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Information loss and the Bohm model

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ABSTRACT The significance of the distinction between pure and mixed states in the context of the Bohm model is discussed, and it is shown that the presence of closed time-like loops does not give rise to any need for the mixed state description, or to any ambiguities, when the Bohm model is used.

1. Introduction

There has been much recent discussion on the apparent loss of information in quantum cosmology due to the presence of black-hole radiation¹⁻⁵. The fact that particles falling into a black hole take information with them means that other particles, with which they are correlated in the quantum wavefunction, cease to be in a pure state, but instead are in a mixed state. The purpose of this note is to endeavour to see what all this might mean in the context of a Bohm, hidden-variable, model. This model, unlike conventional quantum theory, has a clear ontology, so issues regarding the nature of what is, in contrast to what some particular individual might know about it, are more clear. Also, the model in its conventional context is deterministic, so one can ask for example whether the “loss of information” corresponds to some sort of loss of determinism.

Rather than treating the situation of a black hole, we shall here consider the much simpler problem of a flat space which is not globally hyperbolic and which has closed time-like loops. The idea here is that the “information”, rather than falling into a black hole, becomes trapped in the time-like loops. We shall work in 1+1 space, and consider two non-interacting particles, both satisfying the Dirac equation, which are initially in a correlated wavefunction. As we shall see, even such a simple problem reveals interesting features of the nature of the Bohm model, and its relation to the structure of space-time.

First, however, we comment on the distinction between pure and mixed states in the Bohm model.

2. Mixtures and pure states in the Bohm model

Consider two particles, described by variables x and y respectively, and suppose they are in the correlated pure state given by

$$\Psi = \alpha\phi_a(x)\chi_a(y) + \beta\phi_b(x)\chi_b(y), \quad (1)$$

where we suppose that the states $\phi_{a,b}$ and $\chi_{a,b}$ are normalised to unity, and that the χ states are orthogonal:

$$\langle \chi_a | \chi_b \rangle = 0, \quad a \neq b. \quad (2)$$

Then the average value of any observable, O_x , related entirely to the ‘ x ’ system, is given by

$$\langle O_x \rangle = |\alpha|^2 \langle \phi_a | O_x | \phi_a \rangle + |\beta|^2 \langle \phi_b | O_x | \phi_b \rangle. \quad (3)$$

Suppose, on the other hand, that we know the state to be either $\phi_a(x)\chi_a(y)$, with probability $|\alpha|^2$, or $\phi_b(x)\chi_b(y)$, with probability $|\beta|^2$, then, as is well known, we obtain exactly the same result for the average of any variable measured on the ‘ x ’ system. Thus, provided it is in principle impossible to measure anything connected with the ‘ y ’ system, there is no real difference between the mixture and the pure state; the “information” that told us about the correlations has been lost. The difference between pure and mixed states does not then lie in the results of any measurements, but simply in the wavefunction that describes the system, so it only has significance if we give this wavefunction an ontological status.

In the Bohm model, there is no doubt that the trajectories are *real*, i.e., just like those of classical physics. Even if they cannot be observed, they nevertheless exist. Thus, different trajectories correspond to genuinely different realities. The situation with regard to the pure state and mixture, above, is now much clearer, because the two possibilities certainly give different trajectories. Indeed, for the pure state, eq.(1), the velocity, given by the Bohm formula,

$$\begin{aligned} \dot{x} &= \Re \frac{p_x \Psi(x, y)}{m \Psi(x, y)} \\ &= \Re \frac{\alpha \chi_a(y) p_x \phi_a(x) + \beta \chi_b(y) p_x \phi_b(x)}{m \alpha \chi_a(y) \phi_a(x) + m \beta \chi_b(y) \phi_b(x)}, \end{aligned} \quad (4)$$

contains contributions from both terms in the superposition, i.e., interference effects, provided only that y has a value for which both $\chi_a(y)$ and $\chi_b(y)$ are non-zero. On the other hand, for the mixed state, the velocity is given by either

$$\dot{x} = \Re \frac{p_x \phi_a(x)}{m \phi_a(x)} \quad (5a)$$

or

$$\dot{x} = \Re \frac{p_x \phi_b(x)}{m \phi_b(x)}. \quad (5b)$$

It is only when y takes values for which either $\chi_a(y)$ or $\chi_b(y)$ is zero that eq.(4) agrees with one of (5a) or (5b). For this to happen automatically as a result of the orthogonality of the χ states, eq.(2), these would have to be position eigenstates.

Thus, in the Bohm model the question of whether there is “loss of information” is a genuine one, and not just a matter of choice of language. We now turn to the issue of whether it actually happens.

3. Dirac equation in a space with a closed time-like loop

We work with the two-dimensional space shown in fig. 1. This is made from an infinite x, t plane in which there is a cut along $x = 0$ from $t = -\infty$ to $t = t_1 > 0$ and from $t = t_2 > t_1$ to $t = +\infty$, and the region $x < 0$ is then “rolled-up” by identifying $t = t_3 > t_2$ with $t = t_1$. Along the cut we impose a boundary condition that the wavefunction is zero.

On this space we consider fields satisfying the Dirac equation,

$$i\gamma^\mu \partial_\mu \psi = m\psi. \quad (6)$$

For simplicity in obtaining the solutions we take $m = 0$. In our two-dimensional space the spinors have two components, and with the representation,

$$\begin{aligned} \gamma^0 &= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \\ \gamma^1 &= \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \end{aligned} \quad (7)$$

we can write this equation in the form

$$\begin{aligned} \partial_t u &= -\partial_x v \\ \partial_t v &= -\partial_x u, \end{aligned} \quad (8)$$

where u, v are the two components of the Dirac spinor.

We choose the initial conditions, at $t = 0$, $u(x, 0) = f(x)$ and $v(x, 0) = g(x)$, where $f(0) = 0$, $g(0) = 0$ and we take f and g to be odd functions of their arguments. The solution, for $0 < t < t_1$, then becomes

$$u(x, t) = \frac{1}{2}[f(x+t) + f(x-t)] - \frac{1}{2}[g(x+t) - g(|x-t|)], \quad (9)$$

together with a similar equation for v with f and g interchanged.

A simple example is to take $f = g$, which leads to

$$u(x, t) = v(x, t) = \frac{1}{2}[f(x-t) + f(|x-t|)]. \quad (10)$$

Here a small wave-packet, initially around $x = a$, moves, without any change of shape, in the positive x direction, as shown in fig.2.

On the other hand, with $f = -g$, we obtain

$$u = -v = f(x + t) + \frac{1}{2}[f(x - t) - f(|x - t|)], \quad (11)$$

in which a small wave-packet moves to the left, possibly bouncing off the $x = 0$ edge, again as shown in the figure. The figure also shows the behaviour of such a packet in the region $t > t_1$, where there is the possibility of it entering the $x < 0$ region, and hence disappearing down the “black-hole”.

4. Correlated states and information loss

To understand the significance of these solutions we consider two fermions, each described by the above equations, in a correlated state of the form of eq.(1), where the states χ and ϕ are each spinors with $f = -g$ at $t = 0$. We take them to be wave-packets initially sharply concentrated at $x = a, x = b, x = a + c$, and $x = b + c$, as shown in fig.3.

In the fig.3 we have cheated a little by giving the particles a mass. This enables us to use the standard Bohm equation to define their velocities. Clearly the particles will lie in one or the other of the correlated pairs of wavepackets, i.e. those associated with the index a or those associated with the index b .

The crucial question now is how do we define the motion of the particle described by the x variable, at times greater than t_2 . It clearly makes no sense to regard it as correlated with the position of particle y at the same time, since the y particle never “reaches” this time.

In fact, although it does not affect our basic conclusion, it is probably better here to use the retarded Bohm model⁶ in which the correlation is with the other particle at its retarded position, i.e., in general:

$$\dot{x}(t) = \Re \frac{p_x \Psi(x(t), y(t - |x - y|))}{m \Psi(x(t), y(t - |x - y|))}. \quad (12)$$

This expression can be used, with eq.(1) for Ψ , as far as the shaded region in fig.3.

When we reach the shaded region, however, it is not possible to regard the ‘ x ’ particle as being correlated with the ‘ y ’ particle (nothing that happens to the latter can affect it). Hence we should use a single particle wavefunction to describe its motion. Which wavefunction? Here the Bohm model provides an obvious answer. We use the state that evolves in time from that given on the line that bounds the shaded region, namely, $x = t - t_2$. This is

$$\Psi_t(x) = \alpha \chi_a(y(t - |x - y|)) \phi_a(x) + \beta \chi_b(y(t - |x - y|)) \phi_b(x), \text{ on } x = t - t_2. \quad (13)$$

This gives the boundary condition defining the state to be used in the shaded region of fig.3. It is given uniquely provided only that we have a trajectory for the ‘ y ’ particle. Such a trajectory ensures that the “information” that is lost into the “black-hole” is

specified by the initial conditions, and is not random. Hence there is no breakdown of determinism.

5. Discussion

In our completely deterministic Bohm model, where the issues seem very clear, the closed time-like loops cause no problems. There is no real loss of information, and the situation after one particle has fallen into the hole is just as if it no longer exists.

It is of interest to ask what would happen in a hybrid model in which, for example, there is no y trajectory. Such indeed would be the situation, in the standard understanding of the Bohm model, if this variable referred to a boson, for which there are no trajectories. In this case, even without curious topologies, the basic Bohm formula, eq.(4), cannot be used because we do not have a value to insert for y on the r.h.s.(see ref.⁷ for discussion of this). Rather, it seems to be necessary to average over y , and to replace eq.(4) with

$$\dot{x} = \Re \frac{\int dy \Psi^* p_x \Psi}{m \int dy \Psi^* \Psi}. \quad (14)$$

With eq.(1) for the state, this leads to

$$\dot{x} = \Re \frac{|\alpha|^2 \phi_a^* p_x \phi_a + |\beta|^2 \phi_b^* p_x \phi_b}{m |\alpha|^2 |\phi_a|^2 + m |\beta|^2 |\phi_b|^2}. \quad (15)$$

This expression is clearly suitable for use even in the shaded region of fig.3. It is interesting to note that it corresponds neither to a pure or a mixed state. Does this suggest that in the Bohm model there might be testable effects if topological defects like the one we have assumed here actually exist? In particular we note that there is no wavefunction $\psi(x)$ which gives a trajectory of the form given in eq.(15).

It seems clear that, in any case, there are no conceptual problems with understanding the Bohm model in the presence of “black-holes” of the type considered here. Maybe this is another argument in favour of this model. Certainly things do not seem to be so clear if we use either the many-worlds model or a model with explicit collapse.

Work is in progress to extend some of these ideas to real black holes.

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Fig.1. Showing the topology of the two-dimensional space-time considered.

Fig.2. Showing possible tracks of wavepackets with, on the right, $f = g$ initially, and, on the left, $f = -g$ initially.

Fig.3. Showing how information about the wavefunction can be lost when we begin with a correlated state. The shaded region is where the effect of the information loss becomes evident.