On the influence of X-ray galaxy clusters in the fluctuations of the Cosmic Microwave Background

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ABSTRACT
The negative evolution found in X-ray clusters of galaxies limits the amount of available hot gas for the inverse Compton scattering of the Cosmic Microwave Background (the Sunyaev–Zel’dovich effect). Using a parametrisation of the X-ray luminosity function and its evolution in terms of a coalescence model (as presented in the analysis of a flux limited X-ray cluster sample by Edge et al. 1990), as well as a simple virialised structure for the clusters (which requires a gas to total mass fraction $\sim 0.1$ in order to reproduce observed properties of nearby clusters) we show that the Compton distortion $y$ parameter is about two orders of magnitude below the current FIRAS upper limits. Concerning the anisotropies imprinted on arcmin scales they are dominated by the hottest undetected objects. We show that they are negligible ($\Delta T/T \sim 10^{-7}$) at wavelengths $\lambda \sim 1$ mm. At shorter wavelengths they become more important ($\Delta T/T \sim 10^{-6}$ at $\lambda \sim 0.3$ mm), but in fact most clusters will produce an isolated and detectable feature in sky maps. After removal of these signals, the fluctuations imprinted by the remaining clusters on the residual radiation are still much smaller. The conclusion is that X-ray clusters can be ignored as sources of Cosmic Microwave
Background fluctuations.

**Key words:** cosmic microwave background — galaxies: clustering.
1 INTRODUCTION

Clusters of galaxies are the largest gravitationally bound structures in the Universe. Their potential wells, possibly created by still undetected dark matter, contain large amounts of X-ray emitting gas at temperatures of tens of millions of degrees or more. The amount of gas seen is comparable to the mass contained in the member galaxies, but it is doubtful that more than half of the mass of the cluster is contained in these components even when the gas profile is extrapolated to large distances (see, e.g., Briel et al. 1992; Mushotzky 1993).

Since energetic electrons are present in the intracluster medium, they inverse Compton scatter any long wavelength radiation from the background passing through the Cluster (Sunyaev & Zel’dovich 1972). This is particularly important for the Cosmic Microwave Background (CMB) radiation which constitutes an overall background source for all clusters. Clusters are optically thin to the CMB (typically the number of scatterings suffered by a microwave photon in a cluster is \( \tau < 10^{-3} \)), and since their covering factor of the sky is also small, the integrated effect is tiny. Nevertheless recent upper limits on the distortion of the CMB spectrum obtained by the FIRAS instrument on board of COBE (Mather et al, 1993) are indeed very stringent and might challenge models for cluster evolution (Markevitch et al, 1991).

Furthermore, in particular lines of sight where clusters with large amounts of gas are present, the effect might be detectable. This has been the goal of all searches for the Sunyaev-Zel’дович (SZ) effect, using the position of known bright X-ray clusters. The effect consists of a net upscattering of the photons where the average relative change in the frequency of the incoming photon is \( \frac{\Delta \nu}{\nu} \sim 4 \frac{kT_{gas}}{m_e c^2} \) (Rybicky & Lightman, 1979). In fact this results in an approximately constant decrease of the brightness temperature in the Rayleigh–Jeans region of the spectrum and a very steep increase of the temperature at frequencies beyond the CMB blackbody peak. The Sunyaev-Zel’dovich effect there is positive and steeply increasing with frequency. The extent in frequency of this effect at submillimetre wavelengths depends on the temperature of the cluster gas upscattering the background radiation.

A recent result on the X-ray properties of clusters of galaxies is that their X-ray luminosity function evolves negatively (Edge et al. 1990, Gioia et al. 1990a). That means that luminous clusters are underrepresented at high redshifts in comparison with what happens at low redshifts. Quantifying this effect is not an easy task since
it is very much dependent on the sample selection criteria. On the one hand, Edge et al. (1990) take a flux limited all-sky sample \( S(2 - 10\text{keV}) > 1.7 \times 10^{-11} \), with a redshift depth \( z \sim 0.1 \) and find different evolution for clusters above and below a luminosity \( \sim 8 \times 10^{44} \). On the other hand, Gioia et al. (1990a) and Henry et al. (1992) used the *Einstein Observatory* Extended Medium Sensitivity Survey (EMSS; Gioia et al., 1990b) to find also a negative evolution in the cluster X-ray luminosity function. All clusters in the EMSS sample are less luminous than \( 8 \times 10^{44} \) and they span a much larger redshift range. In fact, Gioia et al (1990a) find a significant change in the X-ray luminosity function at \( z \sim 0.3 \). Therefore both studies support the idea of negative evolution although in different luminosity and redshift parts of parameter space (see discussion on these issues by Henry 1992). However both studies suggest that X-ray luminous clusters form by merging of smaller sub-clusters. There is in fact some evidence for this in specific clusters both from the *Einstein Observatory* (Forman et al. 1981; Gioia et al. 1982; Jones & Forman 1992) and from *Rosat* in particular in Coma (White, Briel & Henry 1993, Davis & Mushotzky, 1993, Briel, Henry & Böhringer 1992), Perseus (Schwarz et al. 1992) and A2256 (Briel et al. 1991, Davis & Mushotzky, 1993). Since the X-ray volume emissivity is proportional to the square of the gas density, that means that the amount of gas available for the SZ effect in a typical line of sight is relatively small.

In this paper, we adopt a parametrisation of the X-ray luminosity function that correctly describes the flux-limited X-ray cluster sample considered by Edge et al. (1990) and a simple cluster model where the gas density falls from a constant within the core radius to an \( r^{-2} \) profile up to a maximum radius. We will show that this specific cluster model provides a fairly good description of nearby cluster properties if the gas to total mass ratio is larger than in the standard Cosmology, i.e., we require \( \frac{\rho_{\text{gas}}}{\rho_{\text{tot}}} \sim 0.15 \) when the standard nucleosynthesis value for a flat Universe is 0.06 \( (H_0 = 50 \text{ km s}^{-1} \text{ Mpc}^{-1}) \) will be used throughout the paper). We then compute the induced \( y \) Compton distortion parameter as well as the temperature fluctuations imprinted in the CMB on scales of arcmin at different wavelengths. Our computation indeed assumes that the coalescence model adopted by Edge et al. (1990) can be extrapolated up to higher redshifts. We, however, explore different values for the evolutionary parameter of this model, and the results remain unchanged within a factor of 2 or 3. There are other works, complementary to this one, where the CMB temperature fluctuations
introduced by the SZ effects are predicted for specific theoretical models of large-scale structure of the Universe (Markevitch et al, 1991, 1992, Cole & Kaiser 1988, Makino & Suto 1993, Bartlett & Silk 1993). Here we rely uniquely upon observational X-ray data of clusters and the simplest assumptions and extrapolations to make the appropriate predictions.

In Section 2 we discuss the way in which the clusters are modelled. Section 3 presents our main computations of the SZ effect at different wavelengths. We fully explore the signal produced by these clusters over wavelengths $\lambda \sim > 0.3$ mm (where the contribution of the Galaxy is not expected to be dominant), showing that the hottest ones will be easily detectable at submillimetre wavelengths and therefore that the statistical fluctuations induced in the residual CMB will be negligible at all wavelengths.

2 THE X–RAY CLUSTER POPULATION

2.1 The Cluster Model

In the simple model we adopted, clusters are considered isothermal spheres with an electron density profile:

$$n_e(r) = \begin{cases} n_{e0}, & r \leq r_c \\ \frac{n_{e0} r^2}{r^2_c}, & r_c < r \leq R \end{cases}$$

(1)

where $n_{e0}$ is the electron number density at the center of the cluster, $r_c$ is the core radius and $R$ is the total radius of the cluster out to which gas is present.

Assuming that the clusters are virialized to a radius $R_{vir}$ and that the gas is in hydrostatic equilibrium, from the spherical infall model (Peebles 1980) we can estimate the gas temperature and the virial radius

$$T_{gas} = 2.6 \times 10^8 \left( \frac{M}{10^{15} h_{50}^{-1} M_\odot} \right) \frac{\mu}{R_{vir} (\text{Mpc})} h_{50} K.$$

(2)

$$R_{vir} = \frac{2.71}{(1 + z)} \left( \frac{M}{10^{15} h_{50}^{-1} M_\odot} \right)^{1/3} h_{50}^{-1} \text{Mpc}.$$

(3)

where $m_p$ is the proton mass, $h_{50} = H_0/50$ Mpc$^{-1}$, $\mu$ is the mean molecular weight of the gas which is $\mu \approx 0.6$ for primordial abundances, and $M$ is the mass inside $R_{vir}$.
The ratios \( R/R_{\text{vir}} \) and \( R_{\text{vir}}/r_c \) are introduced in order to take into account the possibility that the gas in the cluster extends beyond the virial radius. Results will be showed for a wide range of these parameters. In particular we will consider a cluster radius ranging from 1 to 2 virial radii and a virial radius going from 5 to 10 core radii, which encompasses most of the reasonable parameters for clusters.

The central density can be found by assuming that the electron density is proportional to the mass density in the cluster:

\[
n_c(r) = \rho(r) \frac{f_{\text{gas}}}{m_p} \left( \frac{X + 1}{2} \right)
\]

where \( f_{\text{gas}} = \Omega_{\text{gas}}/\Omega_0 \) is the fraction of gas to total mass in the cluster and \( \Omega_0 \) will be taken as 1 for simplicity. \( \rho \) is the (total) mass density profile in the cluster and the gas is considered a fully ionized mixture of hydrogen and helium with a mass fraction \( X = 0.76 \) of hydrogen. The above expression indeed assumes that the gas to mass ratio \( f_{\text{gas}} \) is constant independent of environment (see below).

The total cluster mass is,

\[
M = 4\pi r_c^2 \rho_0 \left( R - \frac{2r_c}{3} \right)
\]

whereas the mass within the virial radius is

\[
M_{\text{vir}} = 4\pi r_c^2 \rho_0 \left( R_{\text{vir}} - \frac{2r_c}{3} \right)
\]

These expressions lead to a relation between total mass and virial mass as a function of \( R/R_{\text{vir}} \) and \( R_{\text{vir}}/r_c \) ratios. Comparing eq.(6) with the virial mass obtained from eq.(3) the value of the central density will be given by:

\[
\rho_0 = 2.825 \times 10^{-28} h_{50}^2 \left( \frac{R_{\text{vir}}}{r_c} \right)^3 \left( \frac{R_{\text{vir}}}{r_c} - \frac{2}{3} \right)^{-1} (1 + z)^3 - 3
\]

With these ingredients we can in principle try to reproduce some of the cluster properties. In order to see what are the suitable values for the different parameters involved (and in particular for \( f_{\text{gas}} \)) we have taken a sample of 25 nearby clusters for which we have luminosities and temperatures (David et al. 1993) and also the core radius and gas mass within 3 Mpc (Jones & Forman 1984) and tried to fit the global properties and correlations according to the model presented here. We have checked
that different values of $R/R_{vir}$ do not produce very significant changes in $f_{gas}$ when we try to describe this sample according to our model as far as $R/R_{vir} \sim > 1$. We have consequently assumed $R = R_{vir}$.

We have considered the Luminosity-Temperature relation (Figure 1) and the gas mass within 3 Mpc (Figure 2). Fairly good agreement between expected 2-10 keV luminosity (computed using the measured temperature) and the observed one is achieved if $f_{gas} \sim 0.15$. In Figure 1 we also show the luminosity - temperature relation for an average $R_{vir}/r_c = 5$ to 10 (at the mean redshift of the sample) and this value of $f_{gas}$. Our simplified model shows an approximate $L_X \propto T^\epsilon$ relation for constant $f_{gas}$ with $\epsilon \sim 2$. The value found in the whole David et al. (1993) sample is $\epsilon \sim 3.4$ close to the one found for the EXOSAT sample by Edge & Stewart (1991) which is $\epsilon \sim 2.8$. Although this might imply a change in $f_{gas}$ with mass scale, this point deserves further study.

A similar effect happens in Figure 2 where we plot the expected versus measured gas mass within 3 Mpc. Good agreement is found again for $f_{gas} \sim 0.15$ which is the value adopted here.

In any case the standard Cosmology value $f_{gas} = 0.06$ would clearly underpredict the luminosity for a given temperature as well as the gas mass within 3 Mpc. In fact, there is some recent evidence that bright clusters of galaxies may contain even a larger fraction of baryons to total mass. This is particularly true in the outskirts of some clusters (Briel, Henry & Böhringer 1992) where this ratio may be as large as $\sim > 0.3$ for Coma. If we have still underestimated $f_{gas}$ by some significant amount, the net effect of this in our computations of temperature fluctuations in the CMB is a decrease by the same amount.

2.2 The distribution of Clusters.

Unlike previous work where the distribution of clusters is described in the framework of the Press-Schechter mass function formalism (Cole & Kaiser 1988, Makino & Suto 1993, Bartlett & Silk 1993), we shall be using a simple coalescence model used by Edge et al. (1990) to fit their all sky low redshift cluster distribution. We shall extrapolate this model (allowing the evolutionary parameters to take a wide range of values) to the highest redshifts where clusters are thought to exist ($z \sim 1$).

Detailed work on cosmological models for cluster evolution has been presented
by Kaiser (1991) and Evrard & Henry (1991). They both conclude that standard
Cold Dark Matter scenarios with self similar evolution are unable to explain the shape
of the cluster X-ray luminosity function. In addition different specific cluster models
are built in these works which deviate either from the expected shape of the power
spectrum of the fluctuations or from the self-similar evolution or from both. However,
since it is not our goal to make detailed models for the cluster origin and evolution
we just take a simple parametrisation which is consistent with the available data and
try to extrapolate to higher redshifts keeping in mind that the model parameters can
have large uncertainties.

Using a model in which the growth of clusters is due to the merging of sub-
clusters, Edge et al. (1990) fitted the evolution of the luminosity function needed to
explain the detected deficit of luminous clusters at high redshift. The parametrisation
of the X-ray luminosity function proposed can be written as:

$$\phi(L_x) = \frac{10^{-4}}{B^2} \left( \frac{L_x}{10^{43}} \right)^{-0.6} \left( 1 - \frac{1}{B} \right) \left[ (\frac{L_x}{10^{44}})^{0.4} - 1 \right] \text{Mpc}^{-3} L_{44}^{-1}.$$  

(8)

where $L_x$ is the X-ray luminosity over the 2-10 keV energy band in $^{-1}$, $A = -4.9$, 
$B = b_1 + (b_2/(1+z)^{1.5})$ and $L_{44} = L_x/10^{44}$. Edge et al. (1990) suggest values
$b_1 = 0.923$, $b_2 = 0.82$ However we shall also explore how our results change when $b_2$
ranges from 0.4 to 1.1 keeping $b_1 + b_2 = 1.743$ in order to reproduce de-evolved cluster
luminosity function.

There is no evidence whatsoever for a low luminosity cutoff or for a redshift
cutoff, and in fact there is no need for them given the flatness of the luminosity function
and the fact that at a moderate redshift $z \sim 1$ there are very few clusters. Thus, in
our calculations we extrapolate this function out to a redshift $\sim 1$ where we consider
all clusters begin to form. However, we also checked that extending this high redshift
cutoff to 2 or 3 does not produce any change in our conclusions.

With this parametrisation we want to obtain the differential mass function
per unit comoving volume, since all relevant quantities (gas temperature, density and
cluster radius) can be related to the cluster mass and the parameters $R/R_{vir}$ and
$R_{vir}/r_c$ in the simple cluster model presented in the previous subsection. This requires
having the cluster X-ray luminosity in terms of its mass for a given redshift $z$. In order
to do that we recall that to a good approximation the cluster bremsstrahlung volume emissivity is

\[ e^{ff}_\nu = 6.8 \times 10^{-38} \sum_i Z_i^2 n_e n_i T_{gas}^{-1/2} \exp \left( -\frac{h\nu}{kT_{gas}} \right) \bar{g}_{ff}^{-3-1} \text{Hz}^{-1} \] (9)

where the sum is extended to all the ion species present (hydrogen and helium in a fully ionized mixture), \( n_e \) and \( n_i \) are the electron and the i-ion number density respectively, \( h \) is the Planck constant, \( k \) the Boltzmann constant and \( \bar{g}_{ff} \) is the averaged Gaunt factor which provides quantum mechanical corrections and that is taken as 1 here. The X-ray luminosity is then obtained by integrating the above emissivity over the energy range (2-10 keV in our case) and over the whole cluster volume. The result is

\[ L_x = 3.19 \times 10^{45} \left( \frac{R_{vir}}{r_c} - \frac{2}{3} \right)^{-2/3} \left( \frac{R}{r_c} - \frac{2}{3} \right)^{-4/3} \left( \frac{4}{3} - \frac{r_c}{R} \right) \]

\[ \left( \frac{R_{vir}}{r_c} \right)^3 f_{gas}^2 \left( \frac{X+1}{2} \right)^2 \mu^{-1/2} h_{50}^{7/3} (1+z)^{7/2} m^{4/3} \]

\[ \left[ \exp \left( -\frac{0.24}{\mu h_{50}^{2/3} v^{2/3} (1+z) m^{2/3}} \right) - \exp \left( -\frac{1.21}{\mu h_{50}^{2/3} v^{2/3} (1+z) m^{2/3}} \right) \right]^{-1} \] (10)

where \( v = \frac{R_{vir}/r_c - 2/3}{R/r_c - 2/3} \). Since for a given redshift \( z \), this provides a one to one relation between cluster X-ray luminosity and mass (except for the ratios \( R_{vir}/r_c \) and \( R/R_{vir} \) and \( f_{gas} \)), the number of clusters per unit volume and unit mass \( m = M/10^{15}M_\odot \) at a redshift \( z \) is

\[ \mathcal{N}(m;z) = \phi(L_x;z) \frac{dL_x(m;z)}{dm} \] (11)

This expression will be used to evaluate the Sunyaev-Zel’dovich effect along the line of sight.

As we want to evaluate the SZ effect for the unseen clusters (i.e., those above the X-ray detection threshold will be avoided in a CMB fluctuation analysis) we shall take the minimum redshift as \( z \sim 0.1 \). On the other hand, since we are going to be interested only in \( \sim \) arcmin fluctuations and anisotropies of the CMB radiation,
where the SZ is most relevant, cluster clustering can be ignored in principle. At $z = 1$ (taken here as the maximum redshift), one arcmin translates to $0.5\, h_{50}^{-1}$ Mpc. Since the cluster-cluster spatial correlation function is only fairly well known at separations larger than a few Mpc (Bahcall & Soneira 1983, Sutherland 1988), any extrapolation from larger separations might be completely wrong and therefore we shall adopt the conservative point of view of neglecting it. In Section 4 we present some evidence that any reasonable clustering amplitude at the scales we are working does not quantitatively affect our conclusions.

3 THE SUNYAEV–ZEL’DOVICH EFFECT

3.1 The effect produced by a single cluster

Cosmic Microwave photons which enter a cluster are upscattered by the hot electrons in the intracluster gas through a classical Thomson scattering process since the photon energies are much lower than the electron energies. The scattered photons will have an isotropic distribution provided that we consider both the electrons and the incident photons isotropically distributed.

As the mean number of scatterings suffered by the photons is equal to the gas optical depth, \( \tau = \sigma_T \int dln n_e \approx 10^{-2} - 10^{-3} \) (\( \sigma_T \) being the Thomson cross section and \( n_e \) the number density of electrons), the inverse Compton scattering process can be evaluated under the single scattering approximation. Also since the gas temperature is less than 10 keV for all the clusters detected so far we will also consider non relativistic electrons, with a Maxwellian velocity distribution. Under these circumstances the spectrum of the background radiation along the line of sight observed at \( z = 0 \) is given by

\[
N_{\text{out}}(x, z = 0) = (1 - \tau(z)) N_{\text{in}}(x, z = 0) + \tau(z) \int dx' \overline{G}(x - x'; T_{\text{gas}}) N_{\text{in}}(x', z = 0)
\]  

(12)

where \( x = \ln \left( \frac{h \nu}{k T_r} \right) \), \( \nu \) the frequency of the photon, and \( T_r \) the present temperature of the CMB Radiation which is $2.726 \pm 0.01$ K (Mather et al. 1993). The Green function \( \overline{G}(x - x'; T_{\text{gas}}) \) gives the probability that a photon with incident \( x' \) is scattered to a value \( x \) by the electrons at a temperature \( T_{\text{gas}} \) (Rybicky & Lightman, 1979), \( \tau(z) \) is the gas optical depth and \( N(x, z) \) is the differential number density of photons at redshift \( z \) (number of photons per unit \( x \) per unit volume), i.e.,
\[ N(x, z = 0, T_b) = \frac{8\pi}{c h} \left( \frac{k T_r}{e} \right)^3 \frac{e^{3x}}{\exp \left( \frac{T_r}{T_b} e^x \right) - 1} \]  

(13)

From these changes in the spectrum we may calculate the Sunyaev-Zel’dovich (SZ) change in the CMB temperature expressed as:

\[ \frac{\Delta T}{T} = \frac{T_b - T_r}{T_r} \]  

(14)

\( T_b \) being the brightness temperature of the outgoing spectrum.

In Figure 3 we plot this temperature variation for different representative cases, once the result has been smeared out with a beam of 1 arcmin FWHM. For \( \lambda > 1 \) mm the decrement is negative (the temperature of the radiation which comes out of the cluster is less than the background incoming temperature). Nevertheless, in the Wien part of the spectrum (\( \lambda < 1 \) mm) there is an important increase of the temperature. This is due in part to the low value of the intensity of the unperturbed blackbody spectrum at these wavelengths to be compared with the approximate power-law that the comptonized spectrum develops in that wavelength domain (see Rybicky 

& Lightman 1979). Therefore the best wavelengths to look for the SZ effect are in principle the shortest possible ones, as far as the contamination from the galaxy can be neglected.

3.2 The Compton \( y \) parameter

The Compton distortion parameter defined as \( y = \int dl \frac{k T_{gas}}{m_e c^2} \sigma_T n_e \) where \( m_e \) is the electron mass and the integral is evaluated along the line of sight, can be calculated from the temperature decrement of the background radiation due to the SZ effect in the Rayleigh–Jeans part of the spectrum (\( \lambda > 10 \) cm):

\[ \left. \frac{\Delta T}{T} \right|_{R-J} = -2y \quad \text{thus} \quad <y> = -\frac{<\Delta T >_{R-J}}{2 T_r} \]  

(15)

The \( <\Delta T >_{R-J} \) shift in the radiation temperature is evaluated by averaging the effect produced by a single cluster over the number density of clusters along the line of sight. We checked that the changes in the spectrum are quite small and hence they can be related to the change in the temperature linearly.
\[ N(x, T_b) \approx N(x, T_r) + \frac{\partial N}{\partial T} \bigg|_{T_r} (T_b - T_r) \] (16)

The average temperature shift can be obtained from the average change in the photon number density, which is:

\[
<\Delta N(x)> = <N(x) - N_{in}(x)> = \int_{z_{min}}^{z_{max}} dz \ cH_0^{-1} (1 + z)(1 + \Omega_0 z)^{-1/2} d_A(z) \\
\int_{\Omega} d\Omega \int_{M_{min}}^{M_{max}} dm \ N(m, z) \tau(n, m, z) \left[ \int dx' \ \bar{G}(x - x'; T_{gas}) N_{in}(x') - N_{in}(x) \right]
\] (17)

where \(d_A(z)\) is the angular distance (Weinberg 1972), and in principle we shall use \(M_{min} = 10^{13} \ M_\odot\) and \(M_{max} = 5 \times 10^{16} \ M_\odot\). This last parameter is particularly relevant, since the flattening of the log \(N - \log S\) X-ray cluster counts at low fluxes, reflected in the negative evolution of the luminosity function (Edge et al. 1990), implies that fluctuations will be dominated by the X-ray brightest and, therefore, more massive objects.

In order to make a proper comparison with current COBE/FIRAS upper limits on the \(y\) parameter we have computed eq. (17) from \(z_{min} = 0\) since in those observations no extragalactic sources were avoided. Therefore in Figure 4 we plot \(<y>\) as a function of \(R/R_{vir}\) for different \(R_{vir}/r_c\) ratios. Although there is a slight increase of the \(y\) parameter with \(r_c/R_{vir}\) (which is due to the fact that the smaller this parameter is the larger the central gas density will grow and therefore with less gas the same X-rays are produced) it is always much smaller than the COBE/FIRAS present limits, \(y < 2.5 \times 10^{-5}\) (Mather et al. 1993).

3.3. Induced anisotropies and temperature fluctuations.

Since clusters are extended and since the number of clusters in a typical line of sight is finite (and in fact very small) they imprint fluctuations and correlations in the CMB maps. We are now going to compute these effects by assuming that the space distribution of clusters is uniform at any given redshift. As mentioned before and discussed with some detail in Section 4, cluster clustering is very uncertain on scales comparable to the cluster size (\(~\) few Mpc). However, even with an extrapolation of
the \( r^{-1.8} \) law from larger separations, the effect is negligible on the results presented here.

If \( \theta \) is the angle between two directions represented by the \( \hat{n}_1, \hat{n}_2 \) unit vectors, the angular correlation function for the radiation spectrum is given by

\[
C_N(\theta, x) = \langle (N(\hat{n}_1, x) - \langle N(x) \rangle)(N(\hat{n}_2, x) - \langle N(x) \rangle) \rangle
\]  

(18)

From eq. (18) the angular correlation function is

\[
\begin{align*}
C_N(\theta, x) &= \int_{z_{\text{min}}}^{z_{\text{max}}} dz \, c H_0^{-1}(1 + z)(1 + \Omega_0 z)^{-1/2} d\theta(z) \\
&= \int d\Omega_{\hat{n}_1} \int_{m_{\text{min}}}^{m_{\text{max}}} \left[ \frac{d m \, N(m, z) \, \tau(\hat{n}_1, m, z) \, \tau(\hat{n}_2, m, z)}{(1 + \Omega_0 z)^{-1/2}} \right] \\
&= \left[ \int dx' \, N(x') \, \tau(\hat{n}_1, m, z) \, \tau(\hat{n}_2, m, z) \right] \nonumber \\
\end{align*}
\]  

(19)

Again we use the approximation in eq. (16) to derive the temperature correlation function starting from the correlation in the radiation spectrum:

\[
C_T(\theta, x) = \left[ \left| \frac{\partial N(x, T_b)}{\partial T_b} \right|_{T_b} \right]^{-2} C_N(\theta, x)
\]  

(20)

Due to the finite size of the receiver antenna a smoothing of the intrinsic correlation function is produced. Thus we approximated the antenna by a Gaussian of dispersion \( \sigma \) in order to compare our calculations and the results obtained through real experiments. The resulting correlation function is

\[
C_N(\alpha, \sigma, x) = \frac{1}{2\sigma} \int C_N(\theta, x) \exp \left( -\frac{\alpha^2 + \theta^2}{4\sigma^2} \right) I_0 \left( \frac{\alpha \theta}{2\sigma} \right) \theta \, d\theta
\]  

(21)

where \( C_N(\theta, x) \) is the correlation in the spectrum at a wavelength \( \lambda = h \, c / (k \, T_r \, c^2) \), \( I_0 \) is the modified Bessel function and \( \alpha \) is the beam throw of the telescope. The rms temperature fluctuations can therefore be evaluated as

\[
\left( \frac{\Delta T}{T} \right)_{\text{rms}} = \left[ \frac{\langle (T - \langle T \rangle)^2 \rangle}{T_r} \right]^{1/2} = \frac{\left[ C_T(0, \sigma, x) \right]^{1/2}}{T_r}
\]  

(22)
In Figure 5 we plot the rms fluctuations versus the wavelength for $R/R_{\text{vir}} = 1$, $R/R_{\text{vir}} = 2$ (the extreme ratios) and for two different ratios $R_{\text{vir}}/r_c$ considering an antenna FWHM= 1 arcmin. In the same plot we show some upper limits from observations which correspond to experiments with similar characteristics.

A conclusion can be drawn from Figure 5, the rms fluctuations do not depend very much on the ratios between the total and the virial radius as well as the ratio between the virial and the core radius. However a slight increase occurs in the fluctuations when these ratios are smaller since then the gas density and, therefore, the optical depth are larger.

The other interesting conclusion is that at submillimetre wavelengths ($\lambda < 300\mu$) the SZ effect is quite more important. However, unless we have grossly overestimated the $f_{\text{gas}}$ parameter these SZ fluctuations will never dominate over primordial temperature fluctuations (typically $\sim 10^{-5}$).

In Figure 6 we show the rms temperature fluctuations as a function of the beam FWHM for a wavelength $\lambda = 0.3$ mm, $R = R_{\text{vir}}$, and $R_{\text{vir}} = 5r_c$. These range from $\sim 2$ to $\sim 7 \times 10^{-6}$. We also show in that figure the temperature variation produced by a single cluster at $z = 0.5$ and different temperatures when the beam is aligned with the cluster centre. It is clear that for a $\sim 1$ arcminute beam, even a cluster of $T \sim 1$ keV would be easily detectable since confusion does not operate here (there are $\sim 3$ clusters per square degree in total). These and any hotter clusters will then be easily detected and removed from the CMB fluctuation analysis. If we then recompute the rms fluctuations produced by clusters cooler than $\sim 1$ keV they are of the order of $10^{-8}$.

The situation might be slightly different for larger beam sizes. At 10 arcmin, the rms temperature fluctuations are larger and the effect produced by a single cluster is somewhat diluted. Clusters with temperatures $T \lesssim 3$ keV might be confused, but those with gas temperatures $T \gtrsim 5$ keV would be easily detectable (a $> 10\sigma$ signal at the cluster centre. Again removing these hotter objects, the rms temperature fluctuations in the residual CMB do not exceed $\sim 10^{-7}$.

A general conclusion can then be drawn from this study. Clusters of galaxies produce negligible rms temperature fluctuations in the CMB sky through Sunyaev–Zel’dovich effect at long wavelengths ($\lambda \gtrsim 1$ mm). At shorter wavelengths, the effect is larger but it never produces anything equivalent to confusion noise (which could
contribute to true background noise). Instead, the hottest clusters which dominate these fluctuations, appear as clear signals in sky maps, and will therefore be removed in any CMB fluctuation analysis. The contribution of the undetected objects to the residual CMB fluctuations is negligible.

These conclusions could be different if the evolution undergone by clusters is not well represented by equation (8). In that sense we have evaluated the changes produced in the rms temperature fluctuations when the parameter \( b_2 \) (responsible for the evolution with redshift) in \( B \) takes different values. The results obtained are shown in Figure 7. For both beam sizes (FWHM = 1 arcmin and 10 arcmin) the fluctuations increase as the parameter \( b_2 \) decreases i.e., the fluctuations are larger when negative evolution is weaker. Nevertheless this change is small (a factor of 2-3 at most) and therefore our conclusions remain unchanged for any reasonable value of the parameter \( b_2 \).

4 DISCUSSION

The simple parametrisation of cluster structure and redshift evolution used here leads to the conclusion that confusion noise from the Sunyaev–Zel’dovich effect from these objects does not contribute to the CMB fluctuations. At the shortest wavelengths under study (\( \lambda \sim 0.3 \text{ mm} \)) the hottest clusters which produce the largest fluctuations will be clearly isolated by any instrument aiming at a detection of CMB angular structure on arcmin scales. The less massive undetected clusters will produce negligible background noise. For large beamsizes (\( \sim 10 \text{ arcmin} \)), confusion will be more relevant and only the hottest clusters (\( T_{gas} \sim > 5 \text{ keV} \)) will be detectable. But even in that case, the remaining undetected objects will produce negligible sky noise.

Deviations from the picture presented here might arise for various reasons. First, if the cluster gas has inhomogeneities on top of the smooth profile assumed here, less Sunyaev–Zel’dovich effect will arise, since gas inhomogeneities enhance the X-ray luminosity. The same applies to cooling flow clusters which will produce less Compton scattering than non-cooling flow ones for a given X-ray luminosity.

The next point to take into account is the effect of clustering of clusters in our study. Assuming that the correlation function

\[
\xi(r) = \left( \frac{r}{r_0} \right)^{-1.8}
\]

(23)
(where the correlation length \( r_0 \sim 40 \, h_{50} \, \text{Mpc} \) can be extrapolated down to arbitrarily small separations we can estimate the enhancement of the CMB temperature fluctuations. If we assume that the correlation length corresponds to 80 arcmin at \( z = 1 \) it is easy to estimate that the probability of finding two clusters in the same beam due to their clustering is \( \sim 10^{-3} \) for one arcmin beam and \( \sim < 10 \) per cent for a 10 arcmin beam. That means that on average the clustering can be completely neglected for beamsizes of a few arcmin and that the corrections introduced for a beam size of 10 arcmin are quite small.

Our results heavily rely on the cluster properties obtained from the X-ray data. In particular the flatness of the cluster X-ray source counts implies that the SZ fluctuations on the CMB are dominated by the X-ray brightest and, therefore hottest clusters (Markevitch et al. 1991 also reach the same conclusions). The situation may be quite different if large numbers of faint X-ray clusters were present in the sky, i.e., if the cluster X-ray source counts would raise again at fluxes below \( \sim 10^{-11} \, -2^{-1} \). In that case, for which there is no observational support whatsoever, the SZ effect on the CMB would be larger and indeed more similar to true confusion noise.

So far, available X-ray data on clusters tell us that the SZ effect from these objects both in the spectrum and in the fluctuations of the CMB will not be relevant at any wavelength.

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FIGURE CAPTIONS

Figure 1: X-ray luminosity vs temperature for a sample of 25 clusters. The hollow points denote the observed values, whilst the filled ones the computed luminosities according to our model (errors from the uncertainties in the temperature). The solid lines correspond to the expected relation for $R_{\text{vir}}/r_c = 5$ (below) and $R_{\text{vir}}/r_c = 10$ (above) for $f_{\text{gas}} = 0.15$. Same values for $f_{\text{gas}} = 0.06$ are shown as dotted lines.

Figure 2: Computed gas mass within 3 Mpc for a sample of 25 clusters using the measured temperatures, $R = R_{\text{vir}}$ and $f_{\text{gas}} = 0.15$ as a function of observed gas mass within 3 Mpc (by Jones & Forman 1984).

Figure 3: Temperature variation for 4 different cluster temperatures: 2.8 keV (solid), 4.7 keV (dashed), 6.7 keV (dot-dashed), 8.6 keV (dotted), smeared out with a beam of 1 arcmin FWHM pointing to the cluster centre. A zero-crossing occurs whenever a curve hits the x-axis in this plot. The redshift is $z = 0.5$, $R = R_{\text{vir}}$ and $R_{\text{vir}} = 5r_c$ (see text for details).

Figure 4: Average Compton $< y >$ parameter versus $R/R_{\text{vir}}$ for different $R_{\text{vir}}/r_c$ ratios. Solid line at the top is COBE FIRAS upper limit.

Figure 5: The rms fluctuations for a map with FWHM= 1 arcmin. Solid lines correspond to $R/R_{\text{vir}} = 1$ and $R_{\text{vir}}/r_c = 5, 10$ (from top to bottom). Dotted lines are for $R/R_{\text{vir}} = 2$ and the same for $R_{\text{vir}}/r_c$ ratios. The vertical arrows are upper limits from observations with VLA (Fomalont et al. 1988, with 60 arcsec FWHM, and Fomalont et al. 1993, with a resolution of 80 arcsec), ATCA (Subrahmanyan, Ekers, Sinclair & Silk 1993, with 2.1 arcmin FWHM) and IRAM (Radford 1993, with 55 arcsec FWHM), from right to left. Due to the different beamsizes of each experiment only the upper limit from Fomalont et al. (1988) can be directly compared to the lines plotted here.

Figure 6: The rms temperature fluctuations as a function of the beam FWHM for a wavelength $\lambda = 0.3$ mm, $R = R_{\text{vir}}$, and $R_{\text{vir}} = 5r_c$ (individual points). The lines correspond to the temperature variation produced by a single cluster: 2.8 keV (solid), 4.7 keV (dashed), 6.7 keV (dot-dashed), 8.6 keV (dotted). Asterisks correspond to the fluctuations produced by objects with $T < 1$ keV at a FWHM of 1 arcmin and objects with $T < 5$ keV at a FWHM of 10 arcmin.
Figure 7: The rms temperature fluctuations as a function of the parameter $b_2$ (see text for details) for beams of 1 arcmin FWHM (solid line) and 10 arcmin FWHM (dashed line).