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Abstract

This paper presents the mathematical optimization techniques applied in order to find the most appropriate design for the superconducting bending magnets for CERN's new project, the Large Hadron Collider. It describes the program package ROXIE and the POISOPT routines which are extensions of the FE-program package POISSON. The nature of vector-optimization problems with a number of partly contradictory objectives for the optimal design are explained. These vector-optimization problems lead to a set of Pareto-optimal solutions from which one compromise solution has to be chosen by a decision making process. The paper describes different methods for decision making and suitable optimization procedures.

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Mathematical Optimization of Superconducting Accelerator Magnets

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Abstract - This paper presents the mathematical optimization techniques applied in order to find the most appropriate design for the superconducting bending magnets for CERN's new project, the Large Hadron Collider. It describes the program package ROXIE and the POISOPT routine which is an extension of the finite element program package POISSON. The nature of vector-optimization problems with a number of partly contradictory objectives for the optimal design is explained. These vector-optimization problems lead to a set of Pareto-optimal solutions from which one compromise solution has to be chosen by a decision making process. The paper describes different methods for decision making and suitable optimization procedures.

I. THE COIL BLOCK OPTIMIZATION

The Large Hadron Collider (LHC) project is proposed as the future extension of the CERN accelerator complex. The LHC requires twin aperture superconducting dipoles of highest possible field to guide the proton beams in the existing LEP tunnel of 26.7 km circumference [1]. Superconducting accelerator magnets are characterized by the dominant influence of coil geometry on the field distribution. Their maximum field is determined by the limiting current density in the conductor given by the field strength to which the cable is exposed. The high currents, the very large forces and the remanence effects due to persistent currents make the design for these magnets a difficult optimization problem. While the analytical and numerical methods used for the field computations are basically well established, the task of finding optimum solutions leaves open a wide horizon for methodical developments. For the LHC dipole magnets the design computations start by optimizing the coil geometry. This is achieved by using analytical integration for computing the coil contributions in a yoke which first is considered as having infinite permeability. For these purposes the program package ROXIE (routine for the optimization of magnet X- sections inverse field calculation and end region design) has been developed at CERN.

Mathematical optimization techniques have been applied in order to find a coil block arrangement with low multipole errors, high dipole field and a working point not higher than 95 % with respect to the load line. The main dipoles have Rutherford type trapezoidal superconducting cables of different size for the two layers of the coil. The cables are made of NbTi/Cu composite strands. The position of each cable is evaluated with respect to its height, width and keystone angle. The fact that the keystone of the cables is not sufficient to allow their edges to be positioned on the curvature of a circle, is fully respected. This effect increases with the inclination of the coil blocks versus the radial direction. Each cable is then approximated by line currents carrying the same current located at equally spaced radial and azimuthal positions inside the cable. By this semi-analytical method the grading of the current density due to keystone is accounted for. By applying Biot-Savart's law for each line current, the field is calculated at a given radius in the aperture (10 mm) and in the coil itself. The design variables are the angles φ_i and the inclination angle α_i of the blocks (cf. Fig. 1).

The optimization problem for the dipole cross-section is given by

$$\min \sum_{i=3,5,7,9} p_i \cdot b_i(\vec{X})^2 \quad (1)$$

with $\vec{X} = (\varphi_{i=2,4,5}, \alpha_{i=2,4,5}, I)$ subject to nonlinear constraints

$$B_1(\vec{X}) > 9; \quad r_{JB}^i(\vec{X}) < 95; \quad r_{JB}^o(\vec{X}) < 92 \quad (2)$$

and in addition lower and upper bounds for the design variables arranged in the design variable vector $\vec{X}$. The $b_i(\vec{X})$ are the relative normal multipoles and p_i are weighting factors to compensate for the different sensitivity of the multipoles. Because of the symmetry there appear no skew components. $B_1(\vec{X})$ is the dipole field in Tesla and $r_{JB}(\vec{X})$ the relative performance (in %) with respect to the load line for inner (i) and outer (o) layer. r_{JB} is calculated from the current density in the superconductor and the short sample $B - J$ characteristic curves at 1.8 Kelvin. The constraints imply that the quench starts in the inner layer. As the load line constraints are a function of the current I , it is considered as an additional design variable in the mathematical optimization problem. This nonlinear constrained optimization problem is treated by transformation into a set of unconstrained problems by means of a penalty

Fig. 1: Coil block arrangement for the optimized design together with the design variables

function method [4] and the application of the search routine EXTREM by Jacob [3] which proved to be very efficient with respect to different criteria such as convergence behavior, robustness and accuracy.

The cross-section of the optimal design for the specific cables and an aperture of 56 mm is given in Fig. 1. A design with only five coil blocks could be found which perfectly meets the requirements concerning field quality as given from beam simulations. All previous design studies considered a six block design. The current in each conductor is 9301 A, the inner layer is at 95. % with respect to the load line, the outer layer is at 93.2 %. The number of turns is 98. The main dipole field is 8.9 T, the relative multipoles (in units of 10^{-4}) are $b_3 = 0.1$, $b_5 = -0.031$, $b_7 = -0.017$ and $b_9 = -0.00049$.

II. OPTIMIZATION OF THE IRON YOKE

For the optimization of the iron yoke of the dipole with its saturation effects resulting in field dependent multipole content in the main field, the POISOPT program which is an extension of CERN's version of the finite element program package POISSON is applied. Symbolic input variables for the preprocessor can now be defined which are associated with data such as dimensions for the cross-section of the materials. These symbols can then be used in the polygons for the material definition. The load characteristic is investigated automatically and a list of output data is provided which allows the user to define the objectives and constraints in a user friendly way. Fig. 2. shows a flow chart of the optimization procedure with the POISSON codes. It consists of a calling routine POISOPT containing the optimization algorithms. It reads the user supplied information such as the symbols of the design variables, their lower and upper bounds and starting values etc. from the input file. It calls POISZFK which updates the design variables for the calculations and creates

Fig. 2: Program structure of the extended POISSON code

an objective function from a list of results provided by the POISSON codes. POISSON consists of the meshing routines AUTOMESH and LATTCR, the solver POISCR (using ICCG) and the plotting routines TRIPCR. For each geometry the solver is called with different current densities in the coil regions enabling calculation of the saturation induced multipole content.

A. The objective conflict

The design variables for the optimization of the iron yoke of the superconducting dipole magnets for LHC are the position d_1 and radius r_1 of the holes in the iron insert, the collar radius r_C and the outer yoke radius r_Y . The optimization problem consists of a number of partly contradictory objectives. It is called a vector-optimization problem (VOP). The objectives are:

- A high dipole field B_1
- low variation of the relative quadrupole field component Δb_2 versus excitation
- low variation of the relative sextupole field component Δb_3 versus excitation and
- a small outer yoke radius r_Y in order to reduce the cold mass.

For a high dipole field a small inner yoke radius is advantageous but it conflicts with the demand for small variations of multipoles. For the definition of the "optimal" solution of the VOP we apply the optimality criterion by V. Pareto [2]. Considering the VOP given by the K nonlinear objective functions f arranged in the vector F , "min" $\vec{F}(\vec{X}) = \text{"min"}(f_1(\vec{X}), f_2(\vec{X}), \dots, f_K(\vec{X}))$ subject to additional nonlinear inequality and equality constraints and the design variable vector $\vec{X} = (x_1, x_2, \dots, x_n)$ then a Pareto-optimal (also called efficient or noninferior) solution X^* is given when there exists no solution X for which

$$f_k(X) \leq f_k(X^*) \forall k \in [1, K] \quad (3)$$

$$f_k(X) < f_k(X^*) \text{ for at least one } k \in [1, K] \quad (4)$$

In other words: If we have a design where the improvement of one objective causes the degradation of at least one other objective, then this design is an element of the Pareto-optimal solution set. It is clear that this yields a set of solutions rather than one unique solution. Fig. 3 shows a geometric interpretation for two objectives.

B. Methods for decision making

The objective weighting function is the sum of the weighted objectives and results in the minimization problem

$$\min\{u(\vec{F}(\vec{X})) = \sum_{k=1}^K t_k \cdot f_k(\vec{X})\} \quad (5)$$

with the weighting factors t_k representing the preference of the decision maker. The problem is the choice of the weighting factors in particular when the analyst and the decision maker are two individuals. Furthermore there arises the problem that usually the numerical values of the objectives are of different scale which has to be considered in the weighting factors. This results in a time consuming iterative procedure where in accordance with the results and the sensitivity of the objectives the weighting factors have to be updated and the procedure has to be restarted. The problem with the weighting factors can be overcome by defining the problem in the constraint formulation [4]. Only one of the objectives is minimized and the others are considered by constraints so that we get in a vector notation:

$$\min f_i(\vec{X}) \quad (6)$$

subject to

$$\vec{g}(\vec{X}) - \vec{c} \leq \vec{0} \quad (7)$$

$\vec{g}, \vec{c} \in R^{K-1}$. The c_k represents the minimum request value specified by the user for the k-th objective.

Fig. 3: Solution space with Pareto-optimal solutions

The necessary optimality conditions [4] at the optimum point X^* read in the absence of equality constraints:

$$\nabla_X f_i(X^*) + \vec{\alpha} \nabla_X \vec{g}(X^*) = \vec{0} \quad (8)$$

$$\vec{g}(\vec{X}^*) - \vec{c} = \vec{0} \quad (9)$$

$$\vec{\alpha} > \vec{0} \quad (10)$$

The $\vec{\alpha}$ is the vector of the corresponding Lagrange multipliers. Equations (8) - (10) are the Kuhn-Tucker equations. The solution of the constraint optimization problem is only then Pareto-optimal if all constraints are active ($\vec{g} = \vec{c}$) and the Kuhn-Tucker equations are satisfied (i.e. the Lagrange multipliers are all greater than 0). If the constraints are not active and there are alternative solutions to the constraint problem, then some of them may not be Pareto-optimal, cf. Fig. 3. That is equivalent to a zero weight for the particular objective in the weighting function. The Lagrange multipliers $\vec{\alpha}$ are a measure of the price which has to be paid when the constraint is decreased. This relationship is expressed by the fact that

$$\nabla_c f(\vec{X}^*) = -\vec{\alpha}. \quad (11)$$

The Lagrange multipliers are estimated by minimizing the norm of the left side of (8). A very useful tool which provides the decision maker with a lot of information about the hidden resources of a design is the payoff table. To create this table K individual constraint optimization problems are solved to find the best solution for each of the K objectives. The constraints represent the minimum request of the decision maker on the objectives cf. Table 3. It has to be said that about 150 numerical field calculations had to be performed for each optimization. The large computing time required was supplied by CERN's RISC6000 workstation cluster. Best compromise solutions can then be found by minimizing the "distance" from the in general nonfeasible "perfect solution" cf. Fig.3. The minimization problem reads

$$\min \|\vec{F}^*(\vec{X}) - \vec{F}(\vec{X})\|_p \quad (12)$$

where $\vec{F}^*$ is the vector of the optimal individual solutions, the "perfect solution" taken from the payoff table.

TABLE 1
PAYOFF TABLE FOR THE 4 OBJECTIVES

min r_Y m	max B_1 T	min Δb_2 10^{-4}	min Δb_3 10^{-4}	Design variables mm		
<0.3	>8.8	<1.5	<3.0	r_1	d_1	r_C
254.	8.89	0.93	2.18	25.6	16.8	79.2
268.	8.93	2.02	3.01	12.6	10.0	78.3
295.	8.85	0.03	1.96	5.46	14.6	86.8
258.	8.82	0.28	0.98	21.2	10.0	88.0

The residuals are scaled to the variation of the objectives between their individual best B_k and worst W_k solution taken from the payoff table. By applying different norms e.g. the L_1 , L_2 and L_∞ norm the optimal compromise solutions can be found. The payoff table can also help to set up constraint problems with Pareto-optimal solutions (i.e. finding feasible solutions for constraint problems with active constraints).

III. SOLVING THE OPTIMIZATION PROBLEMS

The formulation of the optimization problem is the most important step in order to guarantee solutions meeting the preferences of the decision maker. For solving the above problems some efficient optimization procedures are briefly described. It is most important to find the minimization method best adopted to the decision making method. It is the **combination** of the appropriate methods which yields an efficient procedure. The objective weighting function allows the immediate application of an algorithm for finding the minimum value of an unconstrained objective function as there are no nonlinear constraints to be considered. The two algorithms applied are the search algorithm EXTREM [3] and the Rosenbrock algorithm [4]. For minimization of the distance function the Levenberg-Marquard algorithm is used. For the constraint formulation (as necessary for the Lagrange-Multiplier estimation or for the setting up of the payoff table) the transformation of the constrained problem into an unconstrained problem by means of a penalty function method is made. The minimization of the transformed problem is then again carried out by means of the algorithm EXTREM.

IV. RESULTS

Table 2 shows the compromise solutions found in the iron yoke optimization. A solution for the objective weighting function $\min\{-1000 \cdot B_1 + 5 \cdot \Delta b_2 + 10 \cdot \Delta b_3 + 10 \cdot r_Y\}$ is given in row 1 of Table 2. A solution for the constraint problem $\max B_1$ subject to $r_Y \leq 260$; $\Delta b_2 \leq 0.5$; $\Delta b_3 \leq 1.7$ is given in row 5 of Table 2. The corresponding Lagrange-multipliers are given in row 6. They indicate that the limiting factor for a further increase in the dipole field (e.g. by reducing the outer yoke radius) is the sextupole variation. The price (dipole field) which has to be paid for a further reduction of the sextupole is 10 times higher than for a further reduction of the quadrupole. The resulting compromise solutions for the distance function problem with the norms L_1 , L_2 and L_∞ are given in row 2,3 and 4 of Table 2. The cross-section of the magnet with the compromise solution given in row 5 is shown in Fig. 4. The variation of the relative quadrupole term is $0.38 \cdot 10^{-4}$ and the variation of the relative sextupole term is $1.70 \cdot 10^{-4}$ for a dipole field varying from 0.6 to 8.92 T. The outer yoke radius could be reduced to 258 mm.

TABLE 2
COMPROMISE SOLUTIONS

	r_Y	B_1	Δb_2	Δb_3	r_1	d_1	r_C
	mm	T	10^{-4}	10^{-4}		mm	
1	265.	8.91	0.09	1.71	13.0	11.7	85.4
2	255.	8.92	0.48	2.20	14.1	10.2	80.1
3	271.	8.89	0.05	1.48	16.5	15.9	87.7
4	270.	8.89	0.64	1.47	12.7	15.4	88.0
5	258.	8.92	0.38	1.70	14.9	10.1	83.0
6	0.01		0.16	2.72			

1) Objective weighting function, 2) L_1 , 3) L_2 ,
4) L_∞ "distance" from the "perfect" solution.
5) Result of constraint formulation. 6) Lagrange-multipliers for solution 5.

With the payoff table and the Lagrange-multiplier estimation a powerful tool for decision making is given where the decision maker is usually not the analyst. The final choice will then also incorporate beam optics considerations, budget, mechanical design and manufacturing.

Fig. 4: Design variables for the iron yoke optimization and the optimized design with field plot

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