Simulations of an electron lens
for Future Circular Colliders:
noise impact and modifications of Landau damping
due to different electron beam profiles

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In this report simulations of an electron lens in proton colliders and effects on the beam dynamics will be presented. In proton colliders an electron lens can be used to provide Landau damping, cleaning of the beam halo and compensation of beam-beam effect.

In the following analysis we will focus only on Landau damping: indeed it can easily provide transverse stability of a high energetic proton beam such as in the FCC (Future Circular Collider) and HE-LHC (High Energy LHC), two future colliders which will be developed at CERN.

We will start with the implementation of different electron beam distributions and we will study the different impact on tune spread for Landau damping. The aim of the second part of the analysis is to discuss the influence on the beam dynamics of a certain displacement of the electron lens with respect to the proton beam orbit. For example the displacement can have a mechanical origin in the case of vibrations or it can be related to the uncertainty of the proton beam centroid position.

In the first chapter we will resume basic concepts of beam dynamics in circular colliders and notions on transverse beam stability and Landau damping. Then, it follows the discussion of the numerical tools (a Poisson Solver and COMBI) used in the simulations, mostly run on EPFL Castor clusters. The upgrades implemented for the analyses will be discussed in details. In order to verify the validity and check the simulations, a benchmark study on convergence is exposed. Finally the simulation results are presented and the conclusions follow.
Before entering in the discussion some basic concepts and notions about Beam Dynamics in synchrotron will be given in the following.

2.1 Beam dynamics

We can divide beam dynamics into two main parts: the transverse dynamics (plane x-y) and the longitudinal dynamics (along the beam direction), as represented in the figure 2.1 below.

In this study we will only consider the effect of the electron lens on the transverse dynamics. One of the most important parameters of this motion is the betatron tune, defined as the number of betatron oscillations in a complete revolution:

\[ Q = \frac{1}{2\pi} \oint_{C} \frac{ds}{\beta(s)} \]  

(2.1)

where \( \beta(s) \) is the betatron function: it is an optical function defined by the lattice of the accelerator. The lattice is the sequence of the magnets in the accelerator.

In the phase space \((x, p)\) the motion of a particle is confined inside an elliptical function whose area is a constant of motion called emittance. The RMS (Root Mean Square) over all the particle trajectories in the phase space defines the beam emittance. A low value of the emittance means that the beam is well confined.

Assuming a gaussian profile for the beam, we can relate the rms of the distribution

\includegraphics[width=0.5\textwidth]{frenet-serret-frame.png}

Figure 2.1: Frenet Serret frame.
\( \sigma \) to the value of the emittance as \( \sigma_x(s) = \sqrt{\epsilon \beta(s)} \) where \( \epsilon \) is the rms emittance and \( \sigma_x \) the sigma of the particles distribution along x-direction where there is no dispersion.[1]

We will use these concepts in the analysis and simulation of the electron lens. We consider the electron lens as a distribution of electrons and if we consider the transverse dynamics, it is on a 2D plane. The electrons distribution leads to a certain potential and a certain electric field seen by the protons in the beam. Since protons are charged particles, they interact with this field. As the beam is ultrarelativistic, the protons see a fixed electric field, so in a first approximation we can safely avoid the study of longitudinal dynamics. The interaction of the protons with the electrons leads to a momentum change, but in \((x,x')\) coordinates, where \(x' = dx/ds\), it can be seen as a kick. From now on, we will refer to the interaction proton beam-electron lens as the kick of the electron lens.

2.2 Electron Lens and Landau damping

The main uses of an electron lens are the following:

- Beam-beam compensation (head-on or long range)
- Landau damping of coherent modes
- Beam collimation and halo cleaning

In this report we will study only the impact on Landau damping since the use of an electron lens has been recently proposed as a solution for beam stability for high energetic proton beams in future circular colliders. [2]

As soon as we consider collective effects in beam dynamics, we need to consider many different phenomena, one of these is related to wake fields. To give the basic idea of this phenomenon, we can think that while the beam travels inside the pipe line, due to the environment geometry, to the finite conductivity and to the image charge effect, there are electromagnetic fields interacting with the beam itself, in particular where there are discontinuities in the pipe line. These represent the wake fields and impedances effects.[3]

This perturbation can give rise to coherent instabilities both in longitudinal and transverse plane.

A way to solve these instabilities is to create a tune spread in the beam.

The physical interpretation of this solution is the following: if an instability occurs, the particles start to oscillate coherently, this can be avoided if the nominal frequency of every particle, according to the particle amplitude, is different (i.e. a tune spread), such that there cannot be a coherent mode excited. This phenomenon is called Landau damping.[4]

2.2.1 State of the art

Any nonlinearity acting on the beam induces a certain tune spread in the beams. For example in LHC (Large Hadron Collider) octupoles magnets are operationally used
to provide sufficient Landau damping of coherent impedance modes. [5]
However, as the beam energy increases, octupoles are less effective. For high energetic proton beams the use of electron lens to provide Landau damping is under discussion as shown is [2]. In fact, in the case of the FCC it would be necessary to increase by a factor of 60 the strength of the LHC octupoles and this would lead to problems in stabilization of octupoles and with the nonlinear magnetic fields.[6]
For high energetic proton beams the use of electron lens to provide Landau damping is under discussion and some studies have recently started with confident results, as shown in [2].
On the opposite, there are no studies about noise due to displacement between the electron lens and the proton beam, but there is the need to quantify and simulate what the effect can be.
CHAPTER 3

PHYSICAL MODEL SIMULATION

The simulation tools used for the following analysis are two:

- Poisson Solver
- COMBI: COherent Multi Bunch Interaction

The first one is used to compute the kick of a given electron beam profile while COMBI simulates its effect on the transverse beam dynamics. The simulation of the electron lens have been implemented for two different situations as reported in table 3.1.

<table>
<thead>
<tr>
<th>Collider</th>
<th>Circumference (m)</th>
<th>Energy (TeV)</th>
<th>Particles per bunch</th>
</tr>
</thead>
<tbody>
<tr>
<td>FCC</td>
<td>97749.14</td>
<td>3.3 (injection energy)</td>
<td>$1.15 \cdot 10^{11}$</td>
</tr>
<tr>
<td>FCC</td>
<td>97749.14</td>
<td>50</td>
<td>$1.15 \cdot 10^{11}$</td>
</tr>
</tbody>
</table>

Table 3.1: Collider specifics.

The parameters used for the electron lens are reported in table 3.2.

<table>
<thead>
<tr>
<th>Proton energy (TeV)</th>
<th>Energy (keV)</th>
<th>Current (A)</th>
<th>Length (m)</th>
<th>Number of electrons ($s^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.3</td>
<td>9</td>
<td>0.5</td>
<td>2.1</td>
<td>$1.18 \cdot 10^{11}$</td>
</tr>
<tr>
<td>50</td>
<td>10</td>
<td>1.5</td>
<td>2.1</td>
<td>$3.36 \cdot 10^{11}$</td>
</tr>
</tbody>
</table>

Table 3.2: Electron lens specifics.

3.1 Poisson Solver

The numerical tool used to generate the potential and the kick distributions starting from a given electron distribution is a Poisson Solver based on IGF (Integrated Green Function) on a 2D-grid.

The algorithm follows these steps:

- Generation of the grid with given discretisation.
- Generation of the chosen distribution.
• Computation of the potential.
• Computation of the field.
• Computation of angular deflections (kick).

Since it exploits the IGF method, this Poisson Solver works with every given distribution and not only with gaussian which was the only one already implemented. The convergence has been checked with respect to the discretisation of the initial grid. The parameters we can tune in the Poisson Solver are the following: dimensions and discretisation of the grid, current intensity of the electron lens (i.e. number of the electrons), energy of the incoming proton beam.

3.1.1 Implementation of different electron beam profiles

Apart the gaussian distribution already existing, the following distributions have been added to the Poisson Solver:

• Lorentzian distribution in the form

\[
\rho(x, y) = \frac{1}{\pi^2 \gamma_x \gamma_y} \frac{1}{1 + \left(\frac{x-x_0}{\gamma_x}\right)^2} \frac{1}{1 + \left(\frac{y-y_0}{\gamma_y}\right)^2}
\]  

(3.1)

• Cylindric hollow profile in the form

\[
\rho(x, y) = \begin{cases} 
\frac{1}{\pi (R_{ext}^2 - R_{int}^2)} & \text{if } R_{int} \leq r \leq R_{ext} \\
0 & \text{otherwise}
\end{cases}
\]

where \( r^2 = (x-x_0)^2 + (y-y_0)^2 \)

(3.2)

• Parabolic profile in the form

\[
\rho(r) = \begin{cases} 
\frac{2}{\pi R_{max}^4} (R_{max}^2 - r^2) & \text{if } r \leq R_{max} \\
0 & \text{otherwise}
\end{cases}
\]

where \( r^2 = (x-x_0)^2 + (y-y_0)^2 \)

(3.3)

• Gaussian distribution with a hole in the form

\[
\rho(r) = \frac{1}{A} e^{-\frac{1}{2} \left(\frac{r}{\sigma}\right)^2} \text{ where } r^2 = (x-x_0)^2 + (y-y_0)^2
\]

(3.4)

where \( A \) is the normalization factor:

\[
A = 2\pi \left\{ \sigma^2 \exp \left[ \frac{1}{2} \left(\frac{R}{\sigma}\right)^2 \right] + R\sigma \sqrt{2\pi} \left[ 1 - \frac{1}{2} \text{erfc}(\frac{R}{\sqrt{2}\sigma}) \right] \right\}
\]

with \( \text{erfc}(x) \) is the complementar error function and \( R \) is the hole radius.\(^1\)

• Exponential profile in the form

\[
\rho(r) = \frac{1}{2\pi \sigma^2} e^{-\frac{r}{\sigma}} \text{ where } r^2 = (x-x_0)^2 + (y-y_0)^2
\]

(3.5)

\(^1\)The complementary error function is defined as \( \text{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-t^2} \, dt \).
All these distributions are normalised because it is possible to scale them just multiplying for the total number of electrons, which is possible to compute from the current \( I \) of the electron lens as

\[
N_e = \frac{I \cdot L}{e\beta c}
\]  

(3.6)

where \( L \) is the length of the electron lens, \( e \) the electron charge, \( \beta \) the relativistic factor of the electrons, \( c \) the speed of light. Usually we consider electron lenses with an energy of 10keV, which leads to a relativistic factor \( \beta \approx 0.19 \).

### 3.2 COMBI: COherent Multi Bunch Interaction

The COMBI code was developed by T. Pieloni and W. Herr at CERN in order to study the coherent beam-beam interaction of multiple bunches. With a self-consistent approach, it is possible to study the different effects that can evolve also in a collider with many interaction points [7]. Then the code has been extended during the years to study different effects on the beam dynamics thanks to the implementations of several actions.

The particles in a bunch are represented by a set of macroparticles, which are treated in a self-consistent way to derive the dynamics of the beam and its main parameters (e.g. the emittances along \( x \) and \( y \)) and which are tracked individually.

The initial parameters are defined in an input file for every simulation run. The different effects are implemented in different actions (e.g. beam-beam head-on collision, impedance, etc.) which can be assigned to a bunch position.

The action which has been considered in this analysis is the electron-lens action. This action will be further described in the following paragraph, together with the modifications made for the studies carried out for this project.

#### 3.2.1 Electron Lens action in COMBI

Concerning the electron lens, there is an action which represents the interaction of the beam with an external electric field, in our case it is the one generated by the electron profile of the electron lens.

The new feature implemented in this action is related to a random displacement of the lens with respect to the proton beam: in fact it is possible to simulate an off-axis interaction and it can be really useful if we want to model noise such as vibrations or uncertainty of the centroid position.

With this implementation it is possible to choose the maximum amplitude of the displacement and its periodicity: i.e. the displacement can be kept constant for a given number of turns, for example if the vibration has a certain frequency. It is important to remember in fact that the revolution frequency \( f_0 \) is, in the case of FCC, \( f_0^{FCC} = 3.067kHz \): so we can simulate the vibrations finding the number of turns related to a particular frequency.

This action has the following arguments which can be chosen in the input file of
the actions sequence:

- $\beta_{electron}[m]$: it is the value of the betatron function at the electron lens.
- $\beta_{rel}$: it is the relativistic beta of the electrons travelling in the electron gun.
- $amp_{x,max}[m], amp_{y,max}[m]$: the maximum displacement that we can assume.
- Logical value (0 or 1) to chose the possibility of simulate the lens with (1) or without (0) the random displacement.
- $N_{per}$: the number of turns the random displacement is kept constant.
In order to check the goodness of the numerical model, a convergence study has been carried out.

4.1 Analytical formula

The only case where there is a formal analytical treatment is the electron lens with gaussian symmetric electron distribution.

The interaction between the beam and the electron lens in fact can be interpreted as a head-on beam beam collision. As shown in [8], it is possible to derive analytically the formula for the non-linear detuning with the amplitude $J$:

$$\Delta Q(J) = \xi \cdot \frac{2}{J} \left[ 1 - I_0 \left( \frac{2}{J} \right) e^{-\frac{2}{J}} \right]$$  \hspace{1cm} (4.1)$$

where we have that $\xi$ is the beam-beam parameter (or the linear beam-beam tune shift) and $J$ the amplitude defined as

$$\frac{\xi}{J} = \frac{-N r_0 \beta}{\frac{4 \pi \gamma \sigma^2}{\epsilon}}$$  \hspace{1cm} (4.2)$$

where $N$ is the number of particles, $r_0$ the proton radius, $\beta$ the betatron function at the collision point, $\gamma$ the relativistic factor, $\sigma$ the beam size at the collision point, $\epsilon$ the particle emittance. In the formula 4.1 $I_0(x)$ is the modified Bessel function of order 0.

The beam-beam parameter $\xi$ represents the maximum tune shift for the particle with $(J_x = 0, J_y = 0)$.

In the case of an electron lens the head-on parameter is given by

$$\xi_{e-lens} = \frac{N_e r_0 \beta_{e-lens}}{4 \pi \gamma \sigma_e^2} (1 + \beta_{rel})$$  \hspace{1cm} (4.3)$$
where in this case we have that $\beta_{rel}$ is the relativistic beta of the electrons, $\beta_{e-lens}$ is the betatron function at the electron lens, $N_e$ the number of electrons, $\sigma_e$ the beam size at the electron lens.

The value of $\xi$ in 4.3 is for an electron lens (in fact we are considering that the two beams are of opposite charge), in the case of proton-proton beam collision there is a minus sign, leading to a negative tune shift instead of a positive one as for the electron lens case.

4.2 Convergence study

Exploiting this formula, we want to check if both the numerical tools used, i.e. the Poisson Solver and COMBI, lead to the correct analytical result.

To do so, the convergence study is in function of the different discretisations of the grid used to compute the kicks of the electron lens. In fact when we generate the kicks in the Poisson solver, one of the parameters we can tune is the discretisation of the grid. In the following treatment $N$ will represent the discretisation number (i.e. $\Delta x = \frac{x_{max} - x_{min}}{N}$).

From the simulations with COMBI, it is possible to find the tune shift in function of the amplitude of different particles, using the FFT on the output file for a given number of turns $N_t$, as explained in the next chapter.

In this convergence study $N_t$ is set equal to 4096. In the graph 4.1 it is possible to see that when $N$ is greater than 100, there is a good agreement with the theoretical behaviour. The error bars represented are due to FFT error, since we simulate only a finite number of turns.

The convergence can be more easily seen if we plot the standard deviations with

![Figure 4.1: Convergence study](image-url)
Figure 4.2: Variance graph

respect to the analytical function, as in the graph 4.2, where we see that for $N$ larger than 100 there is not a real improvement in terms of variance. For the presented simulations we make use of $N=200$ for the grid discretisation to gain in the computational time.
Two main analyses have been carried out: the first one is related to Landau damping of the new electron lens profiles implemented (only the gaussian profile was studied before this project), while the second one is about the displacement between the axis of the beam and the electron lens with a gaussian profile.

Before starting to analyse Landau damping, it is useful to qualitatively discuss the effect of different distributions of the electron beam profile.

All the distributions have a size comparable with the proton beam size at the interaction point (which is $\sigma = 0.28\text{mm}$) and we fix the current of the electron lens to 1.5 A for Landau damping analysis.

### 5.1 Comparison between different beam profiles

A given distribution of electrons is characterised by a potential and by its electric field. The kick felt by the proton beam depends on the electric field, in particular the incoherent kick is proportional to the value of the field in $(x, y)$. In the following figures, from 5.1 to 5.6, we plot for each electron beam profile the distribution in 3D and, in order to better visualise them, the projection along the x-axis of the distribution and of the kick. The current of the electron lens is fixed at 1.5 A and we consider the interaction with a proton beam of energy 50 TeV.

![Figure 5.1: Gaussian distribution (with standard deviation of $\sigma$).](image)
(a) 3D plot of the lorentzian distribution.
(b) 1D plot of the distribution (x-direction).
(c) 1D plot of x-direction kick (along y-axis).

Figure 5.2: Lorentzian distribution (with FWHM=2√ln 2 σ)

(a) 3D plot of the cylindric hollow distribution.
(b) 1D plot of the distribution (x-direction).
(c) 1D plot of x-direction kick (along y-axis).

Figure 5.3: Cylindric Hollow distribution (with R_{hole} = 3 σ).

(a) 3D plot of the parabolic distribution.
(b) 1D plot of the distribution (x-direction).
(c) 1D plot of x-direction kick (along y-axis).

Figure 5.4: Parabolic distribution (with R_{max} = 3 σ).

(a) 3D plot of the gaussian hollow distribution.
(b) 1D plot of the distribution (x-direction).
(c) 1D plot of x-direction kick (along y-axis).

Figure 5.5: Gaussian Hollow distribution (with R_{hole} = 0.9 σ).

Of course since we have the two components of the field in x and y direction, we can represent these components of the kick independently. To give an idea of the
profile, we can see in the picture 5.7a a 3D plot of the kick generated by a gaussian distribution. From the theory we know that the tune shift given to the particle in the centre of the beam is given by the derivative in the origin of the kick profile. In order to better understand the behaviour of the electron lens with different distribution we can look at the profile once we have fixed one axis. Due to symmetric behaviour in $x$ and $y$, we can restrict our analysis to $x$-kick.

5.1.1 Hollow electron lens

In the figure 5.7b we present the different kick profiles of an hollow gaussian distribution with different hole radii which can be used in COMBI for collimation purpose in future studies. We can observe that with small radius the profile is very similar to the gaussian kick, while it approaches the hollow cylindric when the hole radius reaches $3\sigma$, where $\sigma^2$ is the variance of the gaussian as we see in 3.4.

5.2 Tune spread Scan

As we have seen, the basic idea underlying the phenomenon of Landau damping is the detuning with amplitude of different particles in the beam (i.e. the tune spread). We can associate a tune shift to every particle at different amplitude and this leads
to a general tune spread. In the graph 5.8 we can see the kick profile for the different distributions implemented.

In order to more easily think to the interaction between the centre of the beam (between 0 and 1σ) and the electron lens, we can think to linearised kick in the neighbourhood of the origin: so we have the maximum tune shift for the particle at the center of the beam (so called (0,0)-particle) and we can already estimate qualitatively the general tune spread. In fact we can see that the kick generated by the gaussian beam has the maximum slope in the origin and also the maximum amplitude, so we expect that this electron beam profile is the most efficient in term of tune spread.

In order to better represent it, using COMBI it is possible firstly to track every particle at different amplitude and then, through Fourier Tranform (implemented with FFT), compute the tune of each of the tracked particle for a certain number of turns.

In this analysis we consider a proton beam of 50 TeV and an electron lens with current 1.5 A.

![Figure 5.8: Comparison of the kicks generated by different electron beam profiles.](image)

From the FFT we get the oscillation frequency of the particles at different amplitude. We plot the different frequencies in a $Q_x - Q_y$ plot called tune diagram (or tune footprint). The figure 5.9 represents the different footprints generated by different electron beam distributions.

From the tune spread represented in the footprint we can already observe that the most efficient one is given by the gaussian distribution, for which the maximum tune shift of the (0,0) particle is 0.0047.s (considering that the initial tunes are $Q_x = 0.31$ and $Q_y = 0.32$). For completeness, in order to properly study the effectiveness of the electron lens for Landau damping, we can compare the corresponding stability diagrams.[9] Instead of using the coordinates $(x, x')$ we will use the action variables $(J, \psi)$ which can be derived as shown in [1].

As explained in [2], the stability diagram can be computed as

$$D(\Delta q) = - \left( \int \frac{J_x \partial F/\partial J_x}{\Delta q - \delta \nu_x + io} dJ_x dJ_y \right)^{-1}$$  \hspace{1cm} (5.1)

where $F$ is the normalized phase space density as a function of actions $J_x, J_y$, the symbol $io$ stands for an infinitesimally small positive value, $\delta \nu_x = \delta \omega_x / \omega_0$ is the amplitude
dependent tune shift (with \( \omega_0 \) the revolution frequency and \( \delta \omega_z \) is betatron frequency shift related to the electron lens) and \( \Delta q \) is the coherent tune shift.

The stability diagram shows the stability area (below the line) in the complex plane where the coherent modes are stable, while outside of this area they are unstable. The imaginary part is related to the rise time of the instability while the real part with the coherent tune shift induced by the impedance.

This means that the effectiveness of Landau damping (i.e. to stabilise the beam) can be measured as the area below the stability diagram. In the figure 5.10 we can see that the electron lens with gaussian distribution is the most efficient in terms of Landau damping.

Comparing the FWHM of the diagram in the case of 5.10a, we observe that the gaussian beam is more effective than the lorentzian one by a factor 1.4.

5.3 Noise due to displacement

The remaining part of this chapter concerns the simulations of the effect of a gaussian electron lens on the emittance if we consider a random displacement. The simulations presented are for FCC at the injection energy of 3.3 TeV.

The analysis carried out focuses on three different parameters:

- amplitude of the displacement
- current of the electron lens
- periodicity of the displacement.
For each study, two of these parameters are fixed and the third varies. The noise introduced by the displaced electron lens leads to an increase of the emittance. As we know the emittance is a parameter which is related to how the beam is confined and to the momentum dispersion of the particle. From the output file we can track the emittance at every turn and the with the linear regression we can find the slope of the increase. The error bars represent the error of the linear regression.

**5.3.1 Amplitude analysis**

In this analysis, we have varied the amplitude of the displacement between 50nm and 0.95µm. The displacement is random for every turn and the current of the electron lens is 1A. We can see that also for these small amplitudes, we observe an emittance increase per hour of 22-23% in the worst case, that will limit the collider performances. The green line is a second order polynomial interpolation which can be used to predict the emittance growth.

![Amplitude emittance increase (current: 1A).](image)
5.3.2 Current analysis

In this case we have considered a random displacement for every turn, with a fixed maximum displacement of $5\mu m$. The current of the electron lens varies between 1 and $1.1A$, which are typical values for a 3.3 TeV proton beam. Also in this case we observe a quadratic increase of emittance.

5.3.3 Periodicity analysis

In this last case, we want to analyse how the emittance increase changes if the random displacement is kept constant for a given number of turns. From a physical point of view, it means that the beam finds the lens in the same position for $N$ turns: even if it is not centred the emittance should not increase as the previous case. We can see from the graph 5.13 that when the periodicity increases above 2000 turns (which corresponds to a frequency of $2Hz$), the emittance increase is negligible. In this case the electron lens has a current of $1A$ and the maximum displacement is $10\mu m$.
This study has allowed to simulate the behaviour of electron lenses with different distributions and to discuss the different kick profiles.
A discretization threshold for the grid of the Poisson Solver has been found, exploiting the convergence study.

The analysis for the most effective Landau damping distribution has shown that the gaussian distribution provides the largest stability diagram with respect to the other distributions generated, considering the same current for the electron lenses.

With regards to the displacement analysis, we have fixed a threshold on the maximum amplitude to avoid a large increase of emittance, and we have also found quadratic emittance increase either as the maximum amplitude increases either as the current increases. For a displacement of 0.8µm an emittance increase of 15%/h is expected with a random noise applied for each turn. On the other hand if we apply the random displacement every 2000 turns, the impact on emittance is almost negligible, while we observe a strong increase of emittance as the noise frequency increases.

In order to deepen the comprehension and the simulation of electron lens, one of the possible improvements is to model also the longitudinal dynamics due to the interaction with an electron lens. Another possible improvement is to check experimentally the dramatic emittance increase due to the displacement of electron lens, in order to understand if its use is feasible or not.

Moreover, a parametric hollow gaussian distribution has been implemented for future considerations on the halo cleaning by using the COMBI code, but the analysis about the collimation is not taken into account in this report.
REFERENCES


