THE CASIMIR EFFECT AS A POSSIBLE SOURCE
OF COSMIC ENERGY

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Abstract

The energy production due to Casimir effect is considered for the case of a conductive superdense state of matter, which can appear in such cosmological objects as neutron stars, quasars and even the core of the Earth. The energy output produced due to Casimir effect during the creation of a neutron star turns out to be sufficient for explanation of supernova explosions. It is shown that Casimir effect might be a possible source of the huge energy output of quasars. For the Earth’s core, it is found that the energy output is about that needs to heat our planet during the Earth existence.
The aim of the present work is to attract the attention of physicists to another possible source of energy in the cosmic scale, namely the Casimir effect. This effect is well-known in physics (see, e.g., [1-3]). It consists in the energy shift of zero-point vacuum fluctuations of an electromagnetic field due to non-trivial boundary conditions for the field. Usually it is so-called surface Casimir effect, which takes place when the boundary condition is given on a definite surface. An example of the surface Casimir effect is the appearance of a small attractive force between two uncharged metal plates. At the same time, in his work [1] Casimir mentioned a volume effect. Consider the latter in more detail. Let there be zero boundary conditions for an electromagnetic field in a volume $V$ (as if the volume $V$ would be filled by an ideal conductor). Calculate now the shift of energy density of the field for the volume $V$ relative to the case of the absence of the volume. It is equal to the mean value of the 00-component of the stress-energy tensor of the quantum electromagnetic field inside the volume $V$

$$\Delta \epsilon_{vac} = <0|T_{00}|0> = -\hbar c \int_0^\omega \frac{d^3k}{2 (2\pi)^3},$$

(1)

where $\omega = |k|$, $k$ is the wave vector of the electromagnetic field.

One may see that the integral (1) is equal to infinity. So, the reasonable final result should depend on a dimensional cut-off parameter. It is the reason why the surface effect is usually considered, where the final result does not depend, as a rule, on any cut-offs and where the divergences like (1) are canceled.

Nonetheless the volume Casimir effect (VCE) can play an important role in cosmology. In the work [4] it is demonstrated that due to VCE the matter can transfer into a "pseudovacuum" state, which could be associated with a source for the Kerr-Newman metric. Moreover in that paper it is noted that VCE may play a role in the collapse and singularity problems in cosmology.

In the present Letter we are demonstrating that VCE can lead to a huge energy output during compressing a sufficiently big conductive volume. We shall apply it to neutron stars (supernova), quasars and the Earth. In what follows we shall not consider the problem of the energy output due to nuclear reactions. As a rule, in our calculations the energy due to VCE exceeds nuclear energy output (!).

Let us consider a volume $V$ that is filled by a real conductive material. Then we obtain the finite result for $\Delta \epsilon_{vac}$ (1). It is a result of well-known fact (e.g., [2,3,5]) that real conductors are transparent for high-frequency electromagnetic field provided $\omega > \omega_{max} \sim \pi/a$, where $a$ is an of the order of interatomic distance. It should be noted that exact theory for the calculation of VCE in the case of a real conductor is absent as yet. In what follows we shall use two approximate methods. Let us begin with a simple approximation: let any good conductive metal be as an ideal conductor for the $\omega < \omega_{max}$ and as a transparent material for $\omega > \omega_{max}$. Thence one has for the shift of vacuum energy density inside of the volume (due to the presence of the volume)

$$\Delta \epsilon_{vac} = <0|T_{00}|0> = -\hbar c \int_0^{\omega_{max}} \frac{\omega^4}{8\pi^2} \frac{d^3k}{2 (2\pi)^3} = -\frac{\omega_{max}^4}{8\pi^2} \hbar c,$$

(2)

Rigorously speaking, the total energy shift $E$ can be found as a result of integration of $<0|T_{00}|0>$ over the whole space.
\[ E = \int_{\text{inside } V} <0|T_{00}|> d^3x + \int_{\text{outside } V} <0|T_{00}|> d^3x \quad (3) \]

It is easy to see (e.g., [2,6]) that the second term in (3), corresponding to the Casimir energy outside the volume \( V \), is of the order of \( \sim \omega_{\text{max}}^3 \). Therefore it can be omitted in comparing with the first term.

Consider the following situation. Let the value of interatomic (interparticles) distance decrease due to gravitational compression. Then it leads to the following energy creation

\[ \Delta E = E_{\text{before compression}} - E_{\text{after compression}} = \]

\[ = \hbar c \int_0^{\omega_{\text{max}}^{\text{after}}} \frac{d^3k}{(2\pi)^3} \int_{\text{inside } V} d^3x \frac{\omega}{2} - \hbar c \int_0^{\omega_{\text{max}}^{\text{before}}} \frac{d^3k}{(2\pi)^3} \int_{\text{inside } V} d^3x \frac{\omega}{2} \]

\[ = \frac{\pi^2}{8} \hbar c \left[ \frac{V_{\text{after}}}{a_{\text{after}}^4} - \frac{V_{\text{before}}}{a_{\text{before}}^4} \right]. \quad (5) \]

Here we took into account the eqs. (1)-(3).

Let us estimate \( \Delta E \) for the case of the creation of a neutron star (about the latter - see, e.g., [7,8]). It is generally accepted that neutron star appears as a result of gravitational collapse of a massive star core. The radius of a neutron star is about 10-30 Km. Some percents (up to 20\%) of the volume of the neutron star is supposed (see, e.g., [7-9]) to be a proton-electron liquid, which is not only conductive but superconductive (i.e., more close to the ideal conductor). Let us put the radius of the neutron star \( R=20Km \) and the part of the proton-electron liquid \( \delta \) to be equal to 10\% of the volume of the star. Then the volume of (super)conductive material will be \( V \approx 3.4 \times 10^{12}m^3 \). The effective distance between particles inside of the neutron star \( (a_{\text{after}}) \) may be estimated as \( (1 - 5) \times 10^{-15}m \) (it corresponds to the matter density \( 10^{13}g/cm^3 < \rho < 10^{15}g/cm^3 \)). It should be noted that it is not necessary for our purpose that the core of the primordial star would be a conductive. It is important that \( a_{\text{after}} \ll a_{\text{before}} \). Now one can find the energy (4), which is created due to the neutron star arising

\[ \Delta E \approx 2 \times 10^{45}J \left( \frac{\delta}{10\%} \right) \left( \frac{R}{20Km} \right)^3 \left( \frac{3 \times 10^{-15}m}{a_{\text{after}}} \right)^4. \quad (6) \]

In the present time it is generally accepted that collapse to the neutron star leads to supernova explosion. Neutron star is created for a part of second. If we compare the total energy of the most powerful supernova explosion (about \( 10^{43}J \)), one can conclude that the energy (6) is more than sufficiently to produce the explosion.

Let us estimate the temperature that is corresponding to the energy (6). Because the process of the neutron star arising is a rather fast, we shall omit the cooling due to radiation. Assuming that the energy (6) leads to heating the whole matter of the neutron star, one can find
where \( k_B \) is the Boltzmann constant, \( N \) is the number of particles in the star \((\sim R^3/a_{after}^3)\).

The value (6) corresponds to the modern suppositions about the temperature of the beginning of supernova explosion.

Let us estimate the energy output (6) by means of another method (see, e.g., [4, 10, 11]) for VCE calculation. The essay of the method consists in the simulation of dielectric permittivity \((\varepsilon)\) of conductive material by the following frequency dependence

\[
\varepsilon(\omega) = 1 - \frac{4\pi n e^2}{m \omega^2},
\]

where \( e \) is the charge of electron, \( m \) is the mass of particle that is responsible for conductivity, \( n \) their density.

Maxwell equations in media with dielectric permittivity (8) reduce to the wave equations for a vector particle with non-zero mass. As a result of calculations [10, 11], the leading term for the energy is equal to

\[
\Delta E_{vac} \approx -\frac{n e^2 \omega_{max}^2}{2\pi m} \hbar c.
\]

Here the cut-off parameter \( \omega_{max} \) is present as well. It is connected with the fact that equation (8) is not valid for big frequency. We shall put \( \omega_{max} \) to be equal to \( \pi/a \) as above, considering the conductor as a transparent medium for \( \omega > \omega_{max} \). By analogy with (3)-(5), one has for the final energy output

\[
\Delta E \approx \frac{\pi e^2}{2m_e c} \hbar \left[ \frac{N_{after}}{a_{after}^2} - \frac{N_{before}}{a_{before}^2} \right].
\]

where \( m_e \) is the electron mass, \( N \) is the total quantity of electrons in the electron liquid of the star. \( N \) is equal to the quantity of protons in proton liquid, which can be estimated as \( V \rho/(2m_p) \), where \( V \) is the volume of superconductive liquid, \( \rho \) the mass density of the star, \( m_p \) is the proton mass (because the mass of the liquid is gathered by protons mass practically).

Putting the same values as for equation (6), one finds for the energy output in this model

\[
\Delta E \approx 2 \times 10^{15} J \left( \frac{\delta}{10\%} \right) \left( \frac{R}{20Km} \right)^3 \left( 3 \times 10^{-15} m/a_{after} \right)^2.
\]

So, it is practically the same result as (6).

Consider now the quasars (the same for galaxy cores). It is reasonable to assume the existence of process of collapse in the core of the quasar. Let a neutron-star-object be a result of such a collapse. As was demonstrated above, it leads to powerful energy output. In contrast with the previous case there are a strong gravitational wall around the neutron-star-object. If these two factors compensate each other the process will be stationer. The process of addition of new material to the neutron-star-object can continue as soon as the temperature balance
appears between the neutron-star-object and the rest part of quasar. The excess of material of the neutron-star-object could collapse to a black hole, adding more gravitational energy to quasar (the question about conductivity of the matter, which density is near to critical, is unclear for now).

Estimate now how long a quasar can shine due to the process described above. Let we consider the neutron-star-object like the neutron star calculated above. Then a half of the quasar matter could produced the power of quasar radiation to be $\sim 10^{41} J/c$ (maximal observable quasar power [12,13]) for the time

$$t = 1.3 \times 10^6 yr \left( \frac{\delta}{10\%} \right) \left( \frac{R}{20 K m} \right)^3 \left( \frac{3 \times 10^{-15} m}{a_{after}} \right)^{4 or 2},$$

where two values of degrees correspond to two models we are considering, all the notation are the same as for eq.(6), the mass of quasar is taken to be $10^9$ the Sun masses.

Finally, let us consider the not such a big compression as before - look at the Earth. According to the modern theory of the Earth origin, our planet arose as a result of gravitational compression of gas and dust. The important point here is that before the Earth arising there was no such a big pressure as now in the Earth core. At the present, the core undergoes the pressure up $3.7 \times 10^{11} N/m^2$ [12]. Let us consider it consist approximately of the iron. The coefficient of volume compression for the iron is equal to $1.7 \times 10^{11} N/m^2$. Provided the pressure is greater than this value, it can destroy the crystal lattice of iron and compress the metal as "atom to atom". Usual iron has the constant of lattice ($a_{core}$) to equal to $2.87\AA$. After compressing, the distance between the atoms can achieve two crystal radii of the iron atoms, which is equal to $2.52\AA$ ($a_{after}$).

Due to the modern data, the radius of the core, below of that the pressure exceeds the coefficient of volume compression for iron, is about $3 \times 10^8 K m[12]$. So, we should wait for the energy output from this region. Substituting the aforementioned values in the eq.(5) and (6) (where $N$ means the total quantity of electrons inside the volume under consideration), one finds the VCE energy production due to the core compression

$$\Delta E = \left[ 1.2 \times 10^{32} \text{ or } 1.4 \times 10^{30} \right] J$$

Here two values correspond to two models under consideration.

In contrast with the neutron star case, there is essential difference VCE we can not conclude which result is more close to reality.

Consider the question: up to what temperature can this energy heat the whole matter of the Earth? We shall consider the Earth matter consisted of silicon. Then the number of particles (atoms) of the Earth can be estimated as $M_{Earth}/m_{atom}$, where $m_{atom}$ is the atom mass of silicon. So, one has for the temperature

$$T = \Delta E k_B \frac{m_{atom}}{M_{Earth}} \approx \left[ 7 \times 10^4 \text{ or } 8 \times 10^3 \right] K$$

If we consider the known problem (considered a long ago by W.Thomson (Lord Kelvin)) of the cooling down of the Earth, one will find that the Earth must have $T \approx 3 \times 10^4 K$ at the
beginning of its existence. So, one may conclude that VCE can bring an appreciate contribution to heating the Earth.

All these estimations show that the volume Casimir effect should be taken into account in Astrophysics, Cosmology, physics of the Earth. I hope that it also attract the efforts to the elaboration of the theory of the volume Casimir effect.

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References