BB

MZ-TH/94-14
May 1994

INSTITUT FÜR PHYSIK

sw 3422

JOHANNES GUTENBERG UNIVERSITÄT

MAINZ

**THE PROTON-INDUCED DEUTERON BREAK-UP
REACTION ${}^2\text{H}(p,pp)n$**

E.O. Alt and M. Rauh
Institut für Physik
Johannes Gutenberg-Universität
D-55099 Mainz, Germany

CERN LIBRARIES, GENEVA

P00023440

Johannes Gutenberg (Kupferstich von Thuret)

The proton-induced deuteron breakup reaction ${}^2\text{H}(p,pp)n$ [†]

E. O. Alt and M. Rauh

Institut für Physik, Universität Mainz, D-55099 Mainz, Germany

Abstract:

The 'screening and renormalization' approach allows for a mathematically correct incorporation in three-body scattering theory, of the long-ranged Coulomb interaction between charged particles. It is based on first calculating, e.g., the transition amplitudes using screened Coulomb potentials. Then, after renormalization the zero-screening limit, leading to the amplitudes pertaining to unscreened Coulomb potentials, is performed numerically. Within this formalism the proton-induced breakup of deuterons is investigated, with the Coulomb repulsion between the two protons taken into account. Kinematically complete differential cross sections in various kinematic configurations are calculated, and compared with the results of an identical calculation but for the neutron-induced reaction. In this way information on the size of the Coulomb effects in this reaction can be extracted. Comparison is made with experimental data at 13.0 MeV projectile energy. In addition, the quality of approximate ways of including the Coulomb interaction is studied.

[†]Dedicated to Werner Sandhas on the occasion of his 60th birthday

1 Introduction

Ever since the development of the exact few-body theory [1], the investigation of the nucleon-deuteron (Nd) breakup reaction has been at the center of interest, as their principal field of application. This activity started with the pioneering work of Aaron and Amado [2], and their stunning success in describing available experimental data, even though the ansatz used for the nuclear interaction was rather simple. However, due to the enormous numerical difficulties involved, only fairly recently have calculations with so-called realistic nuclear potentials become feasible for breakup cross sections and polarisation observables, cf. [3, 4, 5, 6] and references therein.

The primary motivation for such theoretical investigations is founded on the hope that, by a judicious choice of the kinematic conditions, the effects of various interesting aspects of the fundamental interactions between nucleons, which can not at all, or at least not reliably, be obtained from studies of nucleon-nucleon (NN) scattering or other two-nucleon processes, may be enhanced, possibly even in a selective way. Then, comparison with experimental Nd data would allow for the extraction of valuable information about these topics. In this context, we mention only three-nucleon forces, off-shell behaviour of the NN interaction, but also specific NN on-shell features like the strength of the $^3S_1 - ^3D_1$ mixing, the neutron-neutron scattering length (and through it, information on the validity of the charge-symmetry hypothesis of the nuclear forces), etc.

It is obvious that such an endeavour becomes the more efficient and successful, the more precise the experimental data are which to compare with. Despite great efforts, those available for the neutron-induced breakup reaction are in general neither of desirable precision nor of sufficient quantity for this purpose. Hence, theoretical calculations are usually compared with the plentiful and (nowadays very) accurate proton-deuteron (pd) data, even though the effects of the Coulomb interaction between the two protons either are completely ignored or are taken into account in a rather *ad hoc* manner only (the only exception being the analyses of the near-threshold pd breakup in [7, 8, 9]). Nevertheless, in this way a rather satisfactory overall description of the experimental data is achieved. This is, of course, to be anticipated *a priori* because Coulomb effects are thought of being of the order of 5 – 10% only. On the other hand, it can not be denied that this neglect of the Coulomb repulsion represents a serious deficiency of the theoretical analyses. For, it does prevent the extraction of meaningful results for those of the above mentioned interesting nuclear effects the influence of which on three-body observables does not exceed this Coulomb uncertainty level.

It is clearly of paramount interest to have at one's disposal reliable, quantitative estimates of the Coulomb corrections. Of course, their size can be read off directly whenever the same pd and nd break-up observables are measured at identical kinematic conditions. Such measurements have been performed at 13 MeV [10, 11]: In a space star configuration the Coulomb effects turn out to be sizeable, in a collinear configuration still noticeable, and in a final state interaction configuration small. It is not unplausible to assume that these results are typical, in the sense that quite generally the inclusion of the Coulomb interaction does not affect all breakup observables by the same amount, in all accessible regions of the three-body phase space. Thus, by restricting oneself to those quantities and configurations which are little or not at all influenced by the Coulomb force, in spite of the theoretical neglect of the latter, still sensible and interesting information can be extracted from a comparison of sophisticated nd calculations with pd data.

Of course, for obvious reasons experimental data, from which the size of the Coulomb corrections could be determined, are not available in desirable quantity and quality. Therefore, there is no alternative but to decide on the basis of theoretical calculations whether Coulomb effects are large or not, for a specific observable, for a given kinematic configuration. To achieve this goal, it may even not be necessary to treat the nuclear part of the interaction in as sophisti-

cated manner as is possible nowadays for the neutral-particle reaction (which would go beyond present-day computational facilities anyway).

Already for two-charged particle scattering it is known that the standard methods of ordinary (short-range) scattering theory require considerable modifications if the long-ranged Coulomb force is to be included. This is true, *a fortiori*, in three-particle scattering. By generalizing the various methods developed for the two-body case, several versions of a three-body theory have been proposed which are applicable to Coulomb-like potentials, that is, to potentials possessing a Coulomb tail. The basic idea behind all these attempts (with exception of the one pursued by Kröger *et al* [7, 8, 9]) is to split off the Coulomb interaction part which is of longest range, from the rest of the interaction between the colliding fragments, however in such a form that it can be treated by explicit methods.

The oldest such proposal is due to Veselova [12] (see also [13]). It generalizes the well-known two-body result: The Lippmann-Schwinger (LS) equation is useless for the calculation of, e.g., the (full) scattering amplitude for Coulomb-like potentials. However, it can, in a slightly modified form, be used to calculate the so-called Coulomb-modified short-range amplitude, which is essentially determined by the short-range part of the potential. Similarly, by extracting in the three-body momentum space integral equations the longest-ranged part of the channel interaction and treating it separately in a two-potential type approach, for the remaining, Coulomb-modified short-range amplitudes integral equations could be written down. Since at present their validity is established *below* the bound state breakup threshold only, they are of no help for the calculation of breakup observables.

Another proposal is based on the generalization of the two-body coordinate-space differential equations approach for wave functions. By matching the physical scattering wave function obtained as solution of the three-body Schrödinger or coordinate-space Faddeev equation to appropriate asymptotic boundary conditions [14] (see, however, also [15]), elastic scattering and breakup amplitudes can be extracted. In practical applications use has been made of a partial-wave decomposition of the three-body wave function which, in view of the divergence of Coulomb partial-wave series known already for two-body scattering, may be problematic (for another problem see [16]). Concrete calculations have been sparse and very rough only [17].

The up to now most successful approach, developed by Alt, Sandhas, and Ziegelmann (ASZ) [18, 19], is based on the idea of using screened Coulomb potentials. Then all potentials either are short-ranged, or are being made short-ranged by a screening function. Consequently, all conventional definitions of physical quantities, and their interrelations established in short-range scattering theory, remain valid; also all-practical solution methods developed in this field can be applied. The transition to the unscreened theory is performed at the end in appropriately renormalized quantities like on-shell amplitudes, etc. This approach renders feasible the incorporation of the long-ranged Coulomb force in a mathematically rigorous way. It has already been applied successfully some time ago to the calculation of *pd* elastic scattering cross sections and phase parameters [20, 21, 22] (for energies below the breakup threshold, see also [23, 24, 25]).

It is the purpose of this paper to present a detailed investigation of the applicability of this 'screening and renormalization' approach, to the calculation of the proton-induced deuteron breakup, with inclusion of the Coulomb interaction. First results have been communicated in [26, 27]. Thus, estimates of the size of Coulomb effects can be obtained by comparing cross sections for the neutron- and the proton-induced reaction, for the same nuclear force model. In fact, by switching on and off the Coulomb interaction, *pd* and *nd* breakup processes are treated at the same level of sophistication with respect to the nuclear interaction used.

In sect. 2 we briefly recapitulate the basic idea of the 'screening and renormalization' approach for breakup processes, and its practical realization within the framework of the effective-two body formulation of the three-body theory, developed by Alt, Grassberger, and Sandhas (AGS) [28]. Section 3 contains the specialization of this formalism to the proton-deuteron

breakup reaction. A brief discussion of the numerical problems encountered hereby is included. This implies also questions of convergence of the unscreening procedure. In sect. 4 we first describe the results of detailed convergence tests, which are crucial for the applicability of the whole approach. Comparison with experimental data is then performed for a few kinematic configurations at 13.0 MeV projectile energy. In addition, various aspects of the specific procedure chosen for the calculation of the breakup amplitudes, and tests of several possibilities to approximately include the Coulomb repulsion between the protons, are discussed. At the end we present our conclusions. In the appendices we have collected some interesting details.

We mention that we use throughout natural units, i.e., $\hbar = c = 1$.

2 The 'screening and renormalization' approach

We start with a brief recapitulation of the 'screening and renormalization' approach to three-body breakup reactions with two charged and one neutral particles on which the present calculations are based. As has been mentioned in the Introduction, the underlying idea consists in screening the Coulomb potentials. The fact that now all occurring potentials are short-ranged (either *per se* or because of screening), has two important consequences. Firstly, the standard definitions of on-shell quantities like scattering amplitudes etc and their interrelations are valid. Secondly, all practical methods known in few body scattering with short-ranged potentials can be applied for the solution of the resulting equations; hence in particular also the quasiparticle formalism which turns out to be particularly well suited for this purpose. Of course, at the end the unscreening limit in the physically interesting quantities has to be performed according to well-defined prescriptions.

2.1 The quasiparticle equations for breakup amplitudes

In this subsection we recall the quasiparticle equations for the breakup amplitudes, but in an already specialized form appropriate for the description of the nucleon-induced deuteron breakup reaction.

Consider three particles with masses m_α , $\alpha = 1, 2, 3$. Denote by $U_{0\alpha}$ the breakup operator which leads from a partition $\alpha + (\beta, \gamma)$ of the three particles to a partition with three free particles, characterized by the index "0". These operators satisfy the AGS equations [28]

$$U_{0\alpha}(z) = G_0^{-1}(z) + \sum_{\gamma=1}^3 \bar{\delta}_{\gamma\alpha} U_{0\gamma}(z) G_0(z) T_\gamma(z), \quad (1)$$

and

$$U_{0\alpha}(z) = G_0^{-1}(z) + \sum_{\gamma=1}^3 T_\gamma(z) G_0(z) U_{\gamma\alpha}(z). \quad (2)$$

Herein, $G_0(z) = (z - H_0)^{-1}$ is the resolvent of the free three-body Hamiltonian H_0 . $\bar{\delta}_{\gamma\alpha} = 1 - \delta_{\gamma\alpha}$ denotes the anti-Kronecker symbol. The two-body transition operator T_γ describes scattering in subsystem γ , which consists of the particles α and β ($\gamma \neq \alpha, \beta$), and is given as solution of the usual LS equation with the full interaction V_γ between α and β ,

$$T_\gamma(z) = V_\gamma(z) + V_\gamma(z) G_0(z) T_\gamma(z). \quad (3)$$

It is apparent that (1) represents a coupled set of integral equations for the breakup operators, while the second, equivalent form (2) yields the $U_{0\alpha}$ by means of the arrangement operators $U_{\beta\alpha}$. The latter satisfy the equations

$$U_{\beta\alpha}(z) = \bar{\delta}_{\beta\alpha} G_0^{-1}(z) + \sum_{\gamma=1}^3 \bar{\delta}_{\beta\gamma} T_\gamma(z) G_0(z) U_{\gamma\alpha}(z). \quad (4)$$

Let the initial channel state $|\psi_\alpha\rangle|q_\alpha\rangle$ be given as the product of the bound state wave function $|\psi_\alpha\rangle$ (belonging to the binding energy $\bar{E}_\alpha$) of the pair (β, γ) , and the plane wave $|q_\alpha\rangle$ describing the free motion of particle α relative to the center of mass of this pair. The channel state for the outgoing three free particles consists of plane waves which we choose for convenience as $|p'_3\rangle|q'_3\rangle$. Here, p'_3 denotes the relative momentum of particles 1 and 2. Then the matrix element

$$\mathcal{T}_{0,\alpha}(q'_3, p'_3, q_\alpha; E + i0) = \langle q'_3 | \langle p'_3 | U_{0\alpha}(E + i0) | \psi_\alpha \rangle | q_\alpha \rangle \quad (5)$$

is on the energy shell, i.e. for

$$E = E_\alpha = \frac{q_\alpha^2}{2M_\alpha} + \hat{E}_\alpha = E'_0 = \frac{q_3^2}{2M_3} + \frac{p_3^2}{2\mu_3}, \quad (6)$$

the physical breakup amplitude. Here, $M_\alpha = m_\alpha(m_\beta + m_\gamma)/(m_\alpha + m_\beta + m_\gamma)^{-1}$ and $\mu_\alpha = m_\beta m_\gamma/(m_\beta + m_\gamma)$ are the appropriate reduced masses.

To be specific let us characterize the two identical particles (the two protons in the proton-induced, and the two neutrons in the neutron-induced deuteron breakup, respectively) by the indices 1 and 2, and the odd particle by the index 3. Then the two-body interactions in subsystem γ have the form

$$V_\gamma = V_\gamma^S + \delta_{\gamma 3} V_3^R \quad (7)$$

where V_γ^S is the nuclear part, and V_3^R the screened Coulomb potential which acts between the protons 1 and 2 only. R denotes the screening radius. As has been discussed in [13, 18, 19], by taking into account the results obtained for two particles in [29, 30], the screening and renormalization formalism holds for all reasonably smooth screening functions. For numerical convenience we have chosen an exponential screening function; that is, in coordinate space V_3^R is taken to be of the form

$$V_3^R(r) = \frac{e^2}{r} e^{-r/R}. \quad (8)$$

According to the familiar two-potential formalism, the resulting total T-operator $T_3^{(R)}(z)$ for subsystem 3 can be written as the sum of the (screened) Coulomb T-operator $T_3^R(z)$, which is the solution of the LS equation with potential (8),

$$T_3^R(z) = V_3^R + V_3^R G_0(z) T_3^R(z), \quad (9)$$

and a Coulomb-modified short-range (CMSR) part,

$$T_3^{(R)}(z) = T_3^R(z) + [1 + T_3^R(z) G_0(z)] T_3^{SR}(z) [1 + G_0(z) T_3^R(z)]. \quad (10)$$

The operator $T_3^{SR}(z)$ satisfies the LS equation

$$T_3^{SR}(z) = V_3^S + V_3^S G_3^R(z) T_3^{SR}(z), \quad (11)$$

with $G_3^R(z) = (z - H_0 - V_3^R)^{-1}$ being the (screened) two-body Coulomb resolvent. Equation (11) is a well-defined integral equation, even in the limit $R = \infty$, for short-ranged potentials satisfying the usual restrictions.

The quasiparticle formalism [28] represents a general scheme to reduce three-body operator equations of the type (1) or (2) and (4) to effective two-body equations. However, the resulting expressions become considerably simpler and more transparent if the short-range potentials are purely separable (in fact, with modern approximation techniques this can always be achieved, to any desired degree of accuracy). Since for the practical calculations we have used separable short-range potentials only, we restrict ourselves to this case. That is, we take V_γ^S to be of the form,

$$V_\gamma^S = \sum_r |\chi_{\gamma r}\rangle \lambda_{\gamma r} \langle \chi_{\gamma r}|, \quad \gamma = 1, 2, 3. \quad (12)$$

The form factors $|\chi_{\gamma r}\rangle$ and strength parameters $\lambda_{\gamma r}$ are chosen such as to reproduce the bound state (if existing) and scattering characteristics in subsystem γ in the state with quantum numbers collectively denoted by ' r '. We, moreover, assume for simplicity that the form factors $|\chi_{\gamma r}\rangle$ for different indices ' r ' and ' s ' are orthogonal in the sense that $\langle \chi_{\gamma r} | G_0(z) | \chi_{\gamma s} \rangle \sim \delta_{rs}$ (for example, this condition is realized if the indices r and s belong to different values of quantum

numbers such as subsystem spin). With short-range potentials of the type (12), the explicit solution of (11), and of the ordinary LS equations (3) valid for the T-operators in subsystems 1 and 2, becomes trivial yielding

$$T_{\gamma}^{(R)}(z) = \sum_r |\varphi_{\gamma r}(z)\rangle \Delta_{\gamma r}^{(R)}(z) \langle \varphi_{\gamma r}(z^*)| + \delta_{\gamma 3} T_3^R(z). \quad (13)$$

Here, we have introduced the so-called modified form factors

$$|\varphi_{\gamma r}(z)\rangle = [1 + \delta_{\gamma 3} T_3^R(z) G_0(z)] |\chi_{\gamma r}\rangle \quad (14)$$

which differ from the original form factors for $\gamma = 3$ only, and

$$\Delta_{\gamma r}^{(R)}(z) = \left\{ \lambda_{\gamma r}^{-1} - \langle \chi_{\gamma r} | G_0(z) | \varphi_{\gamma r}(z) \rangle \right\}^{-1}. \quad (15)$$

Note that the label R is suppressed wherever it is not required for clarity.

Let us specifically assume that in the subsystems $\alpha = 1$ and 2 exactly one, and in subsystem $\alpha = 3$ no bound state exists (this corresponds to the present situation where the former consist of the neutron and either one of the protons, and the latter of the two protons). Hence, for $\alpha \neq 3$, those of the form factors $|\varphi_{\alpha d}(z)\rangle = |\chi_{\alpha d}\rangle$ which correspond to a bound state with quantum numbers 'd' (for deuteron), are related at the binding energy $\bar{E}_{\alpha d}$ to the corresponding bound state wave function $|\psi_{\alpha d}\rangle$ via

$$|\psi_{\alpha d}\rangle |q_{\alpha}\rangle = G_0(q_{\alpha}^2/2M_{\alpha} + \bar{E}_{\alpha d} + i0) |\chi_{\alpha d}\rangle |q_{\alpha}\rangle. \quad (16)$$

From this relation follows that a pole of the T-matrix $T_{\alpha}^{(R)}$ due to the bound state in subsystem α in the state 'd' is contained in $\Delta_{\alpha d}^{(R)}(z)$. This is made manifest by writing

$$\Delta_{\alpha d}^{(R)}(z) = \frac{S_{\alpha d}(z)}{(z - Q_{\alpha}^2/2M_{\alpha} - \bar{E}_{\alpha d})} \quad \text{for } \alpha \neq 3, \quad (17)$$

with

$$S_{\alpha d}^{-1}(z) = \langle \chi_{\alpha d} | G_0(z) | \psi_{\alpha d} \rangle. \quad (18)$$

Here, Q_{α} is the relative momentum operator with eigenvalue q_{α} .

Let us introduce the decomposition (13) in the three-body equations (1) or (2) for the breakup, and in (4) for the arrangement operators. From now on we make the dependence on the screening radius R explicit, denoting the operators obtained in this way by $U_{0\alpha}^{(R)}(z)$ and $U_{\beta\alpha}^{(R)}(z)$. Define auxiliary operators $U_{0\alpha}^R(z)$ and $U_{\beta\alpha}^R(z)$ by means of eqs. (1) or (2) and (4), respectively, but with the full subsystem T-operators $T_{\gamma}^{(R)}(z)$ replaced by their non-separable remainders $\delta_{\gamma 3} T_3^R(z)$. Then the equations for the three-body operators $U_{0\alpha}^{(R)}(z)$ can be transformed into equations for effective two-body T-operators

$$T_{p'_3, \alpha m}^{(R)} := \langle p'_3 | U_{0\alpha}^{(R)}(z) G_0(z) | \varphi_{\alpha m}(z) \rangle. \quad (19)$$

For instance, from (1) follows the integral equation

$$T_{p'_3, \alpha m}^{(R)}(z) = \mathcal{V}_{p'_3, \alpha m}^{(R)}(z) + \sum_{\gamma=1}^3 \sum_r T_{p'_3, \gamma r}^{(R)}(z) \mathcal{G}_{0, \gamma r}^{(R)}(z) \mathcal{V}_{\gamma r, \alpha m}^{(R)}(z). \quad (20)$$

The effective breakup potentials $\mathcal{V}_{p'_3, \alpha m}^{(R)}(z)$ are defined as

$$\mathcal{V}_{p'_3, \alpha m}^{(R)}(z) := \langle p'_3 | U_{0\alpha}^R(z) G_0(z) | \varphi_{\alpha m}(z) \rangle, \quad (21)$$

the effective arrangement potentials as

$$\mathcal{V}_{\beta n, \alpha m}^{(R)}(z) := \langle \varphi_{\beta n}(z^*) | G_0(z) U_{\beta \alpha}^R(z) G_0(z) | \varphi_{\alpha m}(z) \rangle, \quad (22)$$

and the effective propagators $\mathcal{G}_{0; \gamma m}^{(R)}(z)$ as

$$\mathcal{G}_{0; \gamma m}^{(R)}(z) = \Delta_{\gamma m}^{(R)}(z). \quad (23)$$

Equation (2), on the other hand, yields the breakup operators by quadrature. The corresponding equation, which will be used exclusively in the following, is written down in a specialized form only, namely that the incoming state is characterized by a bound state in channel α ($\neq 3$), with the quantum numbers of the bound state denoted by 'd',

$$\mathcal{T}_{p'_3, \alpha d}^{(R)}(z) = \mathcal{V}_{p'_3, \alpha d}^{(R)}(z) + \sum_{\gamma=1}^3 \sum_r \mathcal{V}_{p'_3, \gamma r}^{(R)}(z) \mathcal{G}_{0; \gamma r}^{(R)}(z) \mathcal{T}_{\gamma r, \alpha d}^{(R)}(z). \quad (24)$$

The quantities $\mathcal{T}_{\gamma r, \alpha d}^{(R)}(z)$ are the effective two-fragment scattering operators leading from the incoming state to a final state in channel β with bound state quantum numbers 'r'. They are defined in terms of the three-body operators as

$$\mathcal{T}_{\beta n, \alpha d}^{(R)}(z) := \langle \varphi_{\beta n}(z^*) | G_0(z) U_{\beta \alpha}^R(z) G_0(z) | \varphi_{\alpha d}(z) \rangle, \quad (25)$$

and satisfy the effective-two particle, LS-type equations

$$\mathcal{T}_{\beta n, \alpha d}^{(R)}(z) = \mathcal{V}_{\beta n, \alpha d}^{(R)}(z) + \sum_{\gamma=1}^3 \sum_r \mathcal{V}_{\beta n, \gamma r}^{(R)}(z) \mathcal{G}_{0; \gamma r}^{(R)}(z) \mathcal{T}_{\gamma r, \alpha d}^{(R)}(z). \quad (26)$$

All quantities appearing in the above equations are still operators in the space spanned by the relative momenta between the elementary and the composite particle. The on-shell amplitude (5), describing the breakup of the incoming bound system 'd' in channel α , can be found, because of condition (16), as the plane-wave matrix element of the effective breakup operator (19),

$$\mathcal{T}_{0, \alpha d}^{(R)}(q'_3, p'_3, q_\alpha; E + i0) = \langle q'_3 | \mathcal{T}_{p'_3, \alpha d}^{(R)}(E + i0) | q_\alpha \rangle. \quad (27)$$

It can be calculated either by quadrature from the (half off-shell) arrangement amplitudes via (24), or via the integral equation (20) in which the same kernel as in the integral equation (26) for the arrangement amplitudes occurs.

Let us add one important comment. In the present context, the major advantage of using the effective equations (20) or (24) and (26) of the quasiparticle approach over the original equations (1) or (2) and (4) lies in the following fact: The latter are three-body equations with relatively simple kernel, while the former are of the two-body type, albeit with a much more complicated driving term and kernel. In practical applications a partial-wave decomposition has to be performed. For the effective two-body equations this has to be done only with respect to the relative motion of the elementary particle and the correlated pair. That is, the internal motion of the particles within this pair (e.g., the relative motion of the two protons under the influence of the nuclear and Coulomb interaction) is treated in a fully three-dimensional way. When using the original three-body equations, an additional partial-wave analysis concerning the relative motion within the correlated pair (e.g., also of the two protons) has to be performed. In order to get the final amplitudes, partial-wave summations are necessary. For the pp subsystem this is not unproblematic in view of the possible lack of convergence of Coulomb partial-wave series known in two-body scattering.

The separability assumption (12) for the nuclear interaction, and the fact that a nonseparable part in the two-body T-operators occurs in one subsystem, namely in subsystem 3 only, results in considerable simplifications. Namely, the defining equations (1) and (4), with $T_\gamma^{(R)}(z)$ replaced by the nonseparable part $\delta_{\gamma 3} T_3^R(z)$, of the auxiliary operators $U_{0\alpha}^R$ and $U_{\beta\alpha}^R$, respectively, can be solved explicitly [28], yielding

$$U_{0\alpha}^R(z) = G_0^{-1}(z) + \bar{\delta}_{3\alpha} T_3^R(z), \quad (28)$$

and

$$U_{\beta\alpha}^R(z) = \bar{\delta}_{\beta\alpha} G_0^{-1}(z) + \bar{\delta}_{\beta 3} \bar{\delta}_{3\alpha} T_3^R(z). \quad (29)$$

From these solutions one derives the following *exact* and *closed* expressions for the effective breakup and arrangement potentials

$$\mathcal{V}_{p'_3, \alpha m}^{(R)}(z) = \langle p'_3 | [1 + T_3^R(z) G_0(z)] | \chi_{\alpha m} \rangle, \quad (30)$$

and

$$\begin{aligned} \mathcal{V}_{\beta n, \alpha m}^{(R)}(z) &= \bar{\delta}_{\beta\alpha} \langle \chi_{\beta n} | [G_0(z) + (\delta_{\alpha 3} + \delta_{\beta 3} + \bar{\delta}_{\beta 3} \bar{\delta}_{\alpha 3}) G_0(z) T_3^R(z) G_0(z)] | \chi_{\alpha m} \rangle \\ &+ \delta_{\beta\alpha} \bar{\delta}_{\alpha 3} \langle \chi_{\alpha n} | G_0(z) T_3^R(z) G_0(z) | \chi_{\alpha m} \rangle. \end{aligned} \quad (31)$$

The effective breakup potential $\mathcal{V}_{p'_3, \alpha m}^{(R)}(z)$ is graphically depicted in Fig. 1; for the graphical representation of $\mathcal{V}_{\beta n, \alpha m}^{(R)}(z)$ we refer to [22].

It is important to realize that in principle, by using the explicit expressions (30) and (31) for the effective potentials, the breakup amplitudes (24) (and also the arrangement amplitudes (25)) are obtained *without any approximation concerning the Coulomb interaction*, and at the same level of sophistication with which, for $V_3^R = 0$, i.e., if the Coulomb interaction is switched off, the corresponding *nd* breakup amplitude is obtained. Thus, in such a calculation the single approximation consists in assuming the separable form (12) for the nuclear interaction which, however, as mentioned above can be improved upon, e.g. by choosing optimal form factors, separable expansion methods and so on.

2.2 Zero-screening limit

In order to obtain from the solutions of (20) or (24) the physical breakup amplitudes corresponding to unscreened Coulomb potentials, the transition to infinite screening radius must be performed.

It has been proven in [18] that in the renormalized on-shell amplitude the zero-screening limit exists,

$$\lim_{R \rightarrow \infty} Z_R^{-1/2}(p'_3) T_{0, \alpha d}^{(R)}(q'_3, p'_3, q_\alpha; E + i0) Z_{R, \alpha}^{-1/2}(q_\alpha) = T_{0, \alpha d}(q'_3, p'_3; q_\alpha), \quad (32)$$

and yields the physical breakup amplitude corresponding to unscreened Coulomb potentials. The renormalization factors depend on the specific choice of the screening function. For the exponential screening used here, they are given by

$$Z_R(p'_3) = e^{-2i\eta(p'_3)[\ln 2p'_3 R - C]}, \quad (33)$$

$$Z_{R, \alpha}(q_\alpha) = e^{-2i\eta_\alpha(q_\alpha)[\ln 2q_\alpha R - C]}, \quad (34)$$

where $\eta(p'_3) = e^2 \mu_3 / p'_3$ and $\eta_\alpha(q_\alpha) = e^2 M_\alpha / q_\alpha$ are the appropriate Coulomb parameters. $C = 0.5772 \dots$ is the Euler number.

Hence, the following practical procedure for obtaining this amplitude can be used: Calculate by any method available for performing three-body calculations with short-ranged potentials, e.g. by means of the quasiparticle equations (20) or (24), the screened breakup amplitude $T_{0,\alpha d}^{(R)}(q'_3, p'_3, q_\alpha; E + i0)$. Repeat the process for increasing values of R until the renormalized on-shell amplitude becomes independent of R . Then, according to (32), by numerically performing the unscreening limit the breakup amplitude pertaining to unscreened Coulomb potentials has been obtained.

2.3 Practical procedure

For reasons which will become clear later, we have preferred not to proceed directly according to the above prescription. Instead we decompose the breakup amplitude into two terms before performing the zero-screening limit (see [18]). Hereby, use is made of a two-potential representation for the effective arrangement amplitudes $T_{\beta n, \alpha d}^{(R)}(z)$ which is analogous to the two-body formula (10). It results from the fact that the diagonal part of the effective arrangement potentials (22) or (31) can, in analogy to (7) for the genuine two-particle potentials, be quite naturally decomposed into a long-ranged (v_α^R) and a shorter-ranged part, which, for later convenience, we write as

$$\mathcal{V}_{\beta n, \alpha m}^{(R)}(z) = \delta_{\beta\alpha} \bar{\delta}_{\alpha 3} \delta_{nm} \delta_{md} S_{\alpha d}^{-1}(z) v_\alpha^R + \mathcal{V}_{\beta n, \alpha m}^{(SR)}(z). \quad (35)$$

Here, we have taken advantage of the fact that such a decomposition is needed only for those channels $\alpha \neq 3$ in which a bound state 'd' occurs, because only there the corresponding effective Green function $\mathcal{G}_{0,\gamma r}^{(R)}(z)$ has a pole (compare (23) with (17)). The part v_α^R , which in momentum space reads as

$$v_\alpha^R(q'_\alpha, q_\alpha) = \frac{e^2}{4\pi} \left[(q'_\alpha - q_\alpha)^2 + R^{-2} \right]^{-1}, \quad (36)$$

describes pure Coulomb scattering of proton α off the total charge of the other two particles (which coincides in our case with the charge of the other proton) concentrated in their center of mass. It is, therefore, termed 'center-of-mass Coulomb interaction'. Denote by t_α^R the transition operator calculated from v_α^R by means of the LS equation

$$t_\alpha^R(z) = v_\alpha^R + v_\alpha^R \frac{1}{z - Q_\alpha^2/2M_\alpha - \bar{E}_{\alpha d}} t_\alpha^R(z). \quad (37)$$

Note that t_α^R is a genuine two-particle T-operator, acting in the space spanned by the momentum eigenvectors $|q_\alpha\rangle$. Then one obtains from (26) on the half-shell, i.e. for $E = q_\alpha^2/2M_\alpha + \bar{E}_{\alpha d}$ but unconstrained q'_β ,

$$\begin{aligned} T_{\beta n, \alpha d}^{(R)}(q'_\beta, q_\alpha; E + i0) &= \delta_{\beta\alpha} \bar{\delta}_{\alpha 3} \delta_{nm} \delta_{md} S_{\alpha d}^{-1}(q'_\alpha; E + i0) t_\alpha^R(q'_\alpha, q_\alpha; E + i0) \\ &\quad + T_{\beta n, \alpha d}^{(SR)}(q'_\beta, q_\alpha; E + i0), \end{aligned} \quad (38)$$

with $S_{\alpha d}(q'_\alpha; E + i0)$ denoting the momentum representation of (18). The CMSR amplitude $T_{\beta n, \alpha d}^{(SR)}$ is essentially determined by the effective potential part $\mathcal{V}_{\beta n, \alpha m}^{(SR)}$ which comprises all shorter-ranged contributions of $\mathcal{V}_{\beta n, \alpha m}^{(R)}$, while $t_\alpha^R(q'_\alpha, q_\alpha; E + i0)$ is the familiar (screened) Rutherford contribution.

The decomposition (38) induces a similar splitting in the breakup amplitudes,

$$T_{p', \alpha d}^{(R)}(q'_3, q_\alpha; E + i0) = B_{p', \alpha d}^{(R)}(q'_3, q_\alpha; E + i0) + T_{p', \alpha d}^{(SR)}(q'_3, q_\alpha; E + i0), \quad (39)$$

with the CMSR part

$$\begin{aligned} \mathcal{T}_{p'_3, \alpha d}^{(SR)}(q'_3, q_\alpha; E + i0) &= \sum_{\gamma=1}^3 \sum_r \int d^3 q''_\gamma \mathcal{V}_{p'_3, \gamma r}^{(R)}(q'_3, q''_\gamma; E + i0) \mathcal{G}_{0, \gamma r}(q''_\gamma; E + i0) \times \\ &\quad \mathcal{T}_{\gamma r, \alpha d}^{(SR)}(q''_\gamma, q_\alpha; E + i0), \end{aligned} \quad (40)$$

and the driving term

$$\mathcal{B}_{p'_3, \alpha d}^{(R)}(q'_3, q_\alpha; E + i0) = \bar{\delta}_{3\alpha} \int d^3 q''_\alpha \mathcal{V}_{p'_3, \alpha d}^{(R)}(q'_3, q''_\alpha; E + i0) \langle q''_\alpha | q_{\alpha, R}^{(+)} \rangle. \quad (41)$$

Here, $|q_{\alpha, R}^{(+)}\rangle$ is the center-of-mass Coulomb scattering state satisfying the wave function equation analogon of (37). We remark that the special decomposition (35) was chosen just in order to arrive at this form for the driving term, which will be of considerable usefulness for the further development.

Expression (41) can be rewritten by inserting the explicit form (30) for the breakup potential. We start by pointing out that the operator $[1 + T_3^R(z)G_0(z)]$ occurring there acts on the energy shell (6), i.e. for $z = E'_0 \pm i0$, essentially as a MØLLER operator mapping the plane wave $|p'_3\rangle$ onto the corresponding two-particle Coulomb scattering state,

$$[1 + T_3^R(E'_0 \pm i0)G_0(E'_0 \pm i0)] |p'_3\rangle |q'_3\rangle = |p'_{3, R}^{(\pm)}\rangle |q'_3\rangle. \quad (42)$$

Denoting the (screened) momentum-space Coulomb wave functions by $\psi_{p'_{3, R}^{(\pm)}}^{(\pm)}(\mathbf{k}) := \langle \mathbf{k} | p'_{3, R}^{(\pm)} \rangle$ and $\psi_{q_{\alpha, R}^{(+)}}^{(+)}(q'_\alpha) := \langle q'_\alpha | q_{\alpha, R}^{(+)} \rangle$, one finds

$$\mathcal{B}_{p'_3, \alpha d}^{(R)}(q'_3, q_\alpha; E + i0) = \bar{\delta}_{3\alpha} \int d^3 q''_\alpha \psi_{p'_{3, R}}^{(-)*}(\mathbf{k}[q''_\alpha, q'_3]) \chi_{\alpha d}(\mathbf{k}'[q''_\alpha, q'_3]) \psi_{q_{\alpha, R}^{(+)}}^{(+)}(q''_\alpha). \quad (43)$$

The momenta $\mathbf{k}$ and $\mathbf{k}'$ are the usual linear combinations of q''_α and q'_3 ,

$$\mathbf{k}'[q''_\alpha, q'_3] = \varepsilon_{\alpha 3} \left(\frac{\mu_\alpha}{m_\beta} q''_\alpha + q'_3 \right), \quad \mathbf{k}[q''_\alpha, q'_3] = \varepsilon_{3\alpha} \left(q''_\alpha + \frac{\mu_3}{m_\beta} q'_3 \right), \quad (44)$$

For convenience we have introduced the antisymmetric symbol $\varepsilon_{\alpha\beta} = -\varepsilon_{\beta\alpha}$, with $\varepsilon_{\alpha\beta} = 1$ for (α, β) being a cyclic ordering of $(1, 2, 3)$.

We recall that the existence of the zero-screening limit (32) has, in fact, for the first time been proven in [18] in this effective-two body formulation of the three-body theory, on the basis of a decomposition of the type (39). There, it had been shown that, if such a decomposition is inserted in the left-hand side of (32), this limit exists in both terms separately,

$$\begin{aligned} \lim_{R \rightarrow \infty} Z_R^{-1/2}(p'_3) \mathcal{T}_{0, \alpha d}^{(R)}(q'_3, p'_3, q_\alpha; E + i0) Z_{R, \alpha}^{-1/2}(q_\alpha) &= \mathcal{B}_{0, \alpha d}^{(C)}(q'_3, p'_3; q_\alpha) + \mathcal{T}_{0, \alpha d}^{(SC)}(q'_3, p'_3; q_\alpha) \\ &=: \mathcal{T}_{0, \alpha d}(q'_3, p'_3; q_\alpha), \end{aligned} \quad (45)$$

thereby yielding the full unscreened amplitude as their sum. Here, we have introduced the abbreviations

$$\mathcal{B}_{0, \alpha d}^{(C)}(q'_3, p'_3; q_\alpha) = \lim_{R \rightarrow \infty} Z_R^{-1/2}(p'_3) \mathcal{B}_{p'_3, \alpha d}^{(R)}(q'_3, q_\alpha; E + i0) Z_{R, \alpha}^{-1/2}(q_\alpha), \quad (46)$$

and

$$\mathcal{T}_{0, \alpha d}^{(SC)}(q'_3, p'_3; q_\alpha) = \lim_{R \rightarrow \infty} Z_R^{-1/2}(p'_3) \mathcal{T}_{p'_3, \alpha d}^{(SR)}(q'_3, q_\alpha; E + i0) Z_{R, \alpha}^{-1/2}(q_\alpha). \quad (47)$$

The representation (45) is graphically illustrated in Fig. 2.

The CMSR breakup part $\mathcal{T}_{0,\alpha d}^{(SC)}(q_3', p_3'; q_\alpha)$ can be calculated, as described above for the full amplitude, from the screened counterpart (40), by performing the unscreening limit in (47) numerically. The occurring half-off shell CMSR arrangement amplitudes $\mathcal{T}_{\gamma,\alpha d}^{(SR)}(q_3', q_\alpha; E + i0)$ are obtained from the solutions of the integral equations (26) via their definition (38).

In principle, the same procedure can be applied also to the driving term $\mathcal{B}_{0,\alpha d}^{(C)}(q_3', p_3'; q_\alpha)$. Another possibility consists in explicitly performing the limit in (46). Indeed, as shown in [31] the following limits exist

$$\lim_{R \rightarrow \infty} \psi_{p_3', R}^{(-)*}(k) Z_R^{-1/2}(p_3') = \psi_{p_3', C}^{(-)*}(k), \quad (48)$$

$$\lim_{R \rightarrow \infty} \psi_{q_\alpha, R}^{(+)}(q_\alpha') Z_{R, \alpha}^{-1/2}(q_\alpha) = \psi_{q_\alpha, C}^{(+)}(q_\alpha'). \quad (49)$$

Here, $\psi_{p_3', C}^{(-)}(k)$ and $\psi_{q_\alpha, C}^{(+)}(q_\alpha')$ are the usual momentum space Coulomb scattering wave functions for the two outgoing protons and for the centers of mass of the two incoming fragments, respectively. We thus find

$$\mathcal{B}_{0,\alpha d}^{(C)}(q_3', p_3'; q_\alpha) = \bar{\delta}_{3\alpha} \int d^3 q_\alpha'' \psi_{p_3', C}^{(-)*}(k[q_\alpha'', q_3']) \chi_{\alpha d}(k'[q_\alpha'', q_3']) \psi_{q_\alpha, C}^{(+)}(q_\alpha''). \quad (50)$$

A direct evaluation of $\mathcal{B}_{0,\alpha d}^{(C)}(q_3', p_3'; q_\alpha)$ via the right-hand side of (50) is, however, not easily accomplished. For, the Coulomb wave functions in momentum space are not ordinary functions but distributions, in contrast to their coordinate-space counterparts. Hence, it appears to be more promising to calculate the integral (50) in coordinate space. For this purpose we make use of the well-known coordinate-space Coulomb wave functions

$$\psi_{p_3', C}^{(\pm)}(r) = e^{-\pi\eta(p_3')/2} \Gamma(1 \pm i\eta(p_3')) e^{\pm i p_3' \cdot r} {}_1F_1(\mp i\eta(p_3'); 1; \pm i(p_3' r \mp p_3' \cdot r)), \quad (51)$$

and a similar expression for $\psi_{q_\alpha, C}^{(+)}(r)$ where now $\eta_\alpha(q_\alpha)$ enters instead of $\eta(p_3')$. Here, Γ is the Gamma function and ${}_1F_1(a; b; x)$ the confluent hypergeometric function (cf., e.g., [32]). This leads to a six-dimensional integral for (50).

For exponential forms of the form factors,

$$\chi_{\alpha d}(r) = \sqrt{\frac{\pi}{2}} N_{\alpha d} \frac{1}{r} e^{-\beta_{\alpha d} r}, \quad (52)$$

which we have used for our calculations ($N_{\alpha d}$ is a normalization factor), the latter can, by means of the techniques developed in [33], be reduced analytically to a one-dimensional integral over a smooth integrand. Let us write

$$\mathcal{B}_{0,\alpha d}^{(C)}(q_3', p_3'; q_\alpha) = \bar{\delta}_{3\alpha} C_\alpha(p_3', q_\alpha) \times I\left(-q_\alpha, \varepsilon_{\alpha 3} p_3', q_\alpha + \frac{m_\theta}{\mu_\alpha} q_3', \frac{m_\theta}{M_3} q_3' + \varepsilon_{3\alpha} p_3', \beta_\alpha, \eta_\alpha(q_\alpha), \eta(p_3')\right), \quad (53)$$

with

$$C_\alpha(p_3', q_\alpha) = \frac{N_{\alpha d}}{4\pi} e^{-\frac{\pi}{2}[\eta(p_3') + \eta_\alpha(q_\alpha)]} \Gamma(1 + i\eta(p_3')) \Gamma(1 + i\eta_\alpha(q_\alpha)) \quad (54)$$

and

$$I(p_1, p_2, p_1', p_2', \lambda, \eta_1, \eta_2) = \int d^3 r_1 \int d^3 r_2 e^{i(p_1' \cdot r_1 + p_2' \cdot r_2)} \frac{e^{-\lambda|r_1 + r_2|}}{|r_1 + r_2|} \times {}_1F_1(-i\eta_1; 1; i(p_1 r_1 + p_1 \cdot r_1)) {}_1F_1(-i\eta_2; 1; i(p_2 r_2 + p_2 \cdot r_2)). \quad (55)$$

The evaluation of (55) is presented in Appendix A.

Finally, we remark that $B_{0,\alpha d}^{(C)}(q'_3, p'_3; q_\alpha)$, which describes the pure Coulomb breakup of the incoming bound state, has been calculated by numerical methods in [34] for a Born-type study of the reaction $\mu^+d \rightarrow \mu^+pn$.

3 Application to the proton-induced deuteron breakup

3.1 Effective equations

In this section we summarize the equations pertaining to the proton-induced deuteron breakup. Since we found it more convenient, we do not use the isospin formalism. Hence protons and neutrons are treated as distinguishable particles. The various two-nucleon states considered can be classified according to the subsystem total spin j and orbital angular momentum ℓ . We restrict ourselves to taking into account the two-nucleon S-wave states only, for which a simplified notation is used: For neutron-proton states with $\ell = 0$ and $j = 1$ the index 'd' (for deuteron), and with $\ell = 0$ and $j = 0$ the index 's' (for singlet); for the proton-proton state with $\ell = 0$ and $j = 0$ the index 'c'.

That is, our two-nucleon interaction is taken to be of the form

$$V_m(p', p) = \chi_m(|p'|) \lambda_m \chi_m(|p|) P_m, \quad \text{for } m = d, s, \quad (56)$$

which acts in S waves only, whereas

$$V_c(p', p) = \chi_c(|p'|) \lambda_c \chi_c(|p|) P_c + V_3^R(p' - p), \quad (57)$$

due to the presence of the Coulomb force, acts in all partial waves. Here,

$$P_m = \sum_{h_m} |j_m h_m\rangle \langle j_m h_m| \quad (58)$$

are projection operators on subsystem spin j_m . In this context we point out that the restriction to the simple ansatz for the nuclear part in (56) and (57) was forced upon us by the fact that the computing time required, when going over from nd to pd breakup, was found to increase by a factor of the order of 500(!).

As mentioned previously, the (S-wave) form factors χ_m are chosen to be of the Yukawa form (52) in coordinate space. For $m = d, s$, the parameters λ_m and β_m were fitted to the low-energy S-wave parameters of the np system in the triplet and the singlet state. Their explicit values are given in Table 1. The parameters λ_c and β_c , appropriate for the pp 1S_0 state, are determined so as to reproduce the Coulomb-modified nuclear pp effective-range parameters. The corresponding nn effective-range parameters, which are needed for the calculation of the neutron-induced breakup, can be obtained from them under the assumption of charge symmetry. The corresponding values are collected in Table 2.

Choosing the nuclear interaction to act in S waves only has the consequence that the total spin and the total orbital angular momentum of the three-nucleon system are conserved separately. The former can take the values $S = 3/2$ for the quartet, and $S = 1/2$ for the doublet state.

In the incoming channel one of the protons impinges upon the deuteron. Taking into account the identity of the two protons, antisymmetrization leads to the following equation for the determination of the unscreened breakup amplitude for a given value of the total spin S ,

$$T_{0d}^{S(j)}(q', p'; q) = B_{0d}^{(C)S(j)}(q', p'; q) + T_{0d}^{(SC)S(j)}(q', p'; q) \quad (59)$$

where $T_{0d}^{(SC)S(j)}$ is found, according to the prescription (47), as the zero-screening limit

$$T_{0d}^{(SC)}(q', p'; q) = \lim_{R \rightarrow \infty} Z_R^{-1/2}(p') T_{p',d}^{(SR)S(j)}(q', q; E + i0) Z_{R,1}^{-1/2}(q), \quad (60)$$

with

$$T_{p',d}^{(SR)S(j)}(q', q; E + i0) = \sum_{m=d,s,c} \int d^3 q'' \mathcal{V}_{p',m}^{(R)S(j)}(q', q''; E + i0) \times G_{0;m}(q''; E + i0) T_{md}^{(SR)S(j)}(q'', q; E + i0). \quad (61)$$

We have chosen the notation $\mathcal{G}_{0;m} \equiv \mathcal{G}_{0;1m} = \mathcal{G}_{0;2m}$ for $m = d, s$ and $\mathcal{G}_{0;c} = \mathcal{G}_{0;3c}$ (recall (23)). As usual, wherever possible the two-fragment channels of the three-nucleon system are characterized only by the index 'm' of the correlated pair occurring in it. Thus, the incoming channel is labelled by 'd'. Note that all breakup quantities carry an additional index j which specifies, in the doublet channel, whether in the three-free particle state the total spin $S = 1/2$ is obtained via intermediate coupling of a given pair of particles to a spin singlet ($j = s$) or a spin triplet ($j = d$). In the quartet channel, clearly only $j = d$ is possible.

Since spin and orbital parts are not coupled, they can be separated in the (antisymmetrized) effective potential $\mathcal{V}_{p',m}^{(R)S(j)}$ and in the driving term $\mathcal{B}_{0d}^{(C)S(j)}$. Let us write the former as

$$\mathcal{V}_{p',m}^{(R)S(j)}(q', q; E + i0) = \sum_{(\alpha n)} \Lambda_{m(\alpha n)}^{S(j)} \mathcal{V}_{p',\alpha n}^{(R)}(q', q; E + i0), \quad (62)$$

where the sum over (αn) runs over the index pairs (1d), (2d), (1s), (2s), (3c). The explicit expression for $\mathcal{V}_{p',\alpha n}^{(R)}$ is given in (30). For the driving term we similarly write

$$\mathcal{B}_{0d}^{(C)S(j)}(q', p'; q) = \sum_{(\alpha n)} \Lambda_{d(\alpha n)}^{S(j)} \mathcal{B}_{0,\alpha n}^{(C)}(q', p'; q), \quad (63)$$

where now the sum over (αn) runs over the combinations (1d) and (2d) only. $\mathcal{B}_{0,\alpha d}^{(C)}$ is defined in (53). The matrices $\Lambda_{m(\alpha n)}^{S(j)}$, which are essentially spin recoupling coefficients but contain also signs and factors of 2 from the antisymmetrization are collected in Appendix B.

The calculation of the CMSR two-fragment amplitudes $\mathcal{T}_{md}^{(SR)S}$ has been described in detail in [22].

From the total breakup amplitudes $\mathcal{T}_{0d}^{S(j)}(q', p'; q)$ one finds the kinematically complete unpolarized cross section via

$$\frac{d^5 \sigma_{p+d \rightarrow p+p+n}}{d\Omega_1^L d\Omega_2^L dE_S} = \frac{2\pi m_p \rho_S(E_1^L, E_2^L)}{p_p} \left(\frac{2}{3} |\mathcal{T}_{0d}^{\frac{1}{2}(d)}|^2 + \frac{1}{3} |\mathcal{T}_{0d}^{\frac{1}{2}(s)}|^2 + \frac{1}{3} |\mathcal{T}_{0d}^{\frac{1}{2}(s)}|^2 \right). \quad (64)$$

Herein, m_p and p_p are the mass and the laboratory momentum of the projectile, and Ω_1^L, Ω_2^L are the laboratory angles of the detected particles. $\rho_S(E_1^L, E_2^L)$ is the phase space factor, parametrized by means of the arc length E_S along the kinematically allowed curve in a counterclockwise direction (see, e.g., [35]).

We remark once more that, by switching off in the above equations the Coulomb interaction, one arrives at the formalism which allows for the calculation of the neutron-induced breakup reaction, at the same level of sophistication with respect to the nuclear interaction.

3.2 Numerical methods

As mentioned above, the integral I which determines the driving term $\mathcal{B}_{0,\alpha n}^{(C)}$ (compare (53)) could be reduced analytically to a one-dimensional integral with a smooth integrand, and hence can be integrated without problems.

The half-off-shell CMSR elastic scattering amplitudes $\mathcal{T}_{md}^{(SR)S}(q', q; E + i0)$ are found by solving the partial-wave projected integral equations described in [22], for a given value of the screening radius R . Each partial-wave amplitude is then smoothly interpolated by cubic splines as function of the magnitude q' of the off-shell momentum. Afterwards all partial-wave contributions are summed up to yield the CMSR amplitude as function of the three-dimensional off-shell momentum q' . We mention that, as discussed in [22], due to the complexity of the equations, the screened two-particle Coulomb T-matrix T_3^R , which occurs *fully off-shell* in the

effective arrangement potentials (31) and effective Green function (23), had to be approximated by its Born term V_3^R . In this context we recall that, as long as the screening radius R is not too large, V_3^R , and thus also T_3^R even for positive energies, are slowly varying functions of the momenta. In particular, they do not suffer from the unpleasant, typical Coulomb singular behaviour (in contrast to their unscreened counterparts V_3^C and T_3^C which, as stressed repeatedly, never occur in our integral equations). Hence, this approximation is expected to yield reasonably accurate pd elastic scattering amplitudes; that is, the next order terms represent a correction to the Coulomb corrections. In fact, for energies below the breakup threshold, where these typical Coulomb singularities in T_3^C are not present, the accuracy of such an approximation has been verified explicitly in [36, 37], for Coulomb parameters relevant to the present case, by comparing the various effective potential terms with T_3^C kept explicitly, and with T_3^C approximated by V_3^C .

The effective breakup potentials $\mathcal{V}_{p',an}^{(R)S(j)}(q', q; E + i0)$, on the other hand, could be calculated *without any approximation*. This had been possible because, according to their definition (30), the knowledge of only the screened *half-off-shell* two-particle Coulomb amplitude $T_3^R(p'_3, k; p_3^2/2\mu_3 + i0)$ is required. The latter is obtained by solving the three-dimensional LS equation (9) (actually, because of rotational invariance of the screened Coulomb potential (8) only a two-dimensional integral equation needs to be solved). The solution obtained in this way on a discrete lattice (for the magnitude k of the right-hand momentum and angle between k and p'_3) is then interpolated by bi-cubic splines to obtain the continuous function $T_3^R(p'_3, k; p_3^2/2\mu_3 + i0)$. For a given kinematic configuration, i.e. for fixed angles of the detected particles, this has to be done for every point on the kinematically allowed curve for which the values of the outgoing relative momentum p'_3 and corresponding energy $p_3^2/2\mu_3$ are uniquely specified.

When performing the zero-screening limit numerically, the screening radius R has to be increased. However, the larger R is, the more pronounced becomes the quasisingular character of the screened Coulomb potential, and hence of the LS kernel in (9). Despite development of special smoothing techniques, reliable results could be obtained, because of computer memory limitations, for $R \leq 200$ fermi only. Thus, the difficulties connected with the calculation of $T_3^R(p'_3, k; p_3^2/2\mu_3 + i0)$ represent the major limitation in accuracy of the final results for $R \gg 200$ fermi. However, as will be discussed in the next section such large R -values have never been needed.

We mention in parentheses that first attempts to calculate the half-off-shell amplitude $T_3^R(p'_3, k; p_3^2/2\mu_3 + i0)$ by solving the partial-wave projected LS equation and summing up partial-wave contributions, turned out to be unsuccessful, because of the huge number of partial waves required to achieve convergence for large R -values.

When the half-off-shell CMSR elastic scattering amplitudes and the breakup potential are obtained as described above, insertion into (61) yields, via a three-dimensional quadrature, the screened CMSR breakup amplitude. The latter has to be multiplied by the appropriate renormalization factors. Then the whole calculation has to be repeated for increasing R until the renormalized amplitude becomes independent of it. The result of this procedure is the unscreened CMSR breakup amplitude (60). Adding to it the driving term (63) yields, according to (59), the full breakup amplitude corresponding to unscreened Coulomb potentials.

We emphasize once more that, given the nuclear force model represented by eqs. (56) and (57), and provided screening radii not exceeding 200 fermi are sufficient (see next section), the single source of potential lack of accuracy is the above mentioned approximation in the calculation of the pd elastic scattering amplitudes.

It may be of interest to know that there are essentially two factors responsible (with approximately equal weight) for the enormous increase in computing time (by a factor ~ 500 , as already mentioned) when going over from a neutral- to the charged-particle reaction. One concerns the effort which, as described above, is required for the exact calculation of the effective breakup

potential. The other one arises from the fact already mentioned that the total CMSR breakup amplitude is obtained from the breakup potential and elastic CMSR scattering amplitude by an additional three-dimensional quadrature, cf. (61), which can trivially be done in the calculation of the neutral-particle breakup reaction.

4 Results

4.1 Convergence tests

The decisive question determining the practicability of the 'screening and renormalization' approach concerns the magnitude of the screening radius R at which the zero-screening limit is practically reached, the criterion being that the renormalized CMSR part of the breakup amplitude occurring in (60), become independent of R . As discussed in sect. 3, if too large values of R would be needed, the numerical stability in the calculation of $\mathcal{T}_{p,d}^{(SR)S(j)}$ would be very difficult to achieve, because of the quasisingular nature of the screened Coulomb potential.

Many careful tests have shown that the unscreening limit is reached, for proton laboratory energy of 13.0 MeV, in the worst cases at $R \approx 100 - 200$ fermi, for 67.0 MeV already at $R \approx 50 - 100$ fermi. The convergence behaviour depends, however, somewhat on the specific kinematic configuration considered. To see this in detail we show in Table 3 the breakup cross section at 13.0 MeV, as function of R , for four interesting cases, namely a space star (SST), a collinear (COL), a quasifree (QFS) and a final state interaction (FSI) configuration, at those values of the arc length along the kinematically allowed curve at which the kinematic conditions are exactly satisfied. For the chosen set of laboratory angles of the detected protons, the arc length E_S (which uniquely fixes their energies) can be found for these four cases in Table 4. The convergence pattern away from these specific values of E_S can be inferred from Figs. 3 and 4 which contain the cross sections over the whole kinematic curve for the QFS and the SST configuration for the proton angles of Table 4, for $R = 50, 100$ and 200 fermi. It is apparent that convergence is practically reached everywhere for $R \approx 100 - 200$ fermi. A similar behaviour is found for the COL configuration, while for the FSI kinematics the corresponding three curves are practically indistinguishable. They are, therefore, not shown.

In order to study the energy dependence of the convergence rates we have performed a similar investigation at a proton laboratory energy of 67.0 MeV. Table 5 contains cross sections at three configurations, again at those values of arc length at which the kinematic conditions are exactly verified. The corresponding values of the kinematic variables are collected in Table 6. We mention that they have been chosen in accordance with recent and still ongoing measurements ([38, 39] and private communications). As already anticipated from the experience with elastic pd scattering in [22], the higher the energy is, the smaller screening radii are needed. Furthermore, convergence over the whole kinematic curve is again as good as for the specific points chosen for Table 5 and is, therefore, not shown.

In the results to be presented in the following we consequently have always used a screening radius of 100 fermi. This represents a reasonable compromise between the achievable accuracy and the required computing time. It ensures that the unscreening limit for the cross sections has, for all practical purposes, been reached in all kinematic configurations, at all energies considered, at an acceptable expense and accuracy.

This discussion has made it clear that the 'screening and renormalization' approach does provide a practical method for reliably calculating pd breakup cross sections.

4.2 Deuteron breakup at 13.0 MeV

Let us now turn to the discussion of our results for kinematically complete differential cross sections, for a nucleon laboratory energy of 13.0 MeV. (First calculations at 14.1 and 16.0 MeV have already been presented in [26, 27].) As mentioned in the Introduction, this energy is of particular interest since only there published pd [10] as well as nd [11] breakup cross sections, measured *at identical kinematic conditions*, are available. This fact entails two practical consequences. Firstly, the size of the Coulomb effects in this reaction, at the measured kinematic configurations, can be read off directly from the experimental data. Secondly, these data provide

a crucial testing case for theoretical analyses which attempt to describe simultaneously the neutron- and the proton-induced reaction.

We recall that, by switching on and off the Coulomb interaction, our equations yield the amplitudes for the nd and pd breakup reaction, respectively. Thus, even in situations where the magnitude of the individual experimental cross sections is not described satisfactorily (on account of the insufficiency of the nuclear force model used), nevertheless already from a reproduction of their difference conclusions about the relevance, at least of the calculated Coulomb corrections, may possibly be drawn.

Before discussing our results, several comments are appropriate.

(i) As has already been pointed out, the nuclear potentials employed in our calculations are charge-symmetric but not charge-independent. Thus, the experimentally well established charge-independence breaking is taken into account.

(ii) We always include selected cross sections values for the neutron-induced deuteron breakup, obtained in a sophisticated calculation [10] which is based on the Paris potential and takes into account two-nucleon states with total subsystem angular momentum ≤ 3 . They will simply be referred to as 'Paris potential results'. The reason for doing this is twofold. Firstly, comparison with our neutron-reaction cross sections will reveal eventual shortcomings of our nuclear potential model. Secondly, one might hope that at least a qualitative picture of what could be expected from a similarly sophisticated pd breakup calculation, if it could be performed, may be obtainable by simply adding our Coulomb corrections to these Paris potential results.

(iii) For the theoretical cross sections to be shown in the Figures we assume point geometry, i.e. they are not folded with the experimental resolution (as is the case for the Paris potential results). This is adequate for the very accurate ${}^2\text{H}(p,pp)n$, but certainly not for the much less precise ${}^2\text{H}(n,nn) {}^1\text{H}$ reaction data. Consequently, the comparison of the calculated with the measured neutron-reaction cross sections should be considered only under this proviso.

Let us begin with the np -FSI configuration. As can be seen from Fig. 5, experimental pd and nd breakup cross sections practically coincide, with the exception of the FSI peak areas. In other words, Coulomb effects play a negligible role in this configuration. This is also born out by our calculations. The quantitative description of the experimental data is satisfactory. Selected Paris potential results are also shown. They provide an good overall reproduction of the experimental neutron-, and hence also proton-reaction cross section, indicating that, not surprisingly, in particular in the interference region but also at the FSI peaks a more realistic treatment of the NN interaction is important. Thus, because of the experimentally proven, and theoretically confirmed, absence of Coulomb effects, the analysis of the proton-reaction data with the neutron-reaction calculation appears to be justified. Exceptions may be the peak regions, where the peak height is increased by the inclusion of the Coulomb interaction. Adding this fact as a trend to the Paris potential results would even enlarge the difference between 'theoretical' and experimental proton-reaction cross sections. (This conclusion would not be changed by taking into account the finite detector geometry, assuming that it does affect the theoretical pd and nd cross sections in the same way). Whether this supports claims [40] that this discrepancy may represent a signature of three-body forces requires further investigations.

Figure 6 shows a QFS configuration which has been measured [10] only for the reaction ${}^2\text{H}(p,pp)n$. In [26] it was established that at 16.0 MeV the Coulomb corrections lead to an appreciable reduction of the QFS peak height. This is also born out at the present energy. But, in contrast to the 16.0 MeV case where the experimental cross section of [41] could be well reproduced, here our proton-reaction results clearly fall below the data. On the other hand, for ${}^2\text{H}(n,nn) {}^1\text{H}$ the Paris potential and our results practically coincide over the whole QFS peak region, with the exception of the peak height which is increased by the use of a more realistic NN input by approximately 10%. Consequently, by simply increasing the peak height of our proton-reaction cross section would not suffice to reproduce the experimental data. This may

indicate that for the QFS kinematics the effects resulting from the use of a realistic NN potential and from the inclusion of the Coulomb interaction are not even qualitatively additive.

Both proton- and neutron-induced breakup experiments have been performed for the COL configuration shown in Fig. 7. Also here we find sizeable Coulomb effects along the whole kinematic curve, leading to an increase of the proton- over the neutron-reaction cross section. As a result, a nice overall description of the corresponding experimental proton-reaction data is achieved. A similarly satisfactory description of the experimental pd breakup cross section measurements of [42] in a COL configuration at 14.1 MeV had already been found in [27]. Theoretically, at both energies the proton-reaction cross section lies above the one for neutrons, for all arc lengths. Experimentally this may possibly be the case around the COL point. But away from it the experimental trend is just opposite, the neutron-reaction cross section being larger than the one for protons. That is, here we find an interesting, even qualitative disagreement between experiment and theory which deserves further investigations from both theoretical and experimental sides. We point out that also the Paris potential results lie nearly everywhere markedly below the experimental neutron-reaction cross section (in fact, they are in general much closer to the experimental, and thus also to our calculated, cross section for ${}^2\text{H}(p,pp)n$).

For completeness we include in Fig. 8 the SST results [27]. As discussed there, the clear signal of the experiments, namely that the proton-reaction cross section is smaller than the one for neutron-induced breakup, is qualitatively reproduced by our calculations. However, quantitatively not only the absolute cross sections, but also our calculated Coulomb effects are much smaller than those shown by the data. The first observation does not come as a surprise since even the Paris potential results fall well below the experimental cross section for ${}^2\text{H}(n,nn){}^1\text{H}$. Nevertheless, adding to them a Coulomb contribution of the size found in our model would not reproduce the proton-reaction data.

4.3 Pure Coulomb breakup vs. CMSR contribution to the breakup amplitude.

An interesting question concerns the relative importance of the driving term $\mathcal{B}_{0d}^{(C)S(j)}$, which describes the pure Coulomb breakup and could be reduced analytically to a smooth one-dimensional integral, and the CMSR breakup amplitude $\mathcal{T}_{0d}^{(SC)S(j)}$ the calculation of which consumes most of the computing time. As follows from (59), their sum yields the total breakup amplitude $\mathcal{T}_{0d}^{S(j)}$. In Tables 7 and 8 we present their individual contributions for total three-nucleon spin $S = 3/2$ and $S = 1/2$, at 13.0 and 67.0 MeV, respectively. Several interesting observations can be made:

- (i) At the lower energy the real parts of both contributions are frequently of the same order of magnitude, while the imaginary parts differ in most cases by more than one order of magnitude.
- (ii) In the SST and QFS configurations the real parts differ in sign, so that the full amplitude arises by destructive interference. The situation is opposite for the COL and the np -FSI configurations: The two real parts interfere constructively (with one exception). Consequently, there are strong interference effects in the real, but practically none in the imaginary parts of these two breakup contributions.
- (iii) Depending on the configuration, the total Coulomb corrections arise in the various amplitudes from different sources. In some cases they originate mainly from the pure Coulomb breakup (whenever $\mathcal{T}_{0d}^{(SC)S(j)}$ approximately equals the amplitude $\mathcal{T}_{0d}^{S(j)}(nd)$ for the neutron reaction, which is also shown in the Table), in others from the Coulomb effects in the CMSR amplitudes (namely when the pure Coulomb breakup part $\mathcal{B}_{0d}^{(C)S(j)}$ is negligible). In still other cases the (full) pd amplitudes, i.e. the sum of $\mathcal{T}_{0d}^{(SC)S(j)}$ and $\mathcal{B}_{0d}^{(C)S(j)}$, practically coincide with the corresponding nd quantities, indicating that the Coulomb contributions from both these sources are of equal magnitude and essentially cancel each other. Thus, altogether, a rather

complex pattern of the amplitude modifications due to the inclusion of the Coulomb force becomes visible.

(iv) At 67.0 MeV the situation has changed: The driving term contribution has become relatively less important; and for all except one amplitudes the two real parts interfere destructively.

(v) At both energies considered, the quartet channel contributions are distinctly smaller than those for $S = 1/2$. Since the latter are known to be much more sensitive to the nuclear input, the discrepancies between the calculated and measured proton-reaction cross sections at these kinematic points are easily understandable.

The conclusion is that the qualitative behaviour of the two amplitude parts depends on the energy and on the angles of the detected particles in a non-trivial manner.

4.4 Coulomb contribution from the effective breakup potential vs. from the arrangement amplitude

Of practical importance is the question whether the time-consuming evaluation of the breakup potential is really unavoidable. This can be answered by calculating an approximate breakup amplitude where, roughly speaking, the pd breakup potential is replaced everywhere by the expression relevant for the neutron-induced reaction.

To be more precise, let us go back to (24) which expresses the charged-particle breakup amplitude by means of the arrangement amplitude. Substitute there for the Coulomb-modified breakup potential $V_{p'_3, \alpha d}^{(R)}(z)$ the one relevant for the neutral-particle reaction, i.e., by expression (30) with $T_3^R \equiv 0$,

$$V_{p'_3, \alpha d}(z) = \langle p'_3 | \chi_{\alpha m} \rangle. \quad (65)$$

On the energy shell (6), due to energy conservation the plane-wave matrix element of (65) vanishes. (This is most easily seen by replacing the plane waves $\langle q'_3, p'_3 |$ for the outgoing free particles by the equivalent representation $\langle q'_\alpha, p'_\alpha |$, appropriate for an α -channel partition.) Hence, on the energy shell we can define, in analogy to the exact expression (24), the approximate breakup amplitude

$$\begin{aligned} \tilde{T}_{p'_3, \alpha d}^{(R)}(q'_3, q_\alpha; E + i0) : &= \sum_{\gamma=1}^3 \sum_r \int d^3 q''_\gamma V_{p'_3, \gamma r}(q'_3, q''_\gamma; E + i0) G_{0, \gamma r}(q''_\gamma; E + i0) \times \\ &T_{\gamma r, \alpha d}^{(R)}(q''_\gamma, q_\alpha; E + i0), \end{aligned} \quad (66)$$

which contains all Coulomb effects arising from the (full) arrangement amplitude $T_{\gamma r, \alpha d}^{(R)}$, but none from the breakup potential. In other words, the final-state Coulomb distortion present in the exact charged-particle breakup potential (30) is omitted here. We note in parentheses that, because of the simple structure of the neutral-particle breakup potential (65), the integral in (66) can be trivially done.

By repeating the considerations, which for the full pd breakup amplitude lead to the conclusion that the unscreening limit (32) exists, one can convince oneself that also the limit

$$\lim_{R \rightarrow \infty} \tilde{T}_{0, \alpha d}^{(R)}(q'_3, p'_3, q_\alpha; E + i0) Z_{R, \alpha}^{-1/2}(q_\alpha) = \tilde{T}_{0, \alpha d}(q'_3, p'_3; q_\alpha), \quad (67)$$

exists. Note that, because *per constructionem* the charges of the particles in the final state are ignored, no renormalization is required there.

Investigation of the approximate breakup amplitude $\tilde{T}_{0, \alpha d}$ is particularly instructive. For, we only have to recall that, when solving the integral equations (26) for the full arrangement scattering amplitudes $T_{\beta n, \alpha d}^{(R)}$, the approximation $T_3^R \approx V_3^R$ had to be made. In contrast, the effective breakup potential (30) could be evaluated exactly, i.e. without such an approximation.

This has the following important consequence. Whenever it should happen that the pd breakup cross section, as calculated from the approximate amplitude (67), coincides with the one for the neutron-induced breakup, implying that the Coulomb effects from the arrangement amplitude are negligible, *our calculated proton-reaction cross section is effectively exact* (with respect to the Coulomb interaction). Or equivalently, should the proton-reaction cross sections calculated with (67) and with the 'exact' amplitude (32) coincide, all Coulomb effects arise from the arrangement part of the breakup amplitude, and hence are only as accurate as the latter.

In Figs. 9 - 12 we compare at 13.0 MeV the cross sections calculated from the approximate breakup amplitude (67) with those following from the exact amplitudes for the proton- and the neutron-induced breakup, for the four kinematic configurations introduced above. Inspection of Fig. 9 reveals that in the np -FSI configuration, the approximate and the 'exact' proton-reaction cross sections practically coincide, and, as already mentioned earlier, both are very close to the cross section for the neutron-induced breakup, except for the peak heights. The implication is that the Coulomb corrections from breakup potential and from the pd elastic scattering amplitude are individually small; that is, the smallness of the total Coulomb corrections does not arise as the result of a cancellation of the two contributions. We, thus, can conclude that the FSI cross section is practically not affected by the approximation made in the calculation of the pd elastic scattering amplitude, except for the peak height where the possibility of a small effect exists.

For the QFS kinematics it follows from Fig. 10 that the approximate proton-reaction cross section lies above, while the full pd breakup cross section lies below the one for the neutron reaction, by approximately the same amounts. This means that (i) the Coulomb effects both from the breakup potential and from the arrangement part of the breakup amplitude are large, the former being approximately twice as large as the latter, and (ii) both contributions shift the cross section into opposite directions so that the total Coulomb correction arises by destructive interference. A similar situation prevails in the SST configuration shown in Fig. 11. Hence, according to the above discussion the full breakup cross section results for the QFS and the SST kinematics are only moderately affected by the approximation made in the calculation of the pd elastic scattering amplitude.

In the COL configuration, Fig. 12, the situation is different. From the near coincidence of the 'exact' with the approximate pd breakup cross section we can infer that here practically the whole Coulomb correction stems from the arrangement amplitude, and nothing from the Coulomb modifications in the breakup potential. Consequently, the full proton-reaction cross section is only as accurate as the pd elastic scattering amplitude is.

In order to find out how these results depend on the energy, we present in Figs. 13 - 15 a similar investigation, but at 67.0 MeV, for the values of the kinematic variables described in Table 6. At this relatively high energy the picture has completely changed. In all three configurations considered, practically the whole Coulomb corrections arise from the Coulomb modifications of the breakup potential, indicating that the full pd cross sections obtained here are essentially exact. In other words, at such high energies it suffices to take into account the Coulomb final state interaction of the two emerging protons via the breakup potential!

4.5 Approximate evaluation of the effective breakup potential

We have already indicated that the computing time increases dramatically when the Coulomb interaction is included, because of the necessity of the time-consuming calculation of the half off-shell screened two-body Coulomb T-matrix for every point on the kinematic curve, and the subsequent three-dimensional quadrature in (61). Hence, prospects for performing in the near future pd breakup calculations, at the same level of sophistication as is possible nowadays for the neutron-induced reaction, look rather dim. It is, therefore, of interest to see whether some

approximate inclusion of the Coulomb potential may lead to acceptable estimates of the 'exact' Coulomb corrections.

The computing time could be reduced (by approximately 40%) if also in the effective breakup potential the replacement of the two-particle Coulomb amplitude by the Coulomb potential would give sufficiently accurate results. That is, if the breakup potential could be evaluated with the same approximation $T_3^R \approx V_3^R$ which is used for the calculation of the effective arrangement amplitudes.

To check this we have performed such approximate calculations. The corresponding cross section results for ${}^2\text{H}(p,pp)n$ are included in Figs. 9 - 15. At 13.0 MeV, in the np -FSI configuration the approximate and the 'exact' cross section are identical, so that they do not show up as different curves in Fig. 9. In the QFS configuration, Fig. 10, the approximate is rather close to the 'exact' cross section, only weakly underestimating the latter in the peak region. Both cross sections are even closer in the COL configuration shown in Fig. 16. But for the SST kinematics, Fig. 11, the approximate proton-reaction cross section lies nearer to the one for the neutron reaction than to the 'exact' pd result.

At 67.0 MeV, in all three cases investigated and shown in Figs. 13 - 15, the proton-induced breakup cross sections calculated with the full Coulomb T-matrix and with the latter approximated by the Coulomb potential, are very close.

The conclusions are that such a 'Born-type' calculation (i) for the higher energies, gives cross sections very close to the 'exact' ones for all three kinematic configurations investigated, and (ii) for the low energy, cross sections of high accuracy for the np -FSI, of reasonable accuracy for the COL and the QFS, and of insufficient accuracy for the SST configurations. Hence, exploratory investigations with more refined NN potentials might well be attempted by making use of this approximation.

These results may also serve as an indication of the quality of the approximation $T_3^R \approx V_3^R$ in the calculation of the pd elastic scattering amplitude, as function of the projectile energy.

4.6 pp - vs. nn -FSI configuration

The most dramatic effects of the inclusion of the Coulomb interaction are expected to arise in a pp - as compared to a nn -FSI region. Due to the repulsion of the equal charges one anticipates a vanishing of the ${}^2\text{H}(p,pp)n$ cross section at the pp -FSI point, in contrast to a strong enhancement at the nn -FSI point in the corresponding ${}^2\text{H}(n,nn) {}^1\text{H}$ reaction. This expectation is, in fact, corroborated by our calculation, the results of which are shown in Fig. 17. The numerically found vanishing of the pd breakup cross section, among other things, strongly supports our assertion that the zero-screening limit has effectively been reached in our calculations.

In a case of so extreme Coulomb effects the 'Born-type' approximation discussed above is not expected to work any longer. In fact, it still produces a peak at the FSI point, albeit of greatly reduced height. Furthermore, the approximation (67), where the Coulomb effects from the breakup potential are omitted and only those arising from the elastic scattering amplitude are included, already produces a dip at the FSI point, but is quantitatively still too large. Hence, in such a configuration the full pd breakup calculation can not be avoided.

5 Summary and Conclusions

We have used the 'screening and renormalization' approach to charged-particle scattering to provide the first, systematic investigation of the kinematically complete breakup reaction ${}^2\text{H}(p,pp)n$. Since the nuclear force model used is rather simple, in general a quantitative reproduction of the experimental data could not be expected. Nevertheless, at least a qualitative, in some cases even a semiquantitative description of experimental cross sections for the proton-induced deuteron breakup could be achieved. (One puzzle concerning the size of the pd relative to the nd breakup cross section remained.) Of greater interest is, however, that in this way a rather complex pattern of the Coulomb corrections, i.e. of the difference between the breakup cross sections for the proton- and the neutron-induced reaction, was uncovered.

For instance, the dependence of the Coulomb corrections on the kinematic configuration and on the projectile energy was investigated. At the lower energy (13.0 MeV), inclusion of the Coulomb interaction led to an enhancement of the differential cross sections for ${}^2\text{H}(p,pp)n$ with respect to the one for ${}^2\text{H}(n,nn) {}^1\text{H}$ in the np -FSI and the COL configuration, but to a decrease for the QFS, the SST and, of course, for the pp -FSI kinematics. At 67.0 MeV, on the other hand, the pd breakup cross sections came out reduced in comparison to the ones for the neutron reaction in all three configurations considered (COL, QFS and SST). One interesting consequence is that for the COL kinematics, at some intermediate energy proton- and neutron-reaction cross sections must become equal; that is, all Coulomb contributions must cancel each other!

Of interest is the size of the Coulomb corrections found. At 13.0 MeV, the np -FSI cross section turned out to be changed very little by the inclusion of the Coulomb interaction, in contrast to the QFS, COL and SST configurations where Coulomb effects ranging from appreciable to relatively large were found. In the latter three cases, even at the higher energy similarly strong Coulomb corrections could still be observed. That is, the influence of the Coulomb interaction does not die out quickly with the energy.

The following aspects are relevant from the computational point of view, which relied on the splitting of the full breakup amplitude into the pure Coulomb breakup and the Coulomb-modified short-range part. The latter contains the full interplay between the nuclear and the Coulomb interaction. For instance, it has been found that, at the lower energy, these two terms interfere destructively in the QFS and SST, but constructively in the np -FSI and COL configurations. At the higher energy, mainly destructive interference was observed, independent of the configuration.

Another result concerned the relative importance of the Coulomb contributions from the effective breakup potential and from the pd elastic scattering amplitude. Both together yield, via a three-dimensional quadrature, the full breakup amplitude. While the former has been evaluated exactly, in the calculation of the latter the approximation $T_3^R \approx V_3^R$ could not be avoided. From this investigation a 'hierarchy of reliability' (with respect to the inclusion of the Coulomb interaction) of the calculated pd breakup cross sections could be established, according to whether most of the Coulomb corrections came from the (exact) breakup potential or from the (approximate) arrangement amplitude. At 13.0 MeV, breakup cross sections in the np -FSI configuration can be considered to be essentially exact, and for the QFS and SST kinematics only moderately affected by this approximation. In the COL configuration, however, the accuracy of the cross section turns out to be limited by the accuracy with which the pd elastic scattering amplitude can be obtained. At 67.0 MeV, in all three kinematic configurations the calculated cross sections take into account the Coulomb interaction in essentially an exact manner.

Furthermore, the quality of the approximation $T_3^R \approx V_3^R$ in the evaluation of the breakup potential was investigated, with the result that for 13.0 MeV it is inadequate for the SST kinematics, but good to very good for the other three configurations; for 67.0 MeV it leads

to rather accurate cross sections for all three kinematic configurations investigated. That is, in nearly all cases the higher order terms in the iteration of the LS equation for T_3^R lead, as expected, only to a correction of the Coulomb corrections. This suggests a similar quality of this approximation also for the elastic amplitude.

Finally, very drastic Coulomb effects have been found for a pp versus nn -FSI configuration. Here, however, the breakup amplitude must be calculated as exactly as possible; that is, none of the investigated approximations was adequate.

At the end we add the plea for more measurements of neutron- and proton-reaction cross sections at identical kinematic conditions, if possible even outside of the few standard kinematic configurations. As it turns out, calculations which are based on a realistic description of the nuclear but ignore the Coulomb aspect of the interaction, do provide a respectable overall description of proton-induced breakup cross sections in many kinematic regions. But it goes beyond their potentiality to ascertain whether a given discrepancy is due to some (looked for) interesting detail of the nuclear or due to the neglect of the Coulomb forces, unless the former is known to yield a much larger contribution than the latter. Some clarification of this situation could be achieved if (i) the accuracy of our calculations - at least for the difference between the proton- and neutron-reaction cross sections - could be further established, and (ii) the - at least qualitative - additivity of the contributions from a more realistic NN input and from the Coulomb interaction could be verified, or falsified. Both these goals can, however, be accomplished only on the basis of more experimental data.

Acknowledgment

This work was supported by the Deutsche Forschungsgemeinschaft, Project No. Al 250/1-3.

A Evaluation of the Integral I

In eq. (53), the pure breakup amplitude $B_{p'_3, \alpha d}^{(C)}(q'_3, q_\alpha)$ was expressed through the integral

$$I(p_1, p_2, p'_1, p'_2, \lambda, \eta_1, \eta_2) = \int d^3 r_1 \int d^3 r_2 \frac{e^{i(p'_1 \cdot r_1 + p'_2 \cdot r_2)} e^{-\lambda|r_1 + r_2|}}{|r_1 + r_2|} \times \\ {}_1F_1(-i\eta_1; 1; i(p_1 r_1 + p_1 \cdot r_1)) {}_1F_1(-i\eta_2; 1; i(p_2 r_2 + p_2 \cdot r_2)), \quad (A.1)$$

where ${}_1F_1(a; b; z)$ is the confluent hypergeometric function [32].

Here we briefly present a technique to reduce this six-dimensional integral to a one-dimensional one with a smooth integrand by using different integral representations of the confluent and the Gauss hypergeometric functions.

For this purpose, we introduce the auxiliary integral

$$J_{\eta_1, \eta_2}(p_1, p_2, p'_1, p'_2, \lambda, \lambda_1, \lambda_2) = \int d^3 r_1 \int d^3 r_2 \frac{e^{i(p'_1 \cdot r_1 + p'_2 \cdot r_2)} e^{-\lambda|r_1 + r_2|}}{|r_1 + r_2|} \times \\ e^{-\lambda_1 r_1} e^{-\lambda_2 r_2} {}_1F_1(-i\eta_1; 1; i(p_1 r_1 + p_1 \cdot r_1)) {}_1F_1(-i\eta_2; 1; i(p_2 r_2 + p_2 \cdot r_2)), \quad (A.2)$$

which contains two additional parameters λ_1 and λ_2 . Thus, I can be expressed as

$$I(p_1, p_2, p'_1, p'_2, \lambda, \eta_1, \eta_2) = \lim_{\lambda_1 \rightarrow 0} \lim_{\lambda_2 \rightarrow 0} J_{\eta_1, \eta_2}(p_1, p_2, p'_1, p'_2, \lambda, \lambda_1, \lambda_2). \quad (A.3)$$

In the following it is shown how to evaluate the integral J . We start by inserting the integral representation of the confluent hypergeometric function ${}_1F_1$ (see, e.g., [43]),

$${}_1F_1(a; c; z) = \frac{\Gamma(c)}{\Gamma(a)\Gamma(c-a)} \int_0^1 dt t^{a-1} (1-t)^{c-a-1} e^{zt}, \quad (A.4)$$

(Γ is the Gamma function) in (A.2):

$$J_{\eta_1, \eta_2}(p_1, p_2, p'_1, p'_2, \lambda, \lambda_1, \lambda_2) = \frac{1}{\Gamma(-i\eta_1)\Gamma(1+i\eta_1)\Gamma(-i\eta_2)\Gamma(1+i\eta_2)} \times \\ \int_0^1 dt_1 \int_0^1 dt_2 t_1^{-i\eta_1-1} (1-t_1)^{i\eta_1} t_2^{-i\eta_2-1} (1-t_2)^{i\eta_2} \times \\ \int d^3 r_1 \int d^3 r_2 \frac{e^{i(p'_1 + p_1 t_1) \cdot r_1 - (\lambda_1 - i t_1 p_1) r_1} e^{i(p'_2 + p_2 t_2) \cdot r_2 - (\lambda_2 - i t_2 p_2) r_2}}{|r_1 + r_2|} e^{-\lambda|r_1 + r_2|}. \quad (A.5)$$

Expressing the last factor in the integrand of (A.5) by its Fourier transform

$$\frac{e^{-\lambda|r_1 + r_2|}}{|r_1 + r_2|} = \frac{1}{2\pi^2} \int d^3 x \frac{1}{x^2 + \lambda^2} e^{i(r_1 + r_2) \cdot x}, \quad (A.6)$$

we can write

$$J_{\eta_1, \eta_2}(p_1, p_2, p'_1, p'_2, \lambda, \lambda_1, \lambda_2) = \\ C(\eta_1, \eta_2) \int_0^1 dt_1 \int_0^1 dt_2 t_1^{-i\eta_1-1} (1-t_1)^{i\eta_1} t_2^{-i\eta_2-1} (1-t_2)^{i\eta_2} N(t_1, t_2). \quad (A.7)$$

Here we have introduced the abbreviations

$$C(\eta_1, \eta_2) = \frac{1}{\Gamma(-i\eta_1)\Gamma(1+i\eta_1)\Gamma(-i\eta_2)\Gamma(1+i\eta_2)} \quad (A.8)$$

and

$$N(t_1, t_2) = \frac{1}{2\pi^2} \int d^3x \frac{1}{x^2 + \lambda^2} \int d^3r_1 e^{i(\mathbf{x}-\mathbf{x}_1) \cdot \mathbf{r}_1} e^{-\mu_1 r_1} \int d^3r_2 e^{i(\mathbf{x}-\mathbf{x}_2) \cdot \mathbf{r}_2} e^{-\mu_2 r_2}, \quad (\text{A.9})$$

with

$$\mathbf{x}_j = -\mathbf{p}'_j - t_j \mathbf{p}_j, \quad (\text{A.10})$$

$$\mu_j = \lambda_j - it_j p_j, \quad \text{for } j = 1, 2. \quad (\text{A.11})$$

The integrand in (A.9) can be simplified with help of a two-fold parameter differentiation with respect to λ_1 and λ_2 :

$$N(t_1, t_2) = \frac{1}{2\pi^2} \frac{\partial^2}{\partial \lambda_1 \partial \lambda_2} \int d^3x \frac{1}{x^2 + \lambda^2} \times \int d^3r_1 \frac{1}{r_1} e^{i(\mathbf{x}-\mathbf{x}_1) \cdot \mathbf{r}_1} e^{-\mu_1 r_1} \int d^3r_2 \frac{1}{r_2} e^{i(\mathbf{x}-\mathbf{x}_2) \cdot \mathbf{r}_2} e^{-\mu_2 r_2}. \quad (\text{A.12})$$

Use of the Fourier transform

$$\int d^3r_j e^{i\mathbf{Q}_j \cdot \mathbf{r}_j} \frac{1}{r_j} e^{-\mu_j r_j} = \frac{4\pi}{\mu_j^2 + Q_j^2}, \quad j = 1, 2, \quad (\text{A.13})$$

then leads to

$$N(t_1, t_2) = \frac{(4\pi)^2}{2\pi^2} \frac{\partial^2}{\partial \lambda_1 \partial \lambda_2} \int d^3x \frac{1}{x^2 + \lambda^2} \frac{1}{(\mathbf{x} - \mathbf{x}_1)^2 + \mu_1^2} \frac{1}{(\mathbf{x} - \mathbf{x}_2)^2 + \mu_2^2}. \quad (\text{A.14})$$

As was shown in [44], the three-dimensional integral on the right-hand side can be reduced to a one-dimensional one:

$$N(t_1, t_2) = \frac{(4\pi)^2}{2\pi^2} \frac{\partial^2}{\partial \lambda_1 \partial \lambda_2} \int_0^\infty \frac{2\pi^2 ds}{\alpha s^2 + \beta s + \gamma}, \quad (\text{A.15})$$

where the coefficients α , β and γ are defined as

$$\alpha = \mu_2(\sqrt{z_1} + \sqrt{z_2}) [x_1^2 + (\lambda + \mu_1)^2], \quad (\text{A.16})$$

$$\beta = \mu_1 \mu_2 \lambda (2 + z_1 + z_2) + \mu_2 (\lambda^2 + x_1^2 + \mu_1^2) + \mu_1 (\lambda^2 + x_2^2 + \mu_2^2), \quad (\text{A.17})$$

$$\gamma = \mu_1(\sqrt{z_1} + \sqrt{z_2}) [x_2^2 + (\lambda + \mu_2)^2], \quad (\text{A.18})$$

$$z_{1,2} = \frac{(\mathbf{x}_1 - \mathbf{x}_2)^2 + \mu_1^2 + \mu_2^2 \mp \sqrt{[(\mathbf{x}_1 - \mathbf{x}_2)^2 + \mu_1^2 + \mu_2^2]^2 - 4\mu_1^2 \mu_2^2}}{2\mu_1 \mu_2}. \quad (\text{A.19})$$

Instead of performing the last integration as in [44] we make use of an idea developed in [33] in a similar context. Define the auxiliary variables $\mathbf{K}_j := \mathbf{p}_j$ and $\mathbf{P}_j := -\mathbf{p}'_j - \mathbf{p}_j$, for $j = 1, 2$. Then $\mathbf{x}_j$ in (A.10) can be written as

$$\mathbf{x}_j = (1 - t_j) \mathbf{K}_j + \mathbf{P}_j, \quad j = 1, 2. \quad (\text{A.20})$$

After insertion of (A.20) in (A.16) – (A.19) we can, by proceeding exactly as in [33], extract explicitly the dependence on the parameters t_1 and t_2 . This yields

$$N(t_1, t_2) = (4\pi)^2 \frac{\partial^2}{\partial \lambda_1 \partial \lambda_2} \int_0^\infty \frac{ds}{\sigma_{12}(s) t_1 t_2 + \sigma_2(s) t_2 + \sigma_1(s) t_1 + \sigma_0(s)}. \quad (\text{A.21})$$

The polynomials $\sigma_0(s)$, $\sigma_1(s)$, $\sigma_2(s)$, and $\sigma_{12}(s)$, which are then independent of t_1 and t_2 , are defined as follows,

$$\begin{aligned}\sigma_0(s) = & \left[(\lambda_1 + \lambda_2)^2 + (\mathbf{p}'_1 - \mathbf{p}'_2)^2 \right] (s + 2\lambda)s \\ & + 2 \left[\lambda_2(\lambda_1^2 + \lambda^2 + \mathbf{p}_1'^2) + \lambda_1(\lambda_2^2 + \lambda^2 + \mathbf{p}_2'^2) \right] s \\ & + \left[(\lambda_1 + \lambda)^2 + \mathbf{p}_1'^2 \right] \left[(\lambda_2 + \lambda)^2 + \mathbf{p}_2'^2 \right]\end{aligned}\quad (\text{A.22})$$

$$\begin{aligned}\sigma_1(s) = & -2 \left[i(\lambda_1 + \lambda_2)\mathbf{p}_1 + (\mathbf{p}'_2 - \mathbf{p}'_1) \cdot \mathbf{p}_1 \right] (s + 2\lambda)s \\ & -2 \left[i(\lambda_2^2 + \lambda^2 + \mathbf{p}_2'^2)\mathbf{p}_1 + 2\lambda_2(i\lambda_1\mathbf{p}_1 - \mathbf{p}'_1 \cdot \mathbf{p}_1) \right] s \\ & -2 \left[(\lambda_2 + \lambda)^2 + \mathbf{p}_2'^2 \right] \left[i(\lambda_1 + \lambda)\mathbf{p}_1 - \mathbf{p}'_1 \cdot \mathbf{p}_1 \right]\end{aligned}\quad (\text{A.23})$$

$$\begin{aligned}\sigma_2(s) = & -2 \left[i(\lambda_1 + \lambda_2)\mathbf{p}_2 + (\mathbf{p}'_1 - \mathbf{p}'_2) \cdot \mathbf{p}_2 \right] (s + 2\lambda)s \\ & -2 \left[i(\lambda_1^2 + \lambda^2 + \mathbf{p}_1'^2)\mathbf{p}_2 + 2\lambda_1(i\lambda_2\mathbf{p}_2 - \mathbf{p}'_2 \cdot \mathbf{p}_2) \right] s \\ & -2 \left[(\lambda_1 + \lambda)^2 + \mathbf{p}_1'^2 \right] \left[i(\lambda_2 + \lambda)\mathbf{p}_2 - \mathbf{p}'_2 \cdot \mathbf{p}_2 \right]\end{aligned}\quad (\text{A.24})$$

$$\begin{aligned}\sigma_{12}(s) = & -2 [\mathbf{p}_1\mathbf{p}_2 + \mathbf{p}_1 \cdot \mathbf{p}_2] (s + 2\lambda)s \\ & -4 \left[(\lambda_1\mathbf{p}_1 + i\mathbf{p}'_1 \cdot \mathbf{p}_1)\mathbf{p}_2 + (\lambda_2\mathbf{p}_2 + i\mathbf{p}'_2 \cdot \mathbf{p}_2)\mathbf{p}_1 \right] s \\ & +4 \left[i(\lambda_1 + \lambda)\mathbf{p}_1 - \mathbf{p}'_1 \cdot \mathbf{p}_1 \right] \left[i(\lambda_2 + \lambda)\mathbf{p}_2 - \mathbf{p}'_2 \cdot \mathbf{p}_2 \right].\end{aligned}\quad (\text{A.25})$$

After putting (A.21) back into (A.7) and performing some simple manipulations, the integral J becomes

$$J_{\eta_1, \eta_2}(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}'_1, \mathbf{p}'_2, \lambda, \lambda_1, \lambda_2) = \frac{(4\pi)^2}{\Gamma(-i\eta_1)\Gamma(1+i\eta_1)} \frac{\partial^2}{\partial \lambda_1 \partial \lambda_2} \int_0^\infty \mathcal{N}(s) ds, \quad (\text{A.26})$$

with $\mathcal{N}(s)$ defined as

$$\begin{aligned}\mathcal{N}(s) = & \frac{1}{\Gamma(-i\eta_2)\Gamma(1+i\eta_2)} \int_0^1 dt_1 \frac{t_1^{-i\eta_1-1}(1-t_1)^{i\eta_1}}{\sigma_1 t_1 + \sigma_0} \times \\ & \int_0^1 dt_2 \frac{t_2^{-i\eta_2-1}(1-t_2)^{i\eta_2}}{\sigma_1 t_2 + \sigma_0} \left(1 - t_2 \frac{-\sigma_{12}t_1 - \sigma_2}{\sigma_1 t_1 + \sigma_0} \right)^{-1}.\end{aligned}\quad (\text{A.27})$$

To simplify the notation, here and in the following the argument s of the polynomials $\sigma_j(s)$ is suppressed. This last integral can be handled by making use of the following integral representation of the Gauss hypergeometric function ${}_2F_1(a, b; c; z)$ [45],

$${}_2F_1(a, b; c; z) = \frac{\Gamma(c)}{\Gamma(b)\Gamma(c-b)} \int_0^1 dt t^{b-1} (1-t)^{c-b-1} (1-zt)^{-a}, \quad (\text{A.28})$$

which holds provided the conditions $\text{Re}(c) > \text{Re}(b) > 0$, $|\arg(1-z)| < \pi$ are satisfied, as it happens in the present case. The t_2 -integration in (A.27) can now be carried out, yielding

$$\begin{aligned}\mathcal{N}(s) = & \int_0^1 dt_1 \frac{t_1^{-i\eta_1-1}(1-t_1)^{i\eta_1}}{\sigma_1 t_1 + \sigma_0} {}_2F_1 \left(1, -i\eta_2; 1; -\frac{\sigma_{12}t_1 + \sigma_2}{\sigma_1 t_1 + \sigma_0} \right) \\ = & \int_0^1 dt_1 \frac{t_1^{-i\eta_1-1}(1-t_1)^{i\eta_1}}{\sigma_1 t_1 + \sigma_0} \left(1 + \frac{\sigma_{12}t_1 + \sigma_2}{\sigma_1 t_1 + \sigma_0} \right)^{i\eta_2},\end{aligned}\quad (\text{A.29})$$

where in the second step use has been made of

$${}_2F_1(1, a; 1; z) = (1 - z)^{-a}. \quad (\text{A.30})$$

To evaluate the remaining integral it proves advantageous to introduce, instead of t_1 , the new variable y as

$$y = y_s \left(\frac{1}{t_1} - 1 \right) \quad \text{with} \quad y_s := \frac{\sigma_0 + \sigma_2}{\sigma_{12} + \sigma_2 + \sigma_1 + \sigma_0}. \quad (\text{A.31})$$

This transformation maps the interval $0 < t_1 < 1$ onto the straight line

$$C : y \in \{r_s e^{i\varphi_s}, 0 \leq r_s < \infty\} \quad (\text{A.32})$$

in the complex plane, which forms an angle $\varphi_s = \arg(y_s)$ with the positive real axis. Here $r_s = |y_s|$. With the substitution (A.31), the integral in (A.29) can be expressed as

$$\begin{aligned} \mathcal{N}(s) = & \sigma_1^{-i\eta_2-1} (\sigma_1 + \sigma_{12})^{i\eta_2} b^{-(1+i\eta_1)} (1+b)^{1+i\eta_1+i\eta_2} (1+a)^{-(1+i\eta_2)} \times \\ & \int_C dy y^{(1+i\eta_1)-1} (1+y)^{(1+i\eta_2)-1} \left(1 + \frac{a(1+b)}{b(1+a)} y \right)^{-(1+i\eta_2)}, \end{aligned} \quad (\text{A.33})$$

with

$$a = \sigma_0/\sigma_1, \quad b = (\sigma_0 + \sigma_2)/(\sigma_1 + \sigma_{12}). \quad (\text{A.34})$$

Due to the last factor, the integrand in (A.33) has a branch point at

$$y_0 = -\frac{\sigma_1 + \sigma_0}{\sigma_0} y_s = \left| \frac{\sigma_1 + \sigma_0}{\sigma_0} \right| r_s e^{i(\varphi_s + \psi + \pi)}, \quad \text{with} \quad \psi = \arg\left(\frac{\sigma_1 + \sigma_0}{\sigma_0}\right). \quad (\text{A.35})$$

Its location in the complex plane is, however, outside of the area which is enclosed by the positive real axis and the curve C . Thus, by means of *Cauchy's Theorem* the integration along C in (A.33) can be replaced by an integration along the positive real axis,

$$\begin{aligned} \mathcal{N}(s) = & \sigma_1^{-i\eta_2-1} (\sigma_1 + \sigma_{12})^{i\eta_2} b^{-(1+i\eta_1)} (1+b)^{1+i\eta_1+i\eta_2} (1+a)^{-(1+i\eta_2)} \times \\ & \int_0^\infty dy y^{(1+i\eta_1)-1} (1+y)^{(1+i\eta_2)-1} \left(1 + \frac{a(1+b)}{b(1+a)} y \right)^{-(1+i\eta_2)}. \end{aligned} \quad (\text{A.36})$$

This integral can now be performed by using another integral representation of the Gauss hypergeometric function [43]

$${}_2F_1(\alpha, \beta; \gamma; z) = \frac{\Gamma(\gamma)}{\Gamma(\beta)\Gamma(\gamma-\beta)} \int_0^\infty dt t^{\beta-1} (1+t)^{\alpha-\gamma} (1+zt)^{-\alpha}. \quad (\text{A.37})$$

Thus, one finds

$$\begin{aligned} \mathcal{N}(s) = & \sigma_1^{-i\eta_2-1} (\sigma_1 + \sigma_{12})^{i\eta_2} b^{-(1+i\eta_1)} (1+b)^{1+i\eta_1+i\eta_2} (1+a)^{-(1+i\eta_2)} \times \\ & \Gamma(1+i\eta_1)\Gamma(-i\eta_1) {}_2F_1\left(1+i\eta_2, 1+i\eta_1; 1; 1 - \frac{a(1+b)}{b(1+a)}\right). \end{aligned} \quad (\text{A.38})$$

On using (A.34) the following relations can easily be verified

$${}_2F_1(\alpha, \beta; \gamma; z) = (1-z)^{\gamma-\alpha-\beta} {}_2F_1(\gamma-\beta, \gamma-\alpha; \gamma; z), \quad (\text{A.39})$$

$$1 - \frac{a(1+b)}{b(1+a)} = \frac{\sigma_1\sigma_2 - \sigma_0\sigma_{12}}{(\sigma_0 + \sigma_1)(\sigma_0 + \sigma_2)}, \quad (\text{A.40})$$

$$(\sigma_0 + \sigma_2)^{i\eta_2} = (\sigma_1 + \sigma_{12})^{i\eta_2} b^{i\eta_2}, \quad (\text{A.41})$$

$$\sigma_0^{-1-i\eta_2} \left(\frac{\sigma_0}{\sigma_0 + \sigma_1} \right)^{-i\eta_1} = \sigma_1^{-i\eta_2-1} a^{-1-i\eta_1-i\eta_2} (a+1)^{i\eta_1}. \quad (\text{A.42})$$

With their help one obtains the final result for $\mathcal{N}(s)$:

$$\mathcal{N}(s) = \frac{\Gamma(1+i\eta_1)\Gamma(-i\eta_1)}{\sigma_1} \left(\frac{\sigma_0}{\sigma_0+\sigma_1}\right)^{-i\eta_1} \left(\frac{\sigma_0}{\sigma_0+\sigma_2}\right)^{-i\eta_2} \times {}_2F_1\left(-i\eta_1, -i\eta_2; 1; \frac{\sigma_1\sigma_2 - \sigma_{12}\sigma_0}{(\sigma_1+\sigma_0)(\sigma_2+\sigma_0)}\right). \quad (\text{A.43})$$

Introducing now (A.43) into (A.26) yields for J the representation

$$J_{\eta_1, \eta_2}(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}'_1, \mathbf{p}'_2, \lambda, \lambda_1, \lambda_2) = (4\pi)^2 \frac{\partial^2}{\partial \lambda_1 \partial \lambda_2} \int_0^\infty \frac{ds}{\sigma_0(s)} \left[\frac{\sigma_0(s)}{\sigma_0(s) + \sigma_1(s)} \right]^{-i\eta_1} \times \left[\frac{\sigma_0(s)}{\sigma_0(s) + \sigma_2(s)} \right]^{-i\eta_2} {}_2F_1\left(-i\eta_1, -i\eta_2; 1; \frac{\sigma_1(s)\sigma_2(s) - \sigma_{12}(s)\sigma_0(s)}{(\sigma_1(s) + \sigma_0(s))(\sigma_2(s) + \sigma_0(s))}\right). \quad (\text{A.44})$$

The integrand is a smooth and square-integrable function. Hence, it is possible to interchange the two-fold partial differentiations and the integration. Finally, the limits $\lim_{\lambda_1 \rightarrow 0}$ and $\lim_{\lambda_2 \rightarrow 0}$ have to be taken according to (A.3) to obtain a closed formula for the integral $I(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}'_1, \mathbf{p}'_2, \lambda, \eta_1, \eta_2)$ in terms of a one-dimensional integral over a smooth integrand.

B Spin recoupling matrices

The spin recoupling matrices introduced in (62) and (63) can be calculated by standard methods. We point out that in the matrices $\Lambda^{S(n)}$, $n = d, s$ sign factors and a factor 2, both arising from antisymmetrization, have been included.

The columns of the matrices are characterized by the indices $(\alpha m) = (1d), (2d), (1s), (2s), (3c)$, and the rows by $m = d, s, c$.

a) $S = 3/2$. The only non-vanishing elements are

$$\Lambda_{d(1d)}^{\frac{3}{2}(d)} = -\Lambda_{d(2d)}^{\frac{3}{2}(d)} = 1. \quad (\text{B.1})$$

b) $S = 1/2$.

$$\Lambda^{\frac{1}{2}(d)} = \begin{pmatrix} 1 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2}\sqrt{3} & 0 \\ 0 & 0 & 0 & 0 & -\sqrt{3} \end{pmatrix}, \quad \Lambda^{\frac{1}{2}(s)} = \begin{pmatrix} 0 & \frac{1}{2}\sqrt{3} & 0 & 0 & 0 \\ 0 & 0 & 1 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & -1 \end{pmatrix}. \quad (\text{B.2})$$

References

- [1] Faddeev, L. D.: Zh. Eksp. Teor. Fiz. **39** , 1459 (1960) [Sov. Phys. - JETP **12** , 1014 (1961)]
- [2] Aaron, R., Amado, R. D.: Phys. Rev. **150**, 857 (1966)
- [3] Glöckle, W., Witala, H., Cornelius, T.: Few-Body Systems **3**, 1 (1988)
- [4] Frank, T. N., Haberzettl, H., Januschke, T., Kerwath, U., Sandhas, W.: Phys. Rev. **C38**, 1112 (1988)
- [5] Witala, H., Glöckle, W., Cornelius, T.: Phys. Rev. **C39**, 384 (1989)
- [6] Glöckle, W., Witala, H., Cornelius, T.: Nucl. Phys. **A508**, 115c (1990)
- [7] Kröger, H.: J. Math. Phys. **24** 1509 (1983)
- [8] Kröger, H., Nachabe, A. M., Slobodrian, R. J.: Phys. Rev. **C32** 628 (1985)
- [9] Kröger, H., Nachabe, A. M., Slobodrian, R. J.: Phys. Rev. **C33** 1208 (1986)
- [10] Rauprich, G., Lemaitre, S., Niessen, P., Nyga, K. R., Reckenfelderbäumer, R., Sydow, L., Paetz gen. Schieck, H., Witala, H., Glöckle, W.: Nucl. Phys. **A535**, 313 (1991)
- [11] Strate, J., Geissdörfer, K., Lin, R., Bielmeier, W., Cub, J., Ebneith, A., Finckh, E., Friess, H., Fuchs, G., Gebhardt, K., Schindler, S.: Nucl. Phys. **A501**, 51 (1989)
- [12] Veselova, A. M.: Theor. Math. Phys. **3**, 542 (1970); Theor. Math. Phys. **35**, 395 (1978); in: Few Particle Problems in Nuclear Physics, pg. 326. Dubna 1980.
- [13] Alt, E. O., Sandhas, W.: Phys. Rev. **C18**, 1088 (1980)
- [14] Merkuriev, S. P.: Ann. Phys. (NY) **130**, 395 (1980); Acta Phys. Austriaca, Suppl. XXIII, 65 (1981)
- [15] Alt, E. O., Mukhamedzhanov, A. M.: JETP Lett. **56**, 435 (1992); Phys. Rev. **A47**, 2004 (1993)
- [16] Kvitsinskii, A. A., Latypov, D. M.: Sov. J. Nucl. Phys. **53**, 953 (1991)
- [17] Kuperin, Yu. A., Merkuriev, S. P., Kvitsinskii, A. A.: Sov. J. Nucl. Phys. **37**, 857 (1983)
- [18] Alt, E. O., Sandhas, W., Ziegelmann, H.: Phys. Rev. **C17**, 1981 (1978)
- [19] Alt, E. O.: in: Few Body Nuclear Physics (Pisent, G., Vanzani, V., Fonda, L., eds.), pp. 271. Vienna: IAEA 1978.
- [20] Alt, E. O.: in: Few Particle Problems (Mitra, A. N., Slaus, I., Bhasin V. S., Gupta, V. K., ed.), pp. 76. Amsterdam: North Holland (1976)
- [21] Alt, E. O., Sandhas, W., Zankel, H., Ziegelmann, H.: Phys. Rev. Lett. **37**, 1537 (1976)
- [22] Alt, E. O., Sandhas, W., Ziegelmann, H.: Nucl. Phys. **A445**, 429 (1985)
- [23] Berthold, G. H., Zankel, H.: Phys. Rev. **C34**, 1203 (1986)
- [24] Berthold, G. H., Stadler, A., Zankel, H.: Phys. Rev. Lett. **61**, 1077 (1988)
- [25] Berthold, G. H., Stadler, A., Zankel, H.: Phys. Rev. **C41**, 1365 (1990)

- [26] Alt, E. O., Rauh, M.: Few-Body Systems, Suppl. 7, 160 (1994)
- [27] Alt, E. O., Rauh, M.: Phys. Rev. C , (1994)
- [28] Alt, E. O., Grassberger, P., Sandhas, W.: Nucl. Phys. B2, 167 (1967)
- [29] Semon, E. D., Taylor, J. R.: Nuovo Cim. A26, 48 (1975)
- [30] Goodmanson, D. M., Taylor, R. J.: J. Math. Phys. 21, 2202 (1980)
- [31] Gorshkov, V. G.: Sov. Phys.-JETP 13, 1037 (1961)
- [32] Abramowitz, M., Stegun, I. A.: Handbook of Mathematical Functions. New York: Dover Publications 1970.
- [33] Brauner, M., Briggs, J. S., Klar, H.: J. Phys. B: At. Mol. Phys. 22, 2265 (1989)
- [34] Bajzer, Z.: Phys. Lett. 84B, 289 (1979); Few-Body Systems 2, 9 (1987)
- [35] Ohlsen, G. G.: Nucl. Instr. and Meth. 37, 240 (1965)
- [36] Kok, L. P., van Haeringen, H.: Phys. Rev. C21, 512 (1980)
- [37] Kok, L. P., van Haeringen, H.: Czech. J. Phys. B32, 311 (1982)
- [38] Allet, M., Bodek, K., Hajdas, W., Jarczyk, L., Kystrin, St., Lang, J., Müller, R., Navert, S., Naviliat-Cuncic, O., Smyrski, J., Sromicki, J., Strzałkowski, A., Zejma, J.: Helv. Phys. Acta 66, 437 (1993)
- [39] Dechant, B.: Thesis, Ruhr-Universität Bochum, Bochum, 1992
- [40] Witala, H., Cornelius, T., Glöckle, W.: Few-Body Systems 5, 89 (1988)
- [41] Klein, H., Eichner, H., Helten, H.J., Kretzer, H., Prescher, K., Stehle, H., Wohlfarth, W.: Nucl. Phys. A 199, 169 (1973)
- [42] Karus, M., Buballa, M., Helten, J., Laumann, B., Melzer, R., Niessen, P., Oswald, H., Rauprich, G., Schulte-Uebbing, J., Paetz gen. Schieck, H.: Phys. Rev C 31, 1112 (1985)
- [43] Erdelyi, A., Magnus, W., Oberhettinger, F., Triconi, F. G.: Higher Transcendental Functions, Vol. 1. New York: McGraw-Hill 1953
- [44] Lewis jr., R. R.: Phys. Rev. 102 537 (1956)
- [45] Magnus, W., Oberhettinger, F., Soni, R. P.: Formulas and theorems for the special functions of mathematical physics. New York: Springer 1966

np -system	input parameters	parameters of the separable potentials
d	$\tilde{E}_d = 2.226 \text{ MeV}$ $a_{np}^t = 5.41 \text{ fm}$	$\lambda_d = -0.3817 \text{ fm}^{-3}$ $\beta_d = 1.4055 \text{ fm}^{-1}$
s	$a_{np}^s = -23.78 \text{ fm}$ $r_{np}^s = 2.67 \text{ fm}$	$\lambda_s = -0.1542 \text{ fm}^{-3}$ $\beta_s = 1.1771 \text{ fm}^{-1}$

Table 1: Parameters of the separable nuclear S-wave potentials for the np -subsystem.

pp -system	input parameters	calculated quantities	
c	$a_{pp}^{SC} = -7.71 \text{ fm}$ $r_{pp}^{SC} = 2.86 \text{ fm}$	$\lambda_c = -0.1140 \text{ fm}^{-3}$ $\beta_c = 1.075 \text{ fm}^{-1}$	$a_{nn} = -17.85 \text{ fm}$ $r_{nn} = 2.985 \text{ fm}$

Table 2: Parameters of the separable nuclear S-wave potential which, together with Coulomb potential, describes pp -scattering.

	np -FSI	QFS	COL	SST
	$E_S = 10.58 \text{ MeV}$	$E_S = 8.13 \text{ MeV}$	$E_S = 7.71 \text{ MeV}$	$E_S = 6.39 \text{ MeV}$
$R \text{ [fm]}$	$d^5\sigma/d\Omega_1^L d\Omega_2^L dE_S \text{ [mb MeV}^{-1} \text{sr}^{-2}]$			
1	6.40	2.09	0.907	0.711
10	6.46	2.38	0.950	0.761
50	6.73	2.83	0.943	0.759
100	6.77	2.90	0.932	0.756
200	6.79	2.92	0.928	0.755

Table 3: Differential pd -breakup cross section for various kinematic configurations at 13.0 MeV, as function of the screening radius R .

configuration	θ_1^L [deg]	θ_2^L [deg]	ϕ_1^L [deg]	ϕ_2^L [deg]	E_S [MeV]
np -FSI	39.0	62.5	0.0	180.0	3.32, 10.58
QFS	39.0	39.0	0.0	180.0	8.13
COL	50.5	62.5	0.0	180.0	7.71
SST	50.5	50.5	0.0	120.0	6.39

Table 4: Angles of the detected particles and arc length E_S along the kinematically allowed curve at 13.0 MeV where the np -FSI, QFS, COL, and SST conditions are satisfied.

	QFS	COL	SST
	$E_S = 48.2$ MeV	$E_S = 45.7$ MeV	$E_S = 31.4$ MeV
R [fm]	$d^5\sigma/d\Omega_1^L d\Omega_2^L dE_S$ [mb MeV $^{-1}$ sr $^{-2}$]		
1	1.053	0.0107	0.0299
10	1.098	0.0123	0.0306
50	1.106	0.0121	0.0307
100	1.104	0.0119	0.0307
200	1.103	0.0118	0.0307

Table 5: Same as in Fig. 3, but at 67.0 MeV.

configuration	θ_1^L [deg]	θ_2^L [deg]	ϕ_1^L [deg]	ϕ_2^L [deg]	E_S [MeV]
QFS	44.0	44.0	0.0	180.0	48.2
COL	60.0	60.0	0.0	180.0	45.7
SST	54.0	54.0	0.0	120.0	31.4

Table 6: Same as in Fig. 4, but at 67.0 MeV.

	np -FSI	QFS	COL	SST
E_S [MeV]	10.58	8.13	7.71	6.39
$B_{0d}^{(C)\frac{1}{2}(d)}$ [fm $^{-1}$]	$-9.19 - i 1.93$	$0.43 + i 0.19$	$-0.03 - i 0.06$	$-0.45 - i 0.14$
$T_{0d}^{(SC)\frac{1}{2}(d)}$ [fm $^{-1}$]	$-11.17 - i 15.56$	$-1.19 - i 1.62$	$-3.70 - i 12.94$	$0.67 + i 1.70$
$T_{0d}^{\frac{1}{2}(d)}(nd)$ [fm $^{-1}$]	$-11.86 - i 17.22$	$-1.32 - i 1.26$	$-5.57 - i 12.93$	$0.98 + i 1.43$
$B_{0d}^{(C)\frac{1}{2}(d)}$ [fm $^{-1}$]	$7.51 + i 0.25$	$-17.33 - i 4.21$	$6.99 + i 0.32$	$5.66 - i 0.60$
$T_{0d}^{(SC)\frac{1}{2}(d)}$ [fm $^{-1}$]	$-73.38 - i 75.44$	$80.53 + i 36.49$	$9.15 - i 30.38$	$-9.81 - i 41.36$
$T_{0d}^{\frac{1}{2}(d)}(nd)$ [fm $^{-1}$]	$-65.72 - i 73.60$	$64.13 + i 42.41$	$10.29 - i 28.50$	$-7.64 - i 43.04$
$B_{0d}^{(C)\frac{1}{2}(s)}$ [fm $^{-1}$]	$9.64 + i 1.26$	$-10.25 - i 2.54$	$4.05 + i 0.21$	$3.53 + i 0.43$
$T_{0d}^{(SC)\frac{1}{2}(s)}$ [fm $^{-1}$]	$63.52 + i 24.56$	$46.12 + i 21.31$	$0.16 - i 14.61$	$-5.17 - i 23.40$
$T_{0d}^{\frac{1}{2}(s)}(nd)$ [fm $^{-1}$]	$63.12 + i 21.61$	$36.61 + i 24.19$	$1.29 - i 13.12$	$-3.69 - i 23.94$

Table 7: Pure Coulomb vs. CMSR pd breakup amplitude at 13.0 MeV, for different kinematic configurations. The arc length E_S follows from Table 4.

	QFS	COL	SST
E_S [MeV]	48.2	45.7	31.4
$B_{0d}^{(C)\frac{1}{2}(d)}$ [fm $^{-1}$]	$0.000 + i 0.002$	$0.015 + i 0.004$	$0.016 + i 0.007$
$T_{0d}^{(SC)\frac{1}{2}(d)}$ [fm $^{-1}$]	$-0.019 + i 0.008$	$0.004 + i 0.001$	$-0.190 + i 0.093$
$T_{0d}^{\frac{1}{2}(d)}(nd)$ [fm $^{-1}$]	$-0.018 + i 0.010$	$0.002 + i 0.002$	$-0.171 + i 0.109$
$B_{0d}^{(C)\frac{1}{2}(d)}$ [fm $^{-1}$]	$-4.83 - i 0.83$	$0.42 + i 0.14$	$0.22 + i 0.05$
$T_{0d}^{(SC)\frac{1}{2}(d)}$ [fm $^{-1}$]	$25.77 + i 17.38$	$-2.85 + i 0.61$	$-5.54 - i 0.98$
$T_{0d}^{\frac{1}{2}(d)}(nd)$ [fm $^{-1}$]	$24.76 + i 17.48$	$-2.57 + i 0.70$	$-5.47 - i 0.83$
$B_{0d}^{(C)\frac{1}{2}(s)}$ [fm $^{-1}$]	$-2.788 - i 0.478$	$0.236 + i 0.079$	$0.118 + i 0.025$
$T_{0d}^{(SC)\frac{1}{2}(s)}$ [fm $^{-1}$]	$14.88 + i 10.03$	$-1.45 + i 0.33$	$-3.18 - i 0.66$
$T_{0d}^{\frac{1}{2}(s)}(nd)$ [fm $^{-1}$]	$14.30 + i 10.09$	$-1.49 + i 0.39$	$-3.15 - i 0.53$

Table 8: Same as in Fig. 7, but at 67.0 MeV. The arc length E_S follows from Table 6.

Figure 1: Diagrammatical representation of the exact effective breakup potential (30).

Figure 2: Diagrammatical representation of the full breakup amplitude (45) as sum of the pure Coulomb and the CMSR breakup amplitude. The Coulomb distortions between the charged particles in the initial and final state are characterized by shaded blobs.

Figure 3: ${}^2\text{H}(p,pp)n$ cross section at 13.0 MeV proton laboratory energy for a QFS configuration, against the arc length E_S along the kinematically allowed curve, calculated for screening radii $R = 50$ fm (long-dashed line), $R = 100$ fm (dashed line) and $R = 200$ fm (solid line). The arrow indicates the location, where the QFS condition is satisfied.

Figure 4: As in Fig. 3, but for a SST configuration. The space star point is marked by an arrow.

Figure 5: Cross section for ${}^2\text{H}(p,pp)n$ (solid line) vs. ${}^2\text{H}(n,nn){}^1\text{H}$ (dashed line), for a np -FSI configuration at 13.0 MeV, as function of the arc length E_S . The FSI points are marked by arrows. Experimental data: ($\square$) for ${}^2\text{H}(p,pp)n$ [10], ($\circ$) for ${}^2\text{H}(n,nn){}^1\text{H}$ [11]. Full triangles: selected Paris potential results for ${}^2\text{H}(n,nn){}^1\text{H}$ [11].

Figure 6: As in Fig. 5, but for a QFS configuration. The QFS point is indicated by an arrow.

Figure 7: As in Fig. 5, but for a COL configuration. The collinearity point is marked by an arrow.

Figure 8: As in Fig. 5, but for a SST configuration. The arrow marks the space star point.

Figure 9: ${}^2\text{H}(p,pp)n$ cross section at 13.0 MeV for a np -FSI configuration, against the arc length E_S , calculated with the full amplitude (32) (solid line), with the approximate amplitude (67) in which the effective pd breakup potential is replaced by the one for the neutron-induced reaction (long-dashed line), and in the approximation $T_3^R \approx V_3^R$ for the effective breakup-potential (dash-dotted line). The latter falls practically on top of the solid line. For comparison the ${}^2\text{H}(n,nn){}^1\text{H}$ cross section is included (dotted line). The arrows mark the FSI points.

Figure 10: As in Fig. 9, but for a QFS configuration. The arrow indicates the QFS point. Note the blown-up scale.

Figure 11: As in Fig. 9, but for a SST configuration. The arrow marks the point with SST geometry. Note the blown-up scale.

Figure 12: ${}^2\text{H}(p,pp)n$ cross section at 13.0 MeV for a collinear configuration (COL), against the arc length E_S , calculated with the full amplitude (32) (solid line), and with the approximate amplitude (67) in which the effective pd breakup potential is replaced by the one for the neutron-induced reaction (long-dashed line). The neutron-reaction cross section is included for comparison (dotted line). The arrow indicates the COL point.

Figure 13: ${}^2\text{H}(p,pp)n$ cross section at 67.0 MeV for a QFS configuration. See Fig. 9 for the explanation of the different curves. The arrow marks the QFS point.

Figure 14: ${}^2\text{H}(p,pp)n$ cross section at 67.0 MeV for a COL configuration. See Fig. 9 for the explanation of the different curves. The collinearity point is marked by an arrow.

Figure 15: ${}^2\text{H}(p,pp)n$ cross section at 67.0 MeV for a SST geometry. The explanation of the different curves is the same as in Fig. 9. The SST point is shown by an arrow.

Figure 16: As in Fig. 12, except that here the approximate proton-reaction cross section calculated in the approximation $T_3^R \approx V_3^R$ for the effective breakup potential is included (dash-dotted line).

Figure 17: ${}^2\text{H}(p,pp)n$ cross section at 13.0 MeV for a pp -FSI configuration, as function of the arc length E_s . The explanation of the different curves is the same as in Fig. 9. The arrow marks the QFS point.

Alt, Rauh: The proton-induced deuteron breakup reaction ${}^2\text{H}(\text{p},\text{p})\text{n}$: Fig.2

Alt, Rauh: The proton-induced deuteron breakup reaction ${}^2\text{H}(p,pp)n$: Fig.3

Alt, Rauh: The proton-induced deuteron breakup reaction $^2\text{H}(p,pp)n$: Fig.4

Alt, Rauh: The proton-induced deuteron breakup reaction ${}^2\text{H}(p,pp)n$: Fig.5

Alt, Rauh: The proton-induced deuteron breakup reaction $^2\text{H}(p,pp)n$: Fig.5

Alt, Raun: The proton-induced deuteron breakup reaction $^2\text{H}(p,pp)n$: Fig.7

Alt, Rauh: The proton-induced deuteron breakup reaction $^2\text{H}(p,pp)n$: Fig.3

Alt, Rauh: The proton-induced deuteron breakup reaction ${}^2\text{H}(p,pp)n$: Fig.9

Alt, Rauh: The proton-induced deuteron breakup reaction ${}^2\text{H}(p,pp)n$: Fig.10

Alt, Rauh: The proton-induced deuteron breakup reaction $^2\text{H}(p,pp)n$: Fig.11

Alt, Rauh: The proton-induced deuteron breakup reaction $^2\text{H}(p,pp)n$: Fig.12

Alt, Rauh: The proton-induced deuteron breakup reaction $^2\text{H}(p,pp)n$: Fig.13

Alt, Rauh: The proton-induced deuteron breakup reaction $^2\text{H}(p,pp)n$: Fig.14

Alt, Rath: The proton-induced deuteron breakup reaction $^2\text{H}(p,pp)n$: Fig.15

Alt, Rauh: The proton-induced deuteron breakup reaction $^2\text{H}(p,pp)n$: Fig.16

Alt, Rauh: The proton-induced deuteron breakup reaction ${}^2\text{H}(p,pp)n$: Fig.17