiv \text{Re} H_1^0$ and $\phi_2 \equiv \text{Re} H_2^0$, where $H_1^0$ and $H_2^0$ are the neutral components of the Higgs doublets $H_1$ and $H_2$, thus $\phi$ will stand for $(\phi_1, \phi_2)$. Working in the 't Hooft-Landau gauge and in the $D\bar{R}$-scheme, we can write

$$V_{\text{eff}}(\phi, T) = V_0(\phi) + V_1(\phi, 0) + \Delta V_1(\phi, T) + \Delta V_{\text{daisy}}(\phi, T),$$

where

$$V_0(\phi) = m_1^2\phi_1^2 + m_2^2\phi_2^2 + 2m_3^2\phi_1\phi_2 + \frac{g^2 + g'^2}{8}(\phi_1^2 - \phi_2^2)^2,$$

$$V_1(\phi, 0) = \sum_i \frac{n_i}{64\pi^2}m_i^4(\phi)\left[\log \frac{m_i^2(\phi)}{T^2} - \frac{3}{2}\right],$$

$$\Delta V_1(\phi, T) = \frac{T^4}{2\pi^2} \left\{ \sum_i \frac{n_i}{T^2} \left[ \frac{m_i^2(\phi)}{T^2} \right] \right\},$$

$$\Delta V_{\text{daisy}}(\phi, T) = -\frac{T}{12\pi} \sum_i n_i \left[ m_i^4(\phi, T) - m_i^3(\phi) \right].$$
The first term, Eq. (18), is the tree-level potential. The second term, Eq. (19), is the one-loop contribution at $T = 0$: $Q$ is the renormalization scale, where we choose for definiteness $Q^2 = m_{Z}^2$, $m_\phi^2$ is the field-dependent mass of the $i$th particle, and $n_i$ is the corresponding number of degrees of freedom, taken negative for fermions. Since $V_1(\phi, 0)$ is dominated by top ($t$) and stop ($\tilde{t}_1, \tilde{t}_2$) contributions, only these will be included in the following. The third term, Eq. (20), is the additional one-loop contribution due to temperature effects. Here $J_i = J_+(J_-)$ if the $i$th particle is a boson (fermion), and

$$J_\pm(y^2) \equiv \int_0^\infty dx x^2 \log \left( 1 + e^{-\sqrt{y^2 + x^2}} \right).$$

(22)

Since the relevant contributions to $\Delta V_1(\phi, T)$ are due to top ($t$), stops ($\tilde{t}_1, \tilde{t}_2$) and gauge bosons ($W, Z$), only these will be considered in the following. Finally, the last term, Eq. (21), is the correction coming from daisy diagrams. The sum runs over bosons only. As usual, the masses $\overline{m}_i^2(\phi, T)$ are obtained from the $m_i^2(\phi)$ by adding the leading $T$-dependent self-energy contributions, which are proportional to $T^2$.

The relevant degrees of freedom for our calculation are:

$$n_t = 12, \quad n_{\tilde{t}} = 6, \quad n_W = 6, \quad n_Z = 3, \quad n_{W_L} = 2, \quad n_{W_R} = n_{\gamma_L} = 1.$$  

(23)

The field-dependent top mass is

$$m_i^2(\phi) = h_i^2\phi_2^2.$$  

(24)

The entries of the field-dependent stop mass matrix are

$$m_{\tilde{t}_L}^2(\phi) = m_{Q}^2 + m_i^2(\phi) + D_{\tilde{t}_L}^2(\phi),$$  

(25)

$$m_{\tilde{t}_R}^2(\phi) = m_{U}^2 + m_i^2(\phi) + D_{\tilde{t}_R}^2(\phi),$$  

(26)

$$m_X^2(\phi) = h_t(A_t\phi_2 + \mu \phi_1),$$  

(27)

where $m_Q$, $m_U$, and $A_t$ are soft supersymmetry-breaking mass parameters, $\mu$ is a superpotential Higgs mass term, and

$$D_{\tilde{t}_L}^2(\phi) = \left( \frac{1}{2} - \frac{2}{3} \sin^2 \theta_W \right) \frac{g^2 + g^2}{2} (\phi_1^2 - \phi_2^2),$$  

(28)

$$D_{\tilde{t}_R}^2(\phi) = \left( \frac{2}{3} \sin^2 \theta_W \right) \frac{g^2 + g^2}{2} (\phi_1^2 - \phi_2^2).$$  

(29)

are the $D$-term contributions. The field-dependent stop masses are then

$$m_{\tilde{t}_{1,2}}^2(\phi) = \frac{m_{\tilde{t}_L}^2(\phi) + m_{\tilde{t}_R}^2(\phi)}{2} \pm \sqrt{\left[ \frac{m_{\tilde{t}_L}^2(\phi) - m_{\tilde{t}_R}^2(\phi)}{2} \right]^2 + m_X^2(\phi)^2}.$$  

(30)

The corresponding effective $T$-dependent masses, $\overline{m}_{\tilde{t}_{1,2}}^2(\phi, T)$, are given by expressions identical to (30), apart from the replacement

$$m_{\tilde{t}_{L,R}}^2(\phi) \rightarrow \overline{m}_{\tilde{t}_{L,R}}^2(\phi, T) \equiv m_{\tilde{t}_{L,R}}^2(\phi) + \Pi_{\tilde{t}_{L,R}}(T).$$  

(31)

6
The $\Pi_{I_L,R}(T)$ are the leading parts of the $T$-dependent self-energies of $\tilde{t}_{L,R}$,

$$
\Pi_{I_L}(T) = \frac{4}{9} g_s^2 T^2 + \frac{1}{4} g^2 T^2 + \frac{1}{108} g'{}^2 T^2 + \frac{1}{6} h_t^2 T^2 ,
$$

$$
\Pi_{I_R}(T) = \frac{4}{9} g_s^2 T^2 + \frac{4}{27} g^2 T^2 + \frac{1}{3} h_t^2 T^2 ,
$$

where $g_s$ is the strong gauge coupling constant. Only loops of gauge bosons, Higgs bosons and third generation squarks have been included, implicitly assuming that all remaining supersymmetric particles are heavy and decouple. If some of these are also light, the plasma masses for the stops will be even larger, further suppressing the effects of the associated cubic terms, and therefore weakening the first-order nature of the phase transition.

We expect the stops to play an important role in making the transition stronger as they have a large number of degrees of freedom and couple to the Higgses with strength $h_t$. On the other hand as they are coloured scalars their thermal mass is of order $g_s^2 T^2$ as shown above and also they are not protected by any symmetry to have an invariant mass. The final importance they have in the strength of the transition will result from the interplay between these opposite properties.

Finally, the field-dependent gauge boson masses are

$$
m_W^2(\phi) = \frac{g^2}{2}(\phi_1^2 + \phi_2^2), \quad m_Z^2(\phi) = \frac{g^2 + g'^2}{2} (\phi_1^2 + \phi_2^2),
$$

and the effective $T$-dependent masses of the longitudinal gauge bosons are

$$
\overline{m}_{W_L}(\phi, T) = m_W^2(\phi) + \Pi_{W_L}(T) ,
$$

$$
\overline{m}_{Z_L,\gamma L}(\phi, T) = \frac{1}{2} \left[ m_Z^2(\phi) + \Pi_{W_L}(T) + \Pi_{B_L}(T) \right] \\
\pm \sqrt{ \frac{1}{4} \left[ \frac{g^2 - g'^2}{2} (\phi_1^2 + \phi_2^2) + \Pi_{W_L}(T) - \Pi_{B_L}(T) \right]^2 + \left[ \frac{g g'}{2} (\phi_1^2 + \phi_2^2) \right]^2} .
$$

In eqs. (35) and (36), $\Pi_{W_L}(T)$ and $\Pi_{B_L}(T)$ are the leading parts of the $T$-dependent self-energies of $W_L$ and $B_L$, given by

$$
\Pi_{W_L}(T) = \frac{5}{2} g^2 T^2 , \quad \Pi_{B_L}(T) = \frac{47}{18} g'{}^2 T^2 ,
$$

where only loops of Higgs bosons, gauge bosons, Standard Model fermions and third-generation squarks have been included.

We need to analyse the effective potential (17) as a function of $\phi$ and $T$. Before doing this, however, we trade the parameters $m_{W_1}^2, m_{Z_1}^2, m_3^2$ appearing in the tree-level potential (18) for more convenient parameters. To this purpose, we first minimize the zero-temperature effective potential, i.e. we impose the vanishing of the first derivatives of $V_0(\phi) + V_1(\phi, 0)$ at $(\phi_1, \phi_2) = (v_1, v_2)$, where $(v_1, v_2)$ are the one-loop vacuum
expectation values at \( T = 0 \). This allows us to eliminate \( m_1^2 \) and \( m_2^2 \) in favour of \( m_3^2 \) and \( \tan \beta \equiv v_2/v_1 \) in the standard way. Moreover, \( m_3^2 \) can be traded for the one-loop-corrected mass \( m_A^2 \) of the CP-odd neutral Higgs boson [21]. Therefore the whole effective potential (17) is completely determined, in our approximation, by the parameters \((m_A, \tan \beta)\) of the Higgs sector, and by the parameters \((m_t, m_{Q_s}, m_{U_3}, \mu, A_t)\) of the top/stop sector. The same set of parameters also determines the one-loop-corrected masses and couplings of the MSSM Higgs bosons.

The next steps are the computation of the critical temperature and of the location of the minimum of the effective potential at the critical temperature. We define here \( T_0 \) as the temperature at which the determinant of the second derivatives of \( V_{\text{eff}}(\phi, T) \) at \( \phi = 0 \) vanishes:

\[
\det \left[ \frac{\partial^2 V_{\text{eff}}(\phi, T_0)}{\partial \phi_i \partial \phi_j} \right]_{\phi_i = 0} = 0.
\]

(38)

It is straightforward to compute the derivatives in eq. (38) from the previous formulae; the explicit expressions are given in Ref. [22].

Once eq. (38) is solved (numerically) and \( T_0 \) is found, one can minimize (numerically) the potential \( V_{\text{eff}}(\phi, T_0) \) and find the minimum \([v_1(T_0), v_2(T_0)]\). The quantity of interest is indeed, as will be discussed later, the ratio \( v(T_0)/T_0 \), where \( v(T_0) \equiv \sqrt{v_1^2(T_0) + v_2^2(T_0)} \).

Before presenting the numerical results we discuss the experimental constraints on the parameters of the top/stop sector and of the Higgs sector. We treat \( m_{Q_s}, m_{U_3} \) and the other soft mass terms as independent parameters, even if they can be related in specific SUGRA models. We want to be as general as possible and in this way the region of parameters we are able to exclude will be excluded in any particular model.

Direct and indirect searches at LEP imply [23] that \( m_{b_L} \geq 45 \text{ GeV} \), which in turn translates into a bound in the \((m_{Q_s}, \tan \beta)\) plane. Electroweak precision measurements [24] put stringent constraints on a light stop-bottom sector: in first approximation, and taking into account possible effects [25] of other light particles of the MSSM, we conservatively summarize the constraints by \( \Delta \rho(t, b) + \Delta \rho(\tilde{t}, \tilde{b}) < 0.01 \).

We finally need to consider the constraints coming from LEP searches for supersymmetric Higgs bosons [23]. Experimentalists put limits on the processes \( Z \rightarrow hZ^* \) and \( Z \rightarrow hA \), where \( h \) is the lighter neutral CP-even boson. We need to translate these limits into exclusion contours in the \((m_A, \tan \beta)\) plane, for given values of the top/stop parameters. In order to do this, we identify the value of \( BR(Z \rightarrow hZ^*) \), which corresponds to the limit \( m_\phi > 63.5 \text{ GeV} \) on the SM Higgs, and the value of \( BR(Z \rightarrow hA) \), which best fits the published limits for the representative parameter choice \( m_t = 140 \text{ GeV}, m_{Q_s} = m_{U_3} \equiv \tilde{m} = 1 \text{ TeV}, A_t = \mu = 0 \). We then compare those values of \( BR(Z \rightarrow hZ^*) \) and \( BR(Z \rightarrow hA) \) with the theoretical predictions of the MSSM, for any desired parameter choice and after including the radiative corrections associated to top/stop loops [26, 21]. Of course, this procedure is not entirely correct, since it ignores the variations of the efficiencies with the Higgs masses and branching ratios, as well as the possible presence of candidate events at some mass values, but it

8
is adequate for our purposes.

We now present our numerical results, based on the effective potential of eq. (17), concerning the strength of the electroweak phase transition and the condition for preserving the baryon asymmetry.

Particularizing to the MSSM the studies of sphalerons in general two-Higgs models [27], we obtain that

\[ E_{\text{sph}}^{\text{MSSM}} (T) \leq E_{\text{sph}}^{\text{SM}} (T), \]

where, in our conventions,

\[ \frac{E_{\text{sph}}^{\text{SM}} (T)}{T} = 4\sqrt{2\pi} B \left( \frac{\lambda_{\text{eff}}(T)}{4g^2} \right) \frac{v(T)}{T}, \]

and \( B \) is a smoothly varying function whose values can be found in Ref. [6]. For example, \( B(10^{-2}) = 1.67, B(10^{-1}) = 1.83, B(1) = 2.10. \) It can also be shown that

\[ \frac{v(T_D)}{T_D} < \frac{v(T_C)}{T_C} < \frac{v(T_0)}{T_0}, \]

where \( T_C \) is the actual temperature at which the phase transition occurs, satisfying the inequalities

\[ T_0 < T_C < T_D, \]

if \( T_0 \) is defined by (38) and \( T_D \) is the temperature at which there are two degenerate minima.

Finally, the corrections in \( E_{\text{sph}}^{\text{SM}} \) due to \( g' \neq 0 \) have been estimated and shown to be small [28]. Therefore, a conservative bound to be imposed is

\[ R \equiv \frac{v(T_0)}{T_0} \frac{4\sqrt{2\pi} B \left( \frac{\lambda_{\text{eff}}(T_0)}{4g^2} \right)}{45g} > 1. \]

The last point to be discussed is the determination of the value of \( \lambda_{\text{eff}}(T_0) \) to be plugged into eq. (43). The \( B \)-function we use is taken from Ref. [6], where the sphaleron energy was computed using the zero-temperature ‘Mexican-hat’ potential, \( V = \frac{\lambda}{4}(\phi^2 - v^2)^2 \). The sphaleron energy at finite temperature was computed in Ref. [29], where it was proven that it scales like \( v(T) \), i.e.

\[ E_{\text{sph}}^{\text{SM}} (T) = E_{\text{sph}}^{\text{SM}} (0) \frac{v(T)}{v}, \]

with great accuracy. Therefore, to determine the value of \( \lambda_{\text{eff}}(T_0) \) we have fitted \( V_{\text{eff}}(\phi, T_0) \), in the direction of the minimum, to the one-dimensional approximate potential,

\[ V_{\text{eff}}(\phi, T_0) \simeq \frac{1}{4} \lambda_{\text{eff}}(T_0)(\phi^2 - v^2(T_0))^2, \]
The value of $\lambda_{\text{eff}}$ obtained from (45),

$$
\lambda_{\text{eff}}(T_0) = 4 \frac{V_{\text{eff}}(0, T_0) - V_{\text{eff}}[v(T_0), T_0]}{v^4(T_0)},
$$

where all quantities on the right-hand side are calculated numerically from the potential of eq. (17), is then plugged into eq. (43) to obtain our bounds. Of course, the quality of the fit is good only for values of $\phi \lesssim v(T_0)$ but this is precisely the region of interest to determine the sphaleron energy.

Our numerical results are summarized in fig. 2, in the $\left(m_A, \tan \beta \right)$ plane and for two representative values of the top quark mass: $m_t = 130 \text{ GeV}$ (fig. 2a) and $m_t = 170 \text{ GeV}$ (fig. 2b). In each case, the values of the remaining free parameters have been chosen in order to maximize the strength of the phase transition, given the experimental constraints on the top-stop sector. Notice that arbitrarily small values of $m_U$ cannot be excluded on general grounds, even if they are disfavoured by model calculations. Also, we have explicitly checked that, as in Ref. [30], mixing effects in the stop mass matrix always worsen the case. In fig. 2, solid lines correspond to contours of constant $R$: one can see that the requirement of large values of $R$ favours small $\tan \beta$ and $m_A \gg m_Z$. The thick solid line corresponds to the limits coming from Higgs searches at LEP: for our parameter choices, the allowed regions correspond to large $\tan \beta$ and/or $m_A \gg m_Z$. For reference, contours of constant $m_h$ (in GeV) have also been plotted as dashed lines. One can see that, even for third-generation squarks as light as allowed by all phenomenological constraints, only a very small globally allowed region can exist in the $(m_A, \tan \beta)$ plane, and that the most favourable situation is the one already discussed in Ref. [30]. More precisely, the region that is still marginally allowed corresponds to $m_A \gg m_Z$, $\tan \beta \sim 2$, stop and sbottom sectors as light as otherwise allowed, a heavy top, and a light Higgs boson with SM-like properties and mass $m_h \sim 65 \text{ GeV}$, just above the present experimental limit. A less conservative interpretation of the limits from precision measurements, the inclusion of some theoretically motivated constraints on the model parameters, or a few GeV improvement in the SM Higgs mass limit, would each be enough to fully exclude electroweak baryogenesis in the MSSM.

Acknowledgments

The results presented here are based on joint work done in collaboration with A. Brignone, M. Quirós and F. Zwirner. I must say it is always a pleasure to work with them.

References


**Figure captions**

Fig.1: Plot of $\langle \phi(T_c) \rangle / T_c$ versus $m_h$ for $\lambda_S = 0$, $M = 0$, $m_t = 90$, 150 and 175 GeV and $\zeta$ corresponding to $\Lambda = 10^6$ GeV.

Fig.2: Contours of $R$ in the $(m_A, \tan \beta)$ plane, for the parameter choices: a) $m_t = 130$ GeV, $m_{Q_3} = 50$ GeV, $m_{U_3} = 0$ ($m_t \sim 130$ GeV, $m_{Q_3} \sim 50$ GeV), $A_t = \mu = 0$; b) $m_t = 170$ GeV, $m_{Q_3} = 280$ GeV, $m_{U_3} = 0$ ($m_t \sim 330$ GeV, $m_{Q_3} \sim 170$ GeV, $m_{U_3} \sim 280$ GeV), $A_t = \mu = 0$. The region excluded by Higgs searches at LEP is delimited by the thick solid line. For reference, contours of constant $m_h$ (in GeV) are also represented as dashed lines.