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"Quantum Mechanical Evolution of Relativistic Particles"

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Quantum Mechanical Evolution of Relativistic Particles

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This article is dedicated to Jean-Pierre Vigiér

This is a tentative theory of quantum measurement performed on particles with unspecified mass. For such a particle, the center of wave packet undergoes a classical motion which is a precious guide to our approach. The framework is manifestly covariant and a priori nonlocal. It allows for describing an irreversible process which lasts during a nonvanishing lapse of time. The possibility to measure a dynamical variable in an arbitrary state is submitted to discussion. Our picture is most satisfactory if we focus on free particles and constants of the motion. Two-particle systems and measurement of individual observables in a correlated state are considered.

1. Introduction

We would like to study the quantum evolution of relativistic particle systems (in situations where pair creation can be neglected). We mean to consider not only the reversible motion occurring between two successive measurements, but also the irreversible process undergone through a measurement.

In the two-body sector we face well-known conceptual problems associated with the measurement of an individual property in a correlated state (EPR paradox [1]). It is a pleasure to dedicate this work in the honor of Jean- Pierre Vigiér whose enthusiastic involvement in this challenge and in other battles of modern Physics is an example and a permanent stimulation to all of us.

The most popular way of presenting the paradox tacitly assumes that a measurement is something like an event (in a generalized sense: points of spacetime can be replaced by compact regions of spacetime). This approach leads to the following puzzle: The basic axioms of quantum mechanics applied to a gedanken experiment (as well as the result of Aspect experiment) seem to indicate that a measurement performed at point A in spacetime can have a consequence at point B even if A and B are spacelike separated, which seems to be in conflict with the cause-effect relation.

In our opinion the real problem is to understand whether the mechanism which triggers the measurement at point A is random or deterministic. Indeed if the event A itself is the outcome of a deterministic series of events situated in the past, there is an early event, say A_{ret} , of this series which can be (better than A) regarded as the true cause of what happens at point B . After all, the structure of causality is transitive. From this point of view, there is a serious paradox only if the interaction with apparatus is switched on at random ^{*}.

In this spirit we could say that if the decision to perform or not the measurement at point A is made by a human observer, there is perhaps no paradox at all in the absence of metaphysical freedom. Finally one ends up with a problem which pertains to metaphysics rather than logics.

^{*} In realistic situations there are devices which are random *at our scale* but we may suspect that the apparently unpredictable character of the trigger mechanism results from our inability to control all the too many parameters involved in this mechanism.

In order to stay away from philosophical discussions we shall take a pragmatic point of view and give up the notion of event-measurement. Let us assume from the outset that any measurement can concern a region which is *not bounded in space*. At first sight, this idea seems to conflict with our macroscopic perception of an apparatus which occupies a compact region of space. But this apparatus is made of a tremendous number of atoms, of which we have only a statistical knowledge. We cannot hope to control all of them in detail and it is possible that a few of them are not localized. In addition, even if localized, the apparatus interacts with a system which generally is *not* localized.

So we are forced to consider the whole system particles + apparatus in a nonlocal way. Practically this point of view amounts to associate any measurement with a spacelike hypersurface.

Any discussion about measurement in quantum mechanics requires an evolutive description of the system. This matter is perfectly clear in the nonrelativistic case. The time-dependent Schrödinger equation describes the history of the system between successive measurements, whereas the stationary Schrödinger equation is only a reduction of the former, good enough for determining the energy spectrum.

But, in order to speak in general terms about measurement performed at a time or about transition probabilities at a given time, one is forced to invoke the time-dependent Schrödinger equation. If one is only interested in the outcome of a scattering experiment, and provided the in-states are eigenfunctions of the energy, it is possible to forget the time-dependent formulation in favor of the stationary scattering formalism. Though very efficient in so far as calculations are concerned, this simplification is not so fortunate from a conceptual viewpoint. We believe that the time-dependent S -matrix theory, being based upon evolution operators, is more fundamental.

The importance of an evolutive description of quantum mechanics becomes more crucial when relativity is taken into account. There are several formulations. For quantum *field* theory (QFT) it is possible to develop a noncovariant hamiltonian formalism with the coordinate time as evolution parameter, which nonetheless provides relativistic invariant results. Alternatively the Tomonaga-Schwinger formulation uses spacelike surfaces as indicators of evolution and yields (in principle) the same results.

For *particle* dynamics, most part of the literature is concerned with *a priori* sharp masses and the coordinate time is most often employed as evolution parameter.

In contrast, in this work we deliberately choose to consider *particles of unspecified mass* *. This point of view, founded upon the Stückelberg equation [3], offers the advantage of a manifestly covariant hamiltonian formalism, in terms of an abelian evolution group of finite dimension parametrized with scalars [7]. A covariant Hamilton-Jacobi equation follows in the classical limit $\hbar \rightarrow 0$ [8].

Moreover, dealing with off-shell particles permits to define average world-lines (central lines) which would not exist for particles of definite masses [9]. In this work we have decided to take seriously these central lines and to rely on them in order to visualize some individual aspects of the quantum particles. As we shall explain in the next section, they permit to connect spacelike hypersurfaces with the parametrized evolution formalism. In addition they may provide a bridge between the standard formalism of quantum mechanics and the idea of causal interpretation, put forward by de Broglie and Bohm [10] and repeatedly supported by Vigier.

In this framework we propose a *covariant version of the reduction postulate* which attempts to make a step toward a relativistic theory of quantum measurement.

Several approaches toward this goal have been proposed in the past. Hellwig and Kraus [11] have suggested that the change induced by a measurement occurs through a surface limited by a compact domain of space-time and light rays going from it to past infinity (past cone of a domain). More recently Smith and Weingard have constructed relativistic correlated states [12].

* An evolution equation in terms of an invariant scalar parameter was early proposed by Fock [2], but, in contradistinction to Fock, the Stückelberg equation [3] does not involve explicitly the mass, which can be eventually recovered as a constant of the motion. In this approach the evolution parameter has the dimension of a time divided by a mass. Then, equations of the Fock or Stückelberg type have been considered by several authors [4]. See A.Kyprianidis for a review [5]. This kind of equation was re-derived by Vigier *et al.* from stochastic considerations [6].

In a different spirit, the theory of spontaneous collapse considers stochastic reduction models where a state vector is assigned to any spacelike surface [13].

In the present work, we try to remain as close as possible to the conventional axioms of quantum mechanics. We replace pointlike events by spacelike hypersurfaces and explicitly follow the time development of the system between and across the regions where measurement is carried out. Consideration of a finite lapse of time (briefly suggested as a possibility in a footnote of ref.9) is actually a necessity in order to ensure that central lines remain timelike.

Notice that we are interested in the outcome of the measurement but shall not intend to describe what happens *during* this process.

The case of two free (but correlated) particles will be briefly discussed.

2. Central Lines

The concept of wave packet is very useful in quantum mechanics. Unless we accept the causal interpretation of quantum mechanics (in the sense of Bohm, de Broglie and Vigier) there are no trajectories and the center of the wave packet is just a fictitious particle. But this fictitious particle is extremely important for the understanding of what is going on.

Here we have some analogy with the classical concept of center of mass. In nonrelativistic classical mechanics, the center of mass of an extended object is somehow immaterial: for instance it may quite well lay out of the object, as happens for an homogeneous distribution of matter inside a torus. Nevertheless it would be extremely difficult to go without the idea of center of mass when developing the bulk of rational mechanics. Similarly, our approach to quantum mechanics would remain very abstract if we were unable to consider the concept of *center of wave packet*. Actually this concept provides the first contact with locality. In nonrelativistic quantum mechanics it arises as the expectation value of the position operator $\vec{x}$ and the notion of localized states is transparent.

In relativistic quantum mechanics the situation is more delicate. The Newton-Wigner operator [14] is widely considered as the appropriate tool (at least for free particles on the mass shell), but it has been argued by Hegerfeldt that any position operator related with the probability of finding the particle in some volume at time t would violate causality [15].

So we still lack a complete and satisfactory theory of localization.

But the contact with our everyday's experience of the macroscopic world (apparently full of localized objects) can be realized through a relativistic version of the Ehrenfest theorem which assigns to the center of the wave packet the motion of a fictitious particle *.

Notice that any discussion about propagation properties requires some idea of localization: if anything is supposed to travel in space, it is necessary to imagine that it evolves from some finite (and bounded) region to another. Again, in the absence of a complete theory of locality, we are led to rely on the concept of center of wave packet in order to visualize propagation: anticipating the elaboration of a complete localization theory, it is natural to assume that the center of wave packet contains some information about the local behavior of the wave function. This center undergoes classical motion and has a trajectory. In a relativistic framework we are bound to consider its world-line, let us naturally call it the *Central Line*.

Here we come to the question of a precise mathematical formulation. In our opinion a manifestly covariant formalism is necessary. First because noncovariant theories are widely plagued with complicated problems of transformation laws. Then because manifest covariance facilitates the contact with quantum field theories. In order to set up a covariant quantum theory, the main difficulty is to formulate *evolution*. The basic axioms of quantum mechanics are statements about the state of the system "at some time", measurement performed "at some time", and about the evolution of the system, either between measurements or through the measurement process.

We have to treat formally space and time on the same footing, but we must also give a meaning to this expression "at some time".

* The covariant Ehrenfest theorem for a single particle was first stated by L.Hostler [16]. We gave an explicit derivation and extension to mutually interacting particles [9].

The best models for a covariant and evolutive picture of quantum mechanics are the Tomonaga-Schwinger equation in field theory and the Fock-Stückelberg equation for a particle.

The Tomonaga-Schwinger equation is well-known; it rules the evolution of the state vector in QFT in such a way that the indicator of evolution is an arbitrary spacelike hypersurface (σ). We refer to this scheme as the σ -formalism.

The Stückelberg equation is far less popular. It describes a single particle *of unspecified mass*, according to the formula

$$i\partial\Psi/\partial\tau = H\Psi \quad (2.1)$$

where (in momentum representation) the arguments of the wave function are not only the unconstrained component of a four-vector p , but also a scalar parameter τ . It turns out that eq. (2.1) admits the particular solutions $\Psi = e^{-i\tau H}\psi$ provided ψ doesnot depend on τ but satisfies a Klein-Gordon equation of the form $2H = m^2\psi$ (for instance in the absence of external field, H is simply $\frac{1}{2}p^\alpha p_\alpha$). The dependence of Ψ on τ clearly reflects the fact that the mass is not a priori fixed. The mass rather appears as a constant of the motion (in general, constants of the motion are characterized by commutation with H). Though one may question the meaning of this extra parameter, τ is an indicator of evolution which doesnot depend on the observer. Moreover, this formulation of one-particle dynamics amounts to describing the lapse of time by the action of a one-parameter group, like in nonrelativistic quantum mechanics. Extension of this formalism to N-body systems requires an N-parameter abelian group [7,9], which is after all a rather simple generalization of the nonrelativistic structure.

In contrast, the Tomonaga-Schwinger equation, in its more general form, hides the structure of a coarse groupoid (each pair of non intersecting spacelike hypersurfaces corresponds to an element of the groupoid). This departure from a strict group structure (widely overlooked in the litterature, to our knowlege) is perhaps not completely innocent. It is only when a specified foliation of spacetime (one-parameter family of nonintersecting spacelike hypersurfaces) is given that one extracts from the groupoid a subset which is actually a one-parameter group.

Here we are concerned with *particle* dynamics, thus we resort to the eq.(2.1) for a single particle and shall naturally describe the evolution of N -particle systems with the help of N independent parameters, $\tau_1, \tau_2, \dots, \tau_N$. (We refer to this approach as the τ -formalism).

But in order to visualize successive measurements, we shall also consider spacelike hypersurfaces in a way which is similar (but not equivalent) to the Tomonaga-Schwinger theory.

The notion of central line will turn out to be essential in matching the τ -formalism with this kind of σ -formalism.

Let us now precisely define the central lines. *For the moment we consider only the one-body problem.*

The most naive intuition suggests that $q^\alpha = i\partial/\partial p_\alpha$ is a kind of "position" operator. Since the wave function depends on τ , the components of this operator have four expectation values

$$\langle q^\alpha \rangle = \frac{\langle \Psi, q^\alpha \Psi \rangle}{\langle \Psi, \Psi \rangle} \quad (2.2)$$

where the scalar product is, for arbitrary Ψ and Ψ' in $L^2(R^4, d^4p)$

$$\langle \Psi, \Psi' \rangle = \int \Psi^* \Psi' d^4p \quad (2.3)$$

Obviously $\langle q^\alpha \rangle$ depends on τ . For each given $\Psi(p, \tau)$, these four functions of τ define a curve in spacetime. We consider this curve as the world-line of the center of the wave packet, that is to say the *central line* of our particle.

This very simple picture [9] may hurt the habits of many readers, therefore some explanations are in order. Indeed the four quantities q^α obviously differ from the Newton-Wigner operator, and the scalar product involved in eq. is a four-fold integral extended all over the space of timelike future- pointing vectors. Moreover one as the right to ask whether the central line is actually a timelike curve.

Let us first legitimate the normalization formula (2.3). Remember that *in the present theory, the mass of the particle is not a priori specified and the states with definite mass are exceptional* (such states would not be

normalizable in the sense of eq.(2.3). In other words, the particle we consider is off the mass shell. The state of this particle is a superposition of various on-shell states corresponding to different values of the squared mass [17]. Accepting *only positive energy states* defines $L^2(R_+^4, d^4p)$. Omitting the possible dependence on τ we write

$$\Psi(p) = \int_0^\infty \psi(\mu, p) d\mu \quad (2.4)$$

where $\psi(\mu, p)$ is a positive-energy solution of the Klein-Gordon equation corresponding to the mass $m = \sqrt{\mu}$. In other words $(p^2 - \mu)\psi = 0$ and ψ is of the form

$$\psi = \delta(p^2 - \mu)\phi(\mu, p) \quad (2.5)$$

where $\phi(\mu, p)$ is defined only on the positive-energy sheet (η_μ^+) of the hyperboloid of mass $m = \sqrt{\mu}$. Notice that the development (2.4) is unique.

This formulation is by no means novel. It goes back as far as the theory of generalized free fields [18].

I wish to emphasize that the four-dimensional scalar product (2.3) has something to do with the conventional normalization of the solutions of the Klein-Gordon equation : Denote by $(\phi, \phi')_\mu$ the usual scalar product of two functions in $L^2(\eta_\mu^+)$, that is

$$(\phi, \phi')_\mu = \int_{\eta_\mu^+} \phi^* \phi' \frac{d^3p}{p^0}$$

Already some years ago we insisted that if Ψ and Ψ' are associated to ϕ and ϕ' through (2.4), (2.5) and similar formulas in terms of Ψ', ψ', ϕ' , then the off-shell scalar product (2.3) satisfies the *direct integral* formula

$$\langle \Psi, \Psi' \rangle = \int_0^\infty (\phi, \phi')_\mu d\mu \quad (2.6)$$

The use of this formula for defining the scalar product in the one-particle space is well-known in the theory of generalized free fields. But it seems that its identification with the four-fold integral (2.3), the proof of which is given in Appendix, has remained widely ignored for a long time.

Let us see under which conditions the central line is timelike or null.

Its tangent vector is $\frac{d}{d\tau} \langle q^\alpha \rangle$ that is $\langle p^\alpha \rangle$ for a free particle. But $\langle p^\alpha \rangle = \frac{1}{\langle \Psi^*, \Psi \rangle} \int \Psi^* \Psi p^\alpha d^4p$ is obviously timelike and points towards the future because Ψ vanishes outside the half light cone (the restriction to positive energy is essential). This argument ensures that the central line of a free particle is never spacelike.

In the presence of an external potential $A^\mu(q)$ we have $\frac{d}{d\tau} \langle q \rangle = \langle p - eA \rangle$ thus the vector tangent to the central line remains timelike provided that $-eA$ is non-spacelike and future oriented [9]. In other situations it seems that, if the central lines are to keep a physical meaning, the assumption that $L^2(R_+^4)$ is the correct Hilbert space has to be somehow modified. Moreover it may happen that our formalism breaks down when pairs are created. This situation will not arise in the simple applications we have in mind.

A deeper discussion of the general case is out of the scope of this paper. Henceforth we assume that all external fields are "nice" enough to ensure that the central line is homeomorphic to the real line and timelike.

We finally come to the use of q^α instead of the Newton-Wigner position in our definition of the center of wave packet. Notice that the customary Newton-Wigner operator explicitly depends on the mass, and is usually defined as an operator acting on a certain mass shell, labelled by a fixed value of $\mu = m^2$. Let $N_{(\mu)}^k$ be its three components. By direct integration over all the possible mass shells it is possible to construct an operator N^k *acting off shell*. That is to say

$$N^k \Psi = \int_0^\infty N_{(\mu)}^k \psi(\mu, p) d\mu$$

It has been shown by Horwitz and Piron [19] that this construction yields the symmetrized product $\frac{1}{2p^0} M^{k0} + M^{k0} \frac{1}{2p^0}$ where $M^{\alpha\beta} = q^\alpha p^\beta - q^\beta p^\alpha$ is the angular momentum. This construction was obviously expected to commute with the squared mass operator p^2 (which is diagonal with respect to the direct integral). And in fact the symmetrized product of M^{k0} by $\frac{1}{p^0}$ is a constant of the free motion. For this reason its expectation value cannot depend on the evolution parameter τ in the simple case of a free particle. We conclude that the off-shell generalization N^k of the Newton-Wigner operator is *not* a good candidate for defining a word-line in spacetime (in fact it defines a point in a three-plane passing through the origin, which may help defining some kind of localized state, for a free particle).

In contrast the expectation value of the operator q^α (which is, *with respect to the off-shell scalar product*, hermitian as well as N^k is) naturally depends on τ and provides a relativistic version of the Ehrenfest theorem, even in the presence of external field: Let Ω be any dynamical variable and define

$$\langle \Omega \rangle = \frac{\int \Psi^* \Omega \Psi d^4 p}{\int \Psi^* \Psi d^4 p} \quad (2.7)$$

for all Ψ in the domain of Ω , we check easily that

$$i \frac{d}{d\tau} \langle q^\alpha \rangle = \langle [q^\alpha, H] \rangle \quad (2.8)$$

For free particle we get $\frac{d}{d\tau} \langle q^\alpha \rangle = \langle p^\alpha \rangle$

Notice that q^α is not an observable although it has expectation values in the sense of (7): *A measurement of q^α is impossible* since it would sent the particle into some eigenstate of q , and the eigenstates of q contain negative energy contributions whereas we accept only positive-energy states. So the quantities $\langle q^\alpha \rangle$ cannot be interpreted as the average of the outcome of a large number of measurements (for this reason we may hope that q^α will circumvent the Hegerfeldt theorem). We are bound to take these expectation values as a mathematical device, but they are *a posteriori* justified by their nice properties with respect to the central line and the Ehrenfest theorem.

Now what are the localized states? Whatever they are, they cannot be eigenstates of any component of q^α . Any reasonable definition of localized states must agree in the classical limit with our experience of localized macroscopic objects. On the grounds of the Ehrenfest theorem we shall rely on the center of wave packet in order to visualize this classical limit. Therefore in the absence of a complete theory of localization, we shall provisionally assume:

If the particle is in some sense localized in a region, the center of wave packet falls somewhere inside this region.

Caution, this statement is by no means a theorem, not even a conjecture! Rather it is an anticipation of the properties that a nice theory of localisation should exhibit. Whether the direct integral N^k of the Newton-Wigner operators actually meets this requirement is an interesting issue, but we shall not pursue any longer this discussion. Let us leave aside as an open question the problem of localized states. From now on, we shall try (as much as possible) to go without this concept and to develop a formalism of evolution and measurement which is *nonlocal* from the outset.

Meaning of evolution parameter

When $\langle H \rangle$ is strictly positive, which is certainly true for all states when H is a positive operator in $L^2(R_+^4)$, then the evolution parameter τ can be related to the affine parameter on the central line.

Notice that p^2 (which is not positive in $L^2(R_+^4)$) is actually a positive operator in $L^2(R_+^4)$ as obvious if we apply p^2 to Ψ in (2.4). The average squared mass is $2 \langle H \rangle = M^2$. We set $v = p - eA$. In order to exhibit the affine parameter of the central line we introduce the re-scaling

$$M d\gamma = \sqrt{\langle v^2 \rangle} d\tau$$

which defines γ as a function of τ . But (2.8) tells that $\frac{d}{d\tau} \langle q \rangle = \langle v \rangle$. In other words

$$M^2 d\gamma^2 = \left(\frac{d}{d\tau} \langle q^\alpha \rangle \right) \left(\frac{d}{d\tau} \langle q_\alpha \rangle \right) d\tau^2$$

$$M^2 = \left(\frac{d}{d\gamma} \langle q^\alpha \rangle \right) \left(\frac{d}{d\gamma} \langle q_\alpha \rangle \right)$$

This formula shows that γ is affine parameter on the central line. The center of wave packet can be considered as a fictitious particle with mass M , proper time $M\gamma$, and four-velocity $\frac{v^\alpha}{M}$. Notice that for a free particle the ratio $\frac{d\gamma}{d\tau}$ is constant but generally different from unity.

3. N-body systems

For a classical system of N particles, covariant equations of motion naturally involve all the proper times (or suitable generalizations of them). There is a similar situation in the quantum theory. Some years ago we proposed to generalize the Stückelberg equation [7], assuming that the wave function of an N -particle system is of the form $\Psi(p_1, p_2, \dots, p_N, \tau_1, \tau_2, \dots, \tau_N)$ and obeys N coupled first-order differential equations. This point of view was also considered by Horwitz and Rohrlich [8] who developed a detailed covariant technique for calculating the scattering operator. Along this line we write

$$i \frac{\partial \Psi}{\partial \tau_a} = H_a \Psi \quad a = 1, 2, \dots, N \quad (3.1)$$

or equivalently

$$\Psi_{\tau_1 \tau_2 \dots \tau_N} = e^{-i(\tau_1 H_1 + \tau_2 H_2 + \dots + \tau_N H_N)} \Psi_0 \quad (3.2)$$

where $H_1, H_2, \dots, H_N$ are mutually commuting operators and Ψ_0 does not depend on the parameters (it is an initial data). We focus on positive energy states by the requirement that $\Psi_{\tau_1, \tau_2, \dots, \tau_N}$ vanishes if one of its arguments $p_1, p_2, \dots, p_N$ is outside the future light cone. So doing we define a space of states $L^2(R_+^{4N}, d^4 p_1, \dots, d^4 p_N)$ and we normalize through a $4N$ -fold integral $\int \Psi^* \Psi d^4 p_1, \dots, d^4 p_N$. For any dynamical variable Ω , the Ehrenfest theorem now takes the form

$$i \frac{\partial}{\partial \tau_a} \langle \Omega \rangle = \langle [\Omega, H_a] \rangle \quad (3.3)$$

When the particles undergo mutual interactions, there is a well-known complication about the physical co-ordinates. This complication is already present at the classical level and should not be confused with the problem of position operator. In the classical theory the physical positions cannot be canonical co-ordinates [20]. In the presence of a mutual interaction they are related with q_a, p_b through the so-called "position equations" [21] $\{x_a^\mu, H_b\} = 0$ for $a \neq b$. In the off-shell quantum theory it is natural to assume that the operators x_a^α representing the physical co-ordinates obey the commutation relations

$$[x_a^\alpha, H_b] = 0 \quad a \neq b \quad (3.4)$$

If we now consider the N quantities $\Xi_a^\alpha = \langle x_a^\alpha \rangle$ calculated in the state $\Psi_{\tau_1, \dots, \tau_N}$, eqs.(3.3)(3.4) immediately entail that $\partial \Xi_a^\alpha / \partial \tau_b$ vanishes. Thus each expectation value $\langle x_a^\alpha \rangle$ of a coordinate operator depends on a single parameter only, namely τ_a (this result stems from (3.3) applied to x_a^α). As a result, we obtain N central lines representing the average motion of the particles.

In a similar way, the vector $v_a^\alpha = -i[x_a^\alpha, H_a]$ only depends on τ_a , providing an argument which identifies τ_a as the parameter describing the a^{th} central line (this parameter is affine in the particular case of no interaction).

We shall not discuss here which form of the mutual interaction allows for the central lines being timelike, nor the general expression of the operators x_a . We shall assume henceforth that the central lines are homeomorphic to R and timelike. This is certainly true for a system of noninteracting particles. In that case we simply have $x_a = q_a$ and the central lines are straightlines.

The surface functional

Now that we have explained in detail the τ -formalism and its relationship to the central line, we wish to emphasize its natural connection with spacelike three-surfaces.

Indeed some consideration of these surfaces is desirable. Even if we are not systematically seeking an analogy with field theory, it remains true that the image of a spacelike hypersurface dividing spacetime in two regions is the best way to visualize the concepts of "before" and "after" in a covariant way. Notions like transition probability at a time, measurement at a time etc, can be easily formalized in relativity provided we resort to the idea of these three-surfaces. In particular a set of nonintersecting spacelike surfaces very naturally simulates the ordering relation implied by successive measurements.

In view of these considerations it is remarkable that any N -parameter family $\Psi_{\tau_1, \tau_2, \dots, \tau_N}$ satisfying the many-time evolution equations (3.1) induces a functional of surface $\tilde{\Psi}[\sigma]$. In other words, to any spacelike hypersurface (σ) it is possible to associate a state vector constructed as follows:

Let $\Psi_{\tau_1, \dots, \tau_N}$ be given. It satisfies (3.1) or equivalently (3.2), therefore the central lines $\Gamma_1, \Gamma_2, \dots, \Gamma_N$ can be constructed. They are timelike, thus each Γ_a is cut by (σ) in a unique point Ξ_a . The ordered set $\Xi_1, \Xi_2, \dots, \Xi_N$ defines a unique point in the cartesian product $\Gamma_1 \times \Gamma_2 \times \dots \times \Gamma_N$. This product manifold is homeomorphic to R^N and has the parametric equations

$$\begin{aligned} \Xi_1^\alpha &= \langle \Psi_{\tau_1, \tau_2, \dots, \tau_N}, x_1^\alpha \Psi_{\tau_1, \tau_2, \dots, \tau_N} \rangle \\ &\dots\dots\dots \\ \Xi_N^\alpha &= \langle \Psi_{\tau_1, \tau_2, \dots, \tau_N}, x_N^\alpha \Psi_{\tau_1, \tau_2, \dots, \tau_N} \rangle \end{aligned}$$

The correspondance between the sheet $\Gamma_1 \times \Gamma_2 \times \dots \times \Gamma_N$ and R^N is one-to-one, hence the sequence $\Xi_1, \Xi_2, \dots, \Xi_N$ corresponds to a unique set of parameters $\tau_1, \tau_2, \dots, \tau_N$. We define

$$\tilde{\Psi}[\sigma] = \Psi_{\tau_1, \tau_2, \dots, \tau_N}$$

and postulate that $\tilde{\Psi}[\sigma]$ is the state of the system at the "time" labelled by (σ).

So we can formulate evolution also in terms of spacelike hypersurfaces. This facility should not be misleading: There is an important difference between the present scheme and the Tomonaga-Schwinger formalism of QFT. If we fix some initial surface (σ_0) and consider another surface (σ) which is not intersecting, then $\tilde{\Psi}[\sigma]$ is *not linearly* related to $\tilde{\Psi}[\sigma_0]$, in contradistinction to the relation between $\Psi_{\tau_1, \tau_2, \dots, \tau_N}$ and $\Psi_{(\tau_1)_0, (\tau_2)_0, \dots, (\tau_N)_0}$.

Remark. Ch.Bordé has previously related the σ -formalism of Tomonaga and Schwinger with proper time using wave packets around an average *momentum* [22]. In contrast our approach considers wave packets around average *positions*.

4. Ideal Measurement

Spacelike hypersurfaces best describe the ordering relation implied in considering successive measurements. Through the Ehrenfest theorem, the central lines provide the clue to a contact with classical limit and macroscopic world. Therefore we consider it important that the center of wave packet undergo a *timelike shift* in spacetime during the process of measurement. This requirement is impossible to satisfy if measurement is instantaneous. So we are compelled to demand that the region concerned by this process has some finite extension in time. We postulate:

(i) *An ideal measurement takes place within a region limited between two nonintersecting spacelike hypersurfaces. Successive measurements will involve further hypersurfaces, and all of them are non intersecting*

We shall not attempt to describe what happens during the process.

Now consider the measurement of some observable Ω with pure point spectrum. There are projectors Π_n such that

$$\Omega = \sum \omega_n \Pi_n \quad \sum \Pi_n = 1 \quad (4.1)$$

The action of measuring is supposed to start on some hypersurface (Σ) . In the past of (Σ) the wave function is $\Psi_{\tau_1, \dots, \tau_N}$ according to (3.2) and the average positions Ξ_a generate the central lines $\Gamma_1, \Gamma_2, \dots, \Gamma_N$. The central lines respectively cut (Σ) at the points $X_1, X_2, \dots, X_N$. This measurement is applied to the system in the state $\Phi = \tilde{\Psi}[\Sigma] = \Psi_{T_1, T_2, \dots, T_N}$ where the sequence $T_1, T_2, \dots, T_N$ associated with (Σ) is unique. in other words

$$X_a = \langle \Psi_{T_1, T_2, \dots, T_N}, x_a \Psi_{T_1, T_2, \dots, T_N} \rangle = \Gamma_a \cap (\Sigma) \quad (4.2)$$

The reduction postulate will be formulated as follows:

(ii) *An ideal measurement of Ω , started on surface (Σ) , is an irreversible process which drives the system from Φ to an eigenvector of Ω , say $\Phi' = \Pi_n \Phi$ if the value measured is ω_n . The probability to find this value is supposed to be $\frac{\langle \Phi, \Pi_n \Phi \rangle}{\langle \Phi, \Phi \rangle}$ as usual.*

This statement is very similar to its nonrelativistic analog, except for the explicit mention of (Σ) and of the starting of a process which may have some finite extension in time.

The *ideal situation* occurs when each point

$$X'_a = \langle \Phi', x_a \Phi' \rangle \quad (4.3)$$

associated with the final state belongs to the future of X_a , say $X'_a \in \text{fut}.X_a$, in such a way that $X'_1, \dots, X'_N$ lay on some spacelike three-surface (which cannot be unique) *.

As we shall see in a moment, a very consistent picture can be given provided this condition is realized.

But the possibility of this situation depends on the state in which the measurement is performed and also on the observable we consider (remember that $X_1, X_2, \dots, X_N$ are determined by Ω, Π and Φ).

We shall say

(iii) *Definition. The observable Ω is ideally measurable in the state $\Phi \in L^2$ when the points $X'_1, \dots, X'_N$ defined by eq.(4.3) with $\Phi' = \Pi_n \Phi$, provide the above mentioned ideal situation, for all the projectors Π_n present in the decomposition of Ω .*

Now we consider the ideal situation, where Ω is measurable in the state $\Phi = \tilde{\Psi}[\Sigma]$ and assume we find the eigenvalue ω_n .

Then any spacelike surface (Σ') passing through the points $X'_1, \dots, X'_N$ is a good candidate for indicating the end of the measurement process.

Taking Φ' as new initial state we construct a new family labelled by N independent parameters, say

$$\Psi'_{\theta_1, \dots, \theta_N} = e^{-i(\theta_1 H_1 + \dots + \theta_N H_N)} \Phi' \quad (4.4)$$

In other words we make up a new solution of the Stückelberg equation and this solution is supposed to describe the motion *after* measurement.

In this parametrization (where Ψ' reduces to Φ' for $\theta_1 = \theta_2 = \dots = \theta_N = 0$) we have obviously

$$\tilde{\Psi}'[\Sigma'] = \Phi'$$

and only nonnegative values of $\theta_1, \dots, \theta_N$ are relevant. All these values are relevant, unless a new measurement is eventually performed.

* we refer to the future of X_a in a large sense allowing in particular X'_a to be on a null ray originated in X_a or even to coincide with X_a .

In a similar way, only the values $\tau_1 < T_1, \dots, \tau_N < T_N$ are relevant in $\Psi_{\tau_1 \dots \tau_N}$.

Let $\Gamma'_1, \dots, \Gamma'_N$ be the central lines determined by $\Psi'_{\theta_1 \dots \theta_N}$. Letting the parameters take their positive values we define *half-lines* extending from $X'_1, \dots, X'_N$ to the future.

In the same way, the meaningful pieces of $\Gamma_1, \dots, \Gamma_N$ go from $X_1, \dots, X_N$ to past infinity (see Fig.1 for two particles).

The future parts of Γ'_a can be regarded as natural prolongations of the past parts of Γ_a , their matching being realized as follows:

we can join X_a and X'_a by a *timelike* arc; the union of this arc with the past of Γ_a and with the future of Γ'_a constitutes a timelike curve. We consider it as the average world-line of particle a . It generalizes the central line in the cases where one measurement occurs. In this picture the history of the system is piecewise determined. The average world-lines are never taken outside the light cone, though their precise determination within the region of measurement is not possible. Some ambiguity remains about (Σ') ; this surface limiting (to the future) the region of irreversible behavior is not completely defined (even if one formulates the requirement of nonintersection in case of a further measurement starting at some (Σ'')). This dissymmetry between the roles of (Σ) and (Σ') reflects part of the irreversibility of the process occurring between these surfaces. At this point we wish to emphasize that the ambiguity about (Σ') does not affect at all the determination of the further history of the system, described by $\Psi'_{\theta_1 \dots \theta_N}$.

5. Semi-ideal measurement

As it stands, our reduction postulate can be regarded as a definition of ideal measurement. But there is no evidence that all realistic experiments actually fulfill the conditions for an ideal measurement. In particular the requirement that X'_a falls into the future of X_a is probably too restrictive. Therefore some enlargement of our formalism may be desirable. In (ii) we shall replace Φ' by a suitable generalization. For simplicity we begin with the **single-particle problem**.

For a given eigenvalue ω corresponding to the projector Π we consider the curve

$$Y(s) = \langle \Upsilon(s), x \Upsilon(s) \rangle \quad (5.1)$$

where

$$\Upsilon(s) = \Pi e^{-isH} \Phi \quad (5.2)$$

Hence $\Upsilon(0) = \Pi \Phi = \Phi'$. We say that we are in a position to perform a *semi-ideal measurement* of Ω when, for all eigenvalue ω , there exist some *nonnegative* values of s such that $Y(s) \in \text{fut}.X$, as displayed in Fig.2. Let α be the inferior bound of these values. We tentatively suggest this postulate:

(iv) If measurement is made in the state $\Phi = \tilde{\Psi}[\Sigma]$ the probability to find the eigenvalue ω corresponding to projector Π is $\langle \Pi \rangle_\Phi$ as usual. But if we actually find this eigenvalue then the state of the system at the end of the measurement process is given by

$$\Upsilon(\alpha) = \Pi e^{-i\alpha H} \Phi \quad (5.3)$$

It is clear from (4.4) that $\Upsilon(\alpha)$ is eigenstate of Ω with eigenvalue ω . Moreover $\Upsilon(0) = \Phi'$ thus α is simply zero when it happens that $\langle \Phi', x \Phi' \rangle \in \text{fut}.X$ so we recover ideal measurement as a special case.

The region of spacetime affected by irreversibility is limited in the future by some spacelike three-surface (Σ') passing through the point $Y(\alpha)$ and the further evolution is given by $\Psi'_\theta = e^{-i\theta H} \Upsilon(\alpha)$.

In spite of this generalization, the semi-ideal measurement of an arbitrary observable is perhaps not possible in all state of $L^2(R_+^4)$.

Moreover the presence of the term $e^{-i\alpha H}$ in (5.3) entails the following drawback:

Imagine that the state Φ in which we measure the observable Ω is an eigenstate of another observable Λ commuting with Ω . Then $[\Pi, \Lambda] = 0$ and $\Pi \Phi$ is also eigenstate of Λ with the same eigenvalue λ . But in general, the final state $\Upsilon(\alpha)$ as it is given by (5.3) is not anymore eigenstate of Λ unless the measurement is ideal (i.e. unless $\alpha = 0$). This departure from the conventional scenario of measurement theory is a consequence of the term $e^{-i\alpha H}$ in (5.3).

Nevertheless this difficulty disappears when at least Λ is a constant of the motion. Indeed in this particular case $\Lambda \Pi e^{-i\alpha H} \Phi = \Pi e^{-i\alpha H} \Lambda \Phi = \Pi \Lambda e^{-i\alpha H} \omega_1 \Phi = \omega_1 \Lambda \Pi e^{-i\alpha H} \Phi$.

From now on we shall only consider measurements of "good quantum numbers". Actually the constants of the motion enjoy nice properties with respect to semi-ideal measurement.

If we consider a particle which is either free or interacting with a "nice" external electromagnetic field, the following statement can be proven:

Theorem. *For a particle which is either free or in a "nice" electromagnetic field, any constant of the motion with a pure point spectrum is semi-ideally measurable in all states .*

(We recall that any projector present in the spectral decomposition of a constant of the motion is itself a constant of the motion. Constant observables with a pure point spectrum are easily constructed with the help of these projectors).

Proof. Since Ω is a constant of the motion, then $[\Pi, H] = 0$ and (5.3) can be written $\Upsilon(s) = e^{-isH} \Pi \Phi$ which is similar to the solution of (2.1), with $\Pi \Phi$ as initial state. Actually we derive $\frac{d}{ds} \Phi' = -iH \Phi'$ hence

$$i \frac{d}{ds} \langle \Upsilon(s), x \Upsilon(s) \rangle = \langle [x^\alpha, H] \rangle_{\Upsilon(s)}$$

But $[x, H] = i(p - eA)$ and we have assumed that the external field is nice, that is $\langle p - eA \rangle$ is future oriented and timelike or null for all states. So $\frac{d}{ds} Y(s)$ is timelike or null, which means that the curve generated by the points $Y(s)$ is never spacelike. Can we expect that this curve will meet the half light-cone containing the future of X ? Yes if Y moves toward the future when s increases, i.e. provided $\frac{d}{ds} Y(s)$ points toward the future. The question is finally whether the expectation value of $p - eA$ in $\Upsilon(s)$ does point to the future. But the electromagnetic potential we consider ensures that this is true in all state of $L^2(R_+^4)$, which achieves our proof.

Notice that the law postulated in eq. (5.3) is not the only one we could reasonably imagine. For instance an alternative possibility is to assume that $\Upsilon(s) = \Pi e^{-isH} \Pi \Phi$. It reduces to (5.3) when Ω is a constant of the motion.

An important difference between this covariant formulation and the nonrelativistic theory is that an ideal, or semi-ideal, measurement necessarily involves some lapse of time. In the situation presented here, the process undergone by the system is not completely instantaneous, although it is manifestly nonlocal. The timelikeness of the average world lines is an intuitive check that *some kind of causality is preserved*.

Nevertheless, our formulation has a peculiarity which is not present in the nonrelativistic theory: it seems that the ideal measurement of an observable with pure point spectrum is not always possible. More precisely, we are not always in a position to make the ideal description presented above.

6. Two-particle systems and correlated states.

We discuss here the semi-ideal measurement of an observable Ω in a state Φ which is supposed to be eigenstate of another observable Λ . Assume that $[\Omega, \Lambda] = 0$ and both are constants of the motion and have a pure point spectrum. The measurement is started at hypersurface Σ in the state $\tilde{\Psi}[\Sigma] = \Phi(p_1, p_2)$.

If the eigenvalue ω that we find corresponds to the projector Π , let us look at the curves

$$Y_a(s_a) = \langle \Upsilon(s_1, s_2), x_a \Upsilon(s_1, s_2) \rangle \quad a = 1, 2 \quad (6.1)$$

where

$$\Upsilon(s_1, s_2) = \Pi e^{-i(s_1 H_1 + s_2 H_2)} \Phi \quad (6.2)$$

Since Y_a only depends on s_a , an argument similar to that of Section 5 shows that under suitable assumptions * there exist couples of non-negative numbers s_1, s_2 such that $Y_1(s_1) \in \text{fut}.X_1$, $Y_2(s_2) \in \text{fut}.X_2$ and

* For instance this argument holds true when the particles are free or coupled to a "nice" form of external electromagnetic field. More care is needed if mutual interactions are present.

Y_1 spacelike-separated from Y_2 . Let α_1, α_2 be the minimal values of s_1, s_2 satisfying this condition. We postulate that the state of the system after the measurement is

$$\Upsilon(\alpha_1, \alpha_2) = \Pi e^{-i(\alpha_1 H_1 + \alpha_2 H_2)} \Phi \quad (6.3)$$

Obviously $\Upsilon(\alpha_1, \alpha_2)$ is eigenstate of Ω . Moreover $\Upsilon(0, 0) = \Phi'$. When it is possible to have $\alpha_1 = \alpha_2 = 0$ we recover a definition of *ideal* measurement which is a special case. The region affected by the irreversible regime is limited in the future by some spacelike three-surface (Σ') passing through $Y_1(\alpha_1)$ and $Y_2(\alpha_2)$. The subsequent evolution is given by $\Psi'_{\theta_1, \theta_2} = e^{-i(\theta_1 H_1 + \theta_2 H_2)} \Upsilon(\alpha_1, \alpha_2)$ which determines the future pieces of the central lines from $Y_1(\alpha_1)$ and $Y_2(\alpha_2)$ respectively.

We stress that the central lines, completed by the interpolated arcs, respectively $X_1 Y_1(\alpha_1)$ and $X_2 Y_2(\alpha_2)$, are timelike but it is quite possible that the arc $X_1 Y_1(\alpha_1)$ lies in a region spacelike-separated from the arc $X_2 Y_2(\alpha_2)$ (see Fig.3).

The state Φ where we measure Ω is already an eigenstate of Λ with eigenvalue λ . Then an argument similar to that of Section 5 immediately shows that the final state $\Upsilon(\alpha_1, \alpha_2)$ is eigenstate of Λ for the same eigenvalue λ .

Untill now we made no assumption about the way Ω and Λ are related with the individual particles. Here we come to the question of *measuring an individual property in a correlated state*. Define $P = p_1 + p_2$. Suppose that $\Omega = \Omega_1$, setting

$$\Omega_1 = 2\Pi_1 - 1 \quad (6.4)$$

where Π_1 is defined as the multiplication by a characteristic function $g_1(p_1, p_2)$ which takes on the value 1 if $p_1 \in D_1$, zero otherwise, D_1 compact domain inside the future light cone.

Suppose that $\Lambda = 2\Pi_P - 1$ where Π_P is defined as multiplication by a characteristic function g_P which takes on the values 1 if $P \in D$, zero otherwise, D being a suitable domain inside the future light cone.

Now define D_2 as

$$D_2 = \{ p_2 \mid p_2 = a - a_1, a \in D, a_1 \in D_1 \} \quad (6.5)$$

It is obvious that if $P \in D$ and $p_1 \in D_1$, then $p_2 \in D_2$. Provided that D and D_1 are chosen with care, then D_2 is inside the future light cone.

For instance we can fix a constant timelike unit vector u^μ and introduce the euclidian metric $|\xi|_{euc}^2 = 2(\xi \cdot u) - \xi^2$, for all four-vector ξ . Let $\bar{P}$ be a fixed timelike oriented vector, and let $\bar{p}_1$ be another future oriented vector, such that $\bar{P} - \bar{p}_1$ also is future oriented. We set $\bar{p}_2 = \bar{P} - \bar{p}_1$. We can precisely define D (resp. D_1) as the euclidian ball of radius ρ (resp. ρ_1) around the end of $\bar{P}$ (resp. $\bar{p}_1$). Then we easily check from (6.5) that D_2 is included in the euclidian ball of radius $\rho + \rho_1$ around the end of $\bar{p}_2$.

Naturally ρ and ρ_2 are choosen small enough in order to ensure that none of the sets D, D_1, D_2 can cross the light cone.

We notice Ω and Λ have the pure point spectrum $+1, -1$. They are dichotomic observables. Now Ω_1 refers to an individual property of particle 1. Let us set

$$\Omega_2 = 2\Pi_2 - 1$$

where Π_2 is defined as multiplication by g_2 , characteristic function of D_2 . It turns out that any measurement of Ω_1 (performed in an eigenstate of Λ with eigenvalue $+1$) which finds the eigenvalue $\omega_1 = +1$ drives the system into an eigenstate of Ω_2 with eigenvalue $\omega_2 = +1$. Obviously Ω_2 refers to an individual property of particle 2.

Conclusion

We have presented an evolution scheme for particles of indefinite masses. Spacetime is sliced as a sequence of regions determined by the successive measurements. Between these measurements we assume the Stückelberg equation as the foundation of relativistic quantum dynamics.

The reduction postulate is formulated in a non-local covariant way, and we pay much attention to the concept of central line. A different line of thinking was possible. If one doesnot believe that central line have some

physical significance, one may keep postulates (i)(ii), drop definition (iii) and develop some *inequivalent* yet non-local theory where the notion of measurability as defined in Section 4 becomes irrelevant. But our choice was to rely on the concept of central line which provides an intuitive representation of average individual motions.

Our definition of ideal (or semi-ideal) measurement is tailored for preventing the average world-lines to cross the light cone (even through the region of spacetime affected by interaction with the apparatus).

An important difference between this formulation and the nonrelativistic theory is that an ideal or semi-ideal measurement necessarily involves some lapse of time (we could say that the process undergone by the system is not perfectly instantaneous).

We have also this peculiarity: it seems that the ideal measurement of an observable is not always possible and it remains difficult to evaluate to which extent the notion of semi-ideal measurement improves the situation in general.

When Ω is not a good quantum number, definition (iv) is not fully satisfactory. We suspect that *in that case* formula (5.3) requires some modification *.

In contrast everything goes well if *only constants of the motion* are concerned in measurements. We can provisionally accept this limitation, and keep in mind that an eventual extension to nonconstant observables may require investigating what happens during the process of measurement, a question which was avoided throughout this paper.

We have proved that, insofar as free particles and constants of the motion are concerned, all the observables are at least semi-ideally measurable. Looking at two-body systems, this point allows for a covariant description of simple cases where an individual property of one particle is measured in some correlated state. The framework of this description is manifestly covariant and essentially non-local.

Of course, in the example considered here there is very little paradox if any, because the individual observable referring to particle 1 is related to momentum and therefore its measurement can by no means localize particle 2. Since nothing is localized, there is no propagation in the usual sense.

The heuristic representation provided by the central lines shows that the two pieces of average world lines crossing the region where the measurement takes place are in general spacelike separated.

Nevertheless these average world lines remain timelike everywhere. This property is not specific to the example we have discussed. It is implied in the very definition of what we call (semi) *ideal* measurement. Central lines would remain timelike if, instead of Ω_1 , we were going to measure a more sophisticated individual observable allowing for a localization of particle 1. Further work is needed in order to clarify the possibility to construct such an observable (an individual version of the Horwitz-Piron operator might do the job).

Anyhow, it is reasonable to regard the timelikeness of average world lines as indicating that semi-ideal measurements performed in correlated states still preserve some kind of causality.

But the concrete experiments of measurement performed in the real world are certainly much more complex than the ideal schemes we have just imagined.

Appendix

We claim that (2.3) implies (2.6). Inserting (2.4) into (2.3) we compute

$$\langle \Psi, \Psi' \rangle = \int \psi^*(\mu, p) \psi'(\nu, p) d\mu d\nu d^4p$$

But if we parametrize (η_μ^+) by the space components of p , there are uniquely determined functions f, f' in $R^+ \times R^3$ such that

$$\psi(\mu, p) = \delta(p^2 - \mu) f(\mu, \vec{p}) \quad \psi'(\nu, p) = \delta(p^2 - \nu) f'(\nu, \vec{p})$$

* It is a pity that we do not control this case because, in principle, the measurement of a dynamical variable which does not commute with the generators of the motion $H_1, \dots, H_N$, when performed in a state on the mass shell, should drive the state vector slightly *off the mass shell*, and we would appreciate the possibility of describing this interesting situation.

In other words the functions $\phi \in L^2(\eta_\mu^+)$ and $\phi' \in L^2(\eta_\nu^+)$ are explicitly given by $f(\mu, \vec{p})$ and $f'(\nu, \vec{p})$ respectively. We get easily

$$\langle \Psi, \Psi' \rangle = \int \delta(p^2 - \mu) \delta(p^2 - \nu) f^*(\mu, \vec{p}) f'(\nu, \vec{p}) d\mu d\nu d^4p$$

And after integrating over μ and ν ,

$$\langle \Psi, \Psi' \rangle = \int f^*(p^2, \vec{p}) f'(p^2, \vec{p}) d^4p$$

Notice that

$$d^4p = d(p^2) \frac{d^3p}{p_0}$$

Inserting this identity into the previous formula we get

$$\langle \Psi, \Psi' \rangle = \int f^*(p^2, \vec{p}) f'(p^2, \vec{p}) d(p^2) \frac{d^3p}{p_0}$$

Set $p^2 = \lambda$ and Perform the change of variables

$$p^\alpha \rightarrow p^2, \vec{p}.$$

and set $p^2 = \lambda$ hence $p_0 = \sqrt{\vec{p}^2 + \lambda}$. We finally obtain

$$\langle \Psi, \Psi' \rangle = \int_0^\infty d\lambda \int f^*(\lambda, \vec{p}) f'(\lambda, \vec{p}) \frac{d^3p}{\sqrt{\vec{p}^2 + \lambda}}$$

In the last integral of the righthandside we recognize the scalar product on the mass-shell space $L^2(\eta_\lambda^+)$, namely $(\phi(\lambda, p), \phi'(\lambda, p)_\lambda)$ which achieves the proof.

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Fig. 1 Only one space dimension is represented. Central lines can be curved. They get made of pieces of straight lines when particles are free.

Fig.2 Central lines in a semi-ideal measurement. α_1 and $\alpha_2 \geq 0$. The interpolated arcs are just light rays.

Fig 3