Inflation, Supergravity and Superstrings

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Abstract

The positive potential energy required for inflation spontaneously breaks supersymmetry and in general gives any would-be inflaton an effective mass of order the inflationary Hubble parameter thus ruling it out as an inflaton. In this paper I give simple conditions on the superpotential that eliminate some potential sources for this mass and derive a form for the Kähler potential that eliminates the rest. I then show that Kähler potentials of this form can give rise to a new class of models of inflation, the simplest examples of which give a spectral index $1 - n = 1/30$ for the density perturbations and negligible gravitational waves. Finally I point out that Kähler potentials of this form are contained in the Kähler potentials that have been derived from superstrings.

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1 Introduction

The approximate isotropy of the cosmic microwave background radiation implies that the inflation [1] that inflated the observable universe beyond the Hubble radius must occur at an energy scale $V^{1/4} \leq 4 \times 10^{16}$ GeV [2], and it is thought that physics at energies below the Planck scale is described by an effective $N = 1$ supergravity theory [3]. Thus models of inflation should be constructed in the context of supergravity. However, this immediately leads to a problem. The positive potential energy $V$ required for inflation spontaneously breaks supersymmetry which would generally be expected to give effective masses $\sim \sqrt{8\pi V/m_{Pl}} \sim H$ to any would-be inflatons. But inflation requires $V''/V \ll 1$, i.e. the effective mass of the inflaton must be much less than $H$.

Natural inflation [4] avoids this problem by assuming the inflaton corresponds to an angular degree of freedom whose potential is kept flat enough by an approximate compact global symmetry. Solutions to this problem which work for $\phi^a$ chaotic inflation have also been proposed [5, 6], but they rely on forms for the supergravity Kähler potential that have no independent motivation. In this paper I will propose a solution for inflaton fields which are not purely angular degrees of freedom and which uses a well motivated form for the Kähler potential. Some aspects of this solution have been investigated in [7] for the case of false vacuum inflation.

2 Basic Formulae and Notation

I will use the following conventions in this paper: $m_{Pl}/\sqrt{8\pi} = 1$, a prime will denote the derivative with respect to the canonically normalised inflaton field $\sigma$, a bar will denote the hermitian conjugate, $\phi$ will represent a vector whose components $\phi^a$ are complex scalar fields, and subscript $\alpha$ will denote the derivative with respect to $\phi$, so for example $W_\phi$ represents the vector with components $\partial W/\partial \phi^\alpha$.

2.1 Global supersymmetry

In global supersymmetry [3] the scalar kinetic terms are

$$|\partial_\mu \phi|^2,$$

where $\phi = (\phi^1, \phi^2, \ldots)$ and the $\phi^a$ are complex scalar fields. The scalar potential is

$$V = |W_\phi|^2 + \frac{1}{2} \sum_a g_a^2 D_a^2,$$

\footnote{After inflation $V$ disappears and so supersymmetry is restored modulo whatever breaks supersymmetry in our vacuum.}
with

\[ D_a = \bar{\phi} Q_a \phi + \xi_a , \]

where the superpotential \( W(\phi) \) is an analytic function of \( \phi \), \( a \) labels the gauge group generators \( Q_a \), \( g_a \) is the gauge coupling constant, and the real constant \( \xi_a \) is a Fayet-Iliopoulos term that can be non-zero only if \( Q_a \) generates a \( U(1) \) gauge group. The first term is called the \( F \)-term and the second the \( D \)-term. I will assume that the \( F \)-term gives rise to the inflationary potential energy density and that the \( D \)-term is flat along the inflationary trajectory so that it can be ignored during inflation. It may however play a vital role in determining the trajectory and in stabilising the non-inflaton fields.

### 2.2 Supergravity

The scalar fields in a supergravity theory are the coordinates of a Kähler manifold. The metric on a Kähler manifold is

\[ K = \bar{\phi} \phi \]

where the Kähler potential \( K(\phi, \bar{\phi}) \) is a real function of \( \phi \) and its hermitian conjugate \( \bar{\phi} \). The scalar kinetic terms are

\[ \partial_\mu \bar{\phi} K_{\bar{\phi} \phi} \partial^\mu \phi , \]

the \( F \)-term part of the scalar potential is

\[ V_F = e^K \left[ (W_\phi + \bar{W}_\phi) K_{\phi \phi}^{-1} \left( \bar{W}_\phi + W_\phi \right) - 3|W|^2 \right] , \]

and the \( D \)-term part is

\[ V_D = \frac{1}{2} \sum_{a,b} \left( \text{Re} f_{ab} \right)^{-1} D_a D_b , \]

with

\[ D_a = K_{\phi} Q_a \phi + \xi_a , \]

where \( f_{ab}(\phi) \) is an analytic function of \( \phi \) transforming as a symmetric product of two adjoint representations of the gauge group. Only the combination

\[ G(\phi, \bar{\phi}) = K + \ln |W|^2 \]

is physically relevant and we are always free to make a Kähler transformation:

\[ K(\phi, \bar{\phi}) \to K(\phi, \bar{\phi}) - F(\phi) - F(\bar{\phi}) , \quad W(\phi) \to e^{F(\phi)} W(\phi) . \]

### 2.3 Inflation

I assume the effective action during inflation to have the form

\[ S = \int d^4x \sqrt{-g} \left[ -\frac{1}{2} R + g^{\mu \nu} \partial_\mu \bar{\phi} K_{\bar{\phi} \phi} \partial_\nu \phi - V(\phi, \bar{\phi}) \right] , \]
and make the usual flat Robertson-Walker ansatz
\[ ds^2 = dt^2 - a(t)^2 dx^2, \quad \phi = \phi(t). \] (11)

The Hubble parameter \( H \) is defined as \( H \equiv \dot{a}/a \). Inflation [1] requires \( -\dot{H}/H^2 \ll 1 \), or equivalently \( 3H^2 \simeq V \), i.e. the energy density of the universe should be dominated by the scalar potential. The dynamics of the scalar fields then rapidly approaches the slow-roll equations of motion
\[ \ddot{\phi} = -3HK\dot{\phi} V_\phi, \] (12)

and I will assume that they have been attained for all epochs of interest. The canonically normalised inflaton \( \sigma \) is defined by
\[ \frac{1}{2} d\sigma^2 = d\phi K^{\phi\phi}_\phi V_\phi, \] (13)

and combining the slow-roll equations for \( \phi \) and \( \sigma \) gives
\[ \phi' = \frac{K^{\phi\phi}_\phi V_\phi}{V''}, \] (14)

where a prime denotes the derivative with respect to \( \sigma \). The conditions necessary for inflation can be expressed in terms of the potential as
\[ \left( \frac{V'}{V} \right)^2 \ll 1, \quad \left| \frac{V''}{V} \right| \ll 1. \] (15)

3 The Basic Problem

At any point in the space of scalar fields we can make a holomorphic field re-definition such that \( \dot{\phi} = 0 \) and the scalar fields have canonical kinetic terms at that point. Any purely holomorphic terms in the Kähler potential can then be absorbed into the superpotential using a Kähler transformation. Then, in the neighbourhood of that point, the Kähler potential will be
\[ K = |\phi|^2 + \ldots, \] (16)

where \( \ldots \) stand for higher order terms. Therefore from Eq. (5)
\[ V_K = \exp \left( |\phi|^2 + \ldots \right) \times \right\{ \left[ W_\phi + W \left( \bar{\phi} + \ldots \right) \right] (1 + \ldots) \left[ \bar{W}_\phi + \bar{W} (\phi + \ldots) \right] - 3|W|^2 \right\}. \] (17)

\( ^2 \)Strictly speaking \( -\dot{H}/H^2 < 1 \) is all that is required. However realistic models satisfy \( -\dot{H}/H^2 \ll 1 \). See [8] for a more detailed discussion.
Now at $\phi = 0$ the exponential term gives a contribution $V_F$ to the effective mass squared of all scalar fields. Therefore, taking $\phi = 0$ to be during inflation and assuming the inflationary potential energy density is dominated by the $F$-term, the exponential term gives a contribution of 1 to $V''/V$. But $|V''/V| \ll 1$ is necessary for inflation to work. So a successful model of inflation must arrange for a cancellation between the exponential term and the terms inside the curly brackets. This will require fine tuning unless a symmetry is used to enforce it. Natural inflation [4] uses an approximate compact global symmetry. I will use an approximate non-compact global symmetry.

4 A Proposed Solution

Divide the vector of scalar fields $\phi$ into two separate vectors, $\varphi$ and $\psi$: $\phi = (\varphi, \psi)$. Assume that during inflation

$$W = W_\varphi = 0.$$  (18)

Then from Eq. (5)

$$V_F = e^K W_\psi K^{-1}_\psi \bar{W}_\psi,$$  (19)

and so, assuming $V_F$ dominates the inflationary potential energy density, we need $W_\psi \neq 0$ during inflation. Assume the inflaton is contained in $\varphi$. Then, during inflation, $\psi$ is constant and so without loss of generality we can set

$$\psi = 0.$$  (20)

For simplicity, assume that

$$K_{\varphi\psi}|_{\psi=0} = 0,$$  (21)

so that $\varphi$ and $\psi$ have no mixed kinetic terms during inflation. Then, using a Kähler transformation to remove any remaining terms linear in $\psi$ and expanding about $\psi = 0$ we get

$$K = A(\varphi, \bar{\varphi}) + \bar{\psi} B(\varphi, \bar{\varphi}) \psi + \mathcal{O}(\psi^2, \bar{\psi}^2),$$  (22)

where $A$ is real and $B$ is hermitian. Note that Eqs. (18), (21) and (22) become automatic if we impose the symmetry (an R-parity)

$$\psi \rightarrow -\psi, \quad \varphi \rightarrow \varphi, \quad W \rightarrow -W, \quad K \rightarrow K,$$  (23)

which also helps to stabilise $\psi$ at 0 because $V_\psi = 0$ is also automatic. From Eqs. (19) and (22),

$$V_F = e^A W_\psi B^{-1} \bar{W}_\psi,$$  (24)
and so to eliminate the inflaton dependent corrections to the global supersymmetric potential we require

\[ B^{-1} = f(\varphi, \bar{\varphi}) C^{-1}(\chi, \bar{\chi}), \]

and

\[ A = -\ln f(\varphi, \bar{\varphi}) + g(\chi, \bar{\chi}), \]

where \( f \) and \( g \) are real functions, \( C \) is a hermitian matrix and \( \chi \) are non-inflaton \( \varphi \) fields. Thus we obtain the general form for the Kähler potential

\[ K = -\ln \left[ f(\varphi, \bar{\varphi}) - \bar{\psi} C(\chi, \bar{\chi}) \psi \right] + g(\chi, \bar{\chi}) + O(\psi^2, \bar{\psi}^2). \]

### 4.1 An application

Consider the following global supersymmetric model

\[ W = \lambda_1 \psi_1 \chi_1 \varphi + \lambda_2 \psi_2 \chi_2^n, \]

and

\[ D = \Lambda^2 - |\chi_1|^2 - |\chi_2|^2 + |\psi_1|^2 + n |\psi_2|^2. \]

This model is invariant under the R-parity

\[ \psi_1 \rightarrow -\psi_1, \quad \psi_2 \rightarrow -\psi_2, \quad W \rightarrow -W, \]

and the U(1) gauge symmetry\(^3\)

\[ \chi_1 \rightarrow e^{-i\theta} \chi_1, \quad \chi_2 \rightarrow e^{-i\theta} \chi_2, \quad \psi_1 \rightarrow e^{i\theta} \psi_1, \quad \psi_2 \rightarrow e^{i\theta} \psi_2. \]

To obtain the effective potential during inflation we minimise the potential (Eq. (2)) for fixed \( \varphi \) as follows. For \( |\chi_1|^2 + |\chi_2|^2 \leq \Lambda^2 \) the potential is minimised for

\[ \psi_1 = \psi_2 = 0, \]

and so the R-parity ensures that

\[ W = W_\varphi = W_{\chi_1} = W_{\chi_2} = 0. \]

Therefore

\[ V = |W_{\psi_1}|^2 + |W_{\psi_2}|^2 + \frac{1}{2}g^2 D^2, \]

\[ = \lambda_1^2 |\chi_1|^2 |\varphi|^2 + \lambda_2^2 |\chi_2|^{2n} + \frac{1}{2}g^2 \left( \Lambda^2 - |\chi_1|^2 - |\chi_2|^2 \right)^2. \]

\(^3\)For example an ‘anomalous’ U(1) often appears in string theory \([9]\) with \( \Lambda \sim 10^{17} - 10^{18} \) GeV if the dilaton is fixed near its usual value.
Now if

$$|\varphi|^2 \geq \frac{g^2}{\lambda_1^2} \left( \Lambda^2 - |\chi_2|^2 \right),$$

(36)

the potential is minimised for

$$\chi_2 = 0.$$  

(37)

Then

$$V = \lambda_2^2 |\chi_2|^2 + \frac{1}{2} g^2 \left( \Lambda^2 - |\chi_2|^2 \right)^2.$$  

(38)

Now if

$$\frac{\lambda_2 \Lambda^{n-2}}{g} \ll 1,$$

(39)

the potential is minimised for

$$|\chi_2|^2 \simeq \Lambda^2 - \frac{n \lambda_2^2 \Lambda^{2n-2}}{g^2},$$  

(40)

and so

$$V \simeq \lambda_2^2 \Lambda^{2n}.$$  

(41)

Thus, from Eqs. (36) and (40), for

$$|\varphi| \geq \frac{\sqrt{n} \lambda_2 \Lambda^{n-1}}{\lambda_1},$$

(42)

we have a positive potential energy density and a flat potential for the inflaton field $\varphi$.

The above global supersymmetric model satisfies the conditions Eqs. (18) and (20) (Eqs. (32) and (33)) and so if the Kähler potential is of the form of Eq. (27) then the supergravity corrections will not spoil the flatness of the inflaton’s potential (which is exactly flat in this case but there are many possible sources for a small mass for the inflaton; see [7] for some examples). Also, it is easy to check that the supergravity corrections do not spoil the stability of the model.

Alternatively to Eq. (39), if $n = 1$ and $g \Lambda/\lambda_2 < 1$ then the potential is minimised for $\chi_2 = 0$ and

$$V = \frac{1}{2} g^2 \Lambda^4.$$  

(43)

In this case the inflationary potential energy density is dominated by the $D$-term part of the scalar potential which might provide an alternative solution to the problem discussed in Section 3.
5 The Simplest Example and a New Model of Inflation

The simplest example of Eq. (27) is

\[
K = -\ln X, \quad X = 1 - |\phi|^2. \tag{44}
\]

The corresponding Kähler manifold is \( SU(m,1)/(SU(m) \times U(1)) \), where \( m \) is the number of components of \( \phi \). It is a maximally symmetric space with constant riemannian curvature. Such coset spaces form the basis of no-scale supergravity [10], though it is important to note that the Kähler potential in Eq. (44) only corresponds to part of one sector of a no-scale model. Now

\[
K_{\phi\bar{\phi}} = \frac{1}{X^2} \left( X + \phi \bar{\phi} \right), \quad K_{\phi\bar{\phi}}^{-1} = X \left( 1 - \phi \bar{\phi} \right), \tag{45}
\]

and

\[
V_F = |W_\phi|^2 - |W_{\phi\bar{\phi}} - W|^2 - \frac{2}{X} |W|^2. \tag{46}
\]

Let \( \phi = (\varphi, \psi) \), and assume \( W = W_{\psi} = \psi = 0 \). Then the kinetic terms are

\[
\frac{1}{X^2} \partial_{\mu} \bar{\varphi} (X + \varphi \bar{\varphi}) \partial^\mu \varphi, \tag{47}
\]

and the potential is

\[
V_F = |W_{\psi}|^2. \tag{48}
\]

Thus for \( |\varphi| \ll 1 \) we have canonical kinetic terms and the potential has the global supersymmetric form, though with the additional requirements \( W = W_{\psi} = \psi = 0 \). However, for \( |\varphi| \sim 1 \) things get even more interesting as we shall see. Note that because of its non-canonical kinetic term, \( \varphi \) is defined only for \( |\varphi| < 1 \).

5.1 A new model of inflation

From Eqs. (14) and (45)

\[
\varphi' = \frac{K_{\varphi\bar{\varphi}}^{-1} V_{\bar{\varphi}}}{V'} = \frac{X}{V'} \left( 1 - \varphi \bar{\varphi} \right) V_{\bar{\varphi}}. \tag{49}
\]

Therefore

\[
\frac{X'}{X} = -\frac{X}{V'} (V_{\varphi} + \varphi V_{\bar{\varphi}}) = -\frac{X |\varphi| |V_{\phi}|}{V'} \tag{50}
\]

Also

\[
V' = \sqrt{2V_{\varphi}} K_{\varphi\bar{\varphi}}^{-1} V_{\bar{\varphi}} = \sqrt{2XV_{\varphi}} (1 - \varphi \bar{\varphi}) V_{\bar{\varphi}} \geq \sqrt{2} X |V_{\varphi}|. \tag{51}
\]

Inflation requires \( |V'/V| \ll 1 \) and so if, as I will assume here, \( |V_{\varphi}/V| \) is not small\(^4 \) then \( X \ll 1 \) is required for inflation. Therefore during inflation \( |\varphi| \simeq 1 \).

\(^4\)so there is no inflation in the global supersymmetric limit.
Now if there is only one $\varphi$ field, then

$$V' = \sqrt{2} X |V_\varphi|,$$  \hspace{1cm} (52)

and

$$\varphi' = \frac{XV_\varphi}{\sqrt{2} |V_\varphi|} \quad \text{and} \quad \frac{X'}{X} \simeq \frac{V_{\varphi|}}{\sqrt{2} |V_\varphi|}. \hspace{1cm} (53)$$

A standard formula for the number $N$ of $e$-foldings of expansion of the universe until the end of inflation is

$$N \equiv - \int H \, dt \simeq \int \frac{V}{V'} \, d\sigma, \hspace{1cm} (54)$$

and so in our case we get

$$N = \int \frac{V}{V'X'} \, dX = - \int \frac{V}{V_{\varphi|}X^2} \, dX. \hspace{1cm} (55)$$

Now if $V_{\varphi|}$ is not much smaller than $V_{\text{arg} \varphi}$, so that $V_{\varphi|} \sim |V_\varphi|$, then, from Eq. (53), $\varphi$ will be much more slowly varying than $X$ and so

$$N \simeq \frac{V}{V_{\varphi|}} \int \frac{dX}{X^2} \simeq \frac{V}{XV_{\varphi|}}. \hspace{1cm} (56)$$

Therefore from Eq. (52)

$$\frac{V'}{V} = \frac{\sqrt{2} |V_\varphi|}{NV_{\varphi|}} = \frac{1}{\sqrt{2} N} \sqrt{1 + \left(\frac{V_{\text{arg} \varphi}}{V_{\varphi|}}\right)^2}, \hspace{1cm} (57)$$

and also from Eq. (52)

$$\frac{V''}{V} \simeq \frac{X'V'}{XV} = - \frac{1}{N}. \hspace{1cm} (58)$$

Therefore, using the standard formula, the spectral index of the density perturbations produced during inflation is

$$n = 1 - 3 \left(\frac{V'}{V}\right)^2 + 2 \frac{V''}{V} \simeq 1 - \frac{2}{N} = 1 - \frac{1}{30}, \hspace{1cm} (59)$$

which is the same as $\phi^2$ chaotic inflation. However, using another standard formula, the ratio of gravitational waves to density perturbations is

$$R = 6 \left(\frac{V'}{V}\right)^2 = \frac{3}{N^2} \left[1 + \left(\frac{V_{\text{arg} \varphi}}{V_{\varphi|}}\right)^2\right] \simeq 10^{-3}, \hspace{1cm} (60)$$

compared with $R = 6/N = 0.1$ for $\phi^2$ chaotic inflation.
This result can be understood in more conventional terms as follows. For the case of only one $\varphi$ field, Eq. (47) reduces to

$$\frac{1}{X^2} |\partial \varphi|^2.$$  \hspace{1cm} (61)

For simplicity assume the phase of $\varphi$ is constant. Then the canonically normalised inflaton $\sigma$ is given by

$$|\varphi| = \tanh \frac{\sigma}{\sqrt{2}}.$$  \hspace{1cm} (62)

Now during inflation $\sigma \gg 1$ and so

$$|\varphi| \simeq 1 - 2e^{-\sqrt{2}\sigma}.$$  \hspace{1cm} (63)

Therefore

$$V \simeq V_{|\varphi|=1} - 2 \frac{dV}{d|\varphi|}{|_{|\varphi|=1}} e^{-\sqrt{2}\sigma}.$$  \hspace{1cm} (64)

The coefficient of the exponential can be absorbed by the redefinition

$$\tilde{\varphi} = \varphi - \frac{1}{\sqrt{2}} \ln \frac{2}{V}{|_{|\varphi|=1}},$$  \hspace{1cm} (65)

and so we get the inflationary potential\(^5\)

$$V = V_1 \left(1 - e^{-\sqrt{2}\tilde{\varphi}}\right),$$  \hspace{1cm} (66)

which has only one free parameter $V_1 = V_{|\varphi|=1}$ and that is determined by the COBE normalisation to be $V_1 = (6 \times 10^{15} GeV)^4 = (600 GeV)^{4/3}$. The latter naively suggests that the origin of $V_1$ may be associated with gaugino condensation.

### 6 Another Example

Another example of Eq. (27) is

$$K = -\ln \left(1 - \sum_{a=1}^{m} \phi^a \bar{\phi}^a + \frac{1}{4} \left| \sum_{a=1}^{m} \phi^a \phi^a \right|^2 \right).$$  \hspace{1cm} (67)

The corresponding Kähler manifold is $SO(m, 2)/(SO(m) \times SO(2))$. For the case of $m = 2$ with $\varphi = \phi^1$ and $\psi = \phi^2$ we get

$$K = -\ln \left[\left(1 - \frac{1}{2} |\varphi|^2\right)^2 - |\psi|^2\right] + O\left(\psi^2, \bar{\psi}^2\right),$$  \hspace{1cm} (68)

\(^5\)Note that this is the potential during inflation ($\tilde{\varphi} \gg 1$). When inflation ends ($\tilde{\varphi} \sim 1$), the neglected, model dependent (i.e. $W$ dependent) terms become important.
and setting $\psi = 0$ gives the Kähler potential for $\varphi$

$$K = -2 \ln \left(1 - \frac{1}{2} |\varphi|^2\right),$$  \hspace{1cm} (69)

Then proceeding as in Section 5.1 also gives

$$n = 1 - \frac{2}{N} = 1 - \frac{1}{30},$$  \hspace{1cm} (70)

but gives a slightly larger

$$R = \frac{6}{N^2} \left[1 + \frac{1}{2} \left(\frac{V_{\arg \varphi}}{V_{|\varphi|}}\right)^2\right] \sim 10^{-2.5}. \hspace{1cm} (71)$$

Also the inflationary potential for $\arg \varphi$ fixed is

$$V = V_1 \left(1 - e^{-\sigma}\right), \hspace{1cm} (72)$$

with $V_1 = V|_{|\varphi| = \sqrt{2}} = (7 \times 10^{15}\text{ GeV})^4$.

Alternatively, for $m = 3$ with $\varphi = (\phi^1 + i\phi^2)/\sqrt{2}$, $\chi = (\phi^1 - i\phi^2)/\sqrt{2}$ and $\psi = \phi^3$ we get

$$K = - \ln \left[\left(1 - |\varphi|^2\right)\left(1 - |\chi|^2\right) - |\psi|^2\right] + \mathcal{O}\left(\psi^2, \bar{\psi}^2\right),$$ \hspace{1cm} (73)

and setting $\chi = \psi = 0$ gives the Kähler potential for $\varphi$

$$K = - \ln \left(1 - |\varphi|^2\right),$$ \hspace{1cm} (74)

and so we get the same results as in Section 5.1.

7 Superstring Examples

7.1 Orbifold compactifications

The Kähler potential of the untwisted sector of the low-energy effective supergravity theory derived from orbifold compactification of superstrings always contains [11]

$$K = - \ln Y - \sum_{i=1}^{3} \ln X_i,$$ \hspace{1cm} (75)

with

$$Y = S + \bar{S},$$ \hspace{1cm} (76)

and

$$X_i = T_i + \bar{T}_i - |\phi_i|^2,$$ \hspace{1cm} (77)
where $S$ is the dilaton, $T_i$ are the untwisted moduli associated with the radii of compactification, and $\phi_i$ are the untwisted matter fields associated with $T_i$. The $\phi_i$ may contain some continuous Wilson line moduli [12, 13]. The $F$-term part of the scalar potential derived from this Kähler potential is

$$V_F = \frac{1}{Y \prod_{i=1}^{3} X_i} \times$$

$$\left[ |YW_S - W|^2 + \sum_{i=1}^{3} \left( |X_i W_{T_i} - W|^2 + X_i \left| W_{\phi_i} + \bar{\phi}_i W_{T_i} \right|^2 \right) - 3|W|^2 \right].$$

Now if we divide the scalar fields into $\psi$, $\varphi$ and $\chi$ fields as defined in Section 4 as follows$^6$

$$T_3 \in \varphi,$$

$$\phi_3^a \in \psi \text{ or } \varphi,$$

$$S, T_1, T_2, \phi_1^a \text{ and } \phi_2^a \in \chi \subset \varphi,$$

and assume that during inflation

$$W = W_\varphi = \psi = 0,$$

then the inflationary potential energy density will be given by

$$V = \frac{1}{Y \prod_{i \neq 3} X_i} \sum_{\phi_3^a \in \psi} \left| W_{\phi_3^a} \right|^2,$$

and we see that the inflaton, which will be some combination of $T_3$ and the $\phi_3^a \in \phi$, receives no supergravity corrections to its global supersymmetric potential [7]. Furthermore, if $T_3$ is constant then we get the model described in Section 5. However, having $X_3 \ll 1$ may take us beyond the range of validity of the effective field theory given by Eq. (75).

### 7.2 More orbifold compactifications

The Kähler potential of the $Z_4$ and $Z_6$ orbifolds contains [11, 14]

$$K = - \ln \det \left( T + \bar{T} \right),$$

where $T$ is a $2 \times 2$ complex matrix whose diagonal elements correspond to two of the $T_i$ moduli of Section 7.1, and whose off-diagonal elements are other (1,1) moduli. This Kähler potential can also arise in orbifolds with continuous Wilson lines, in which case one of the diagonal elements corresponds to one of the $T_i$ moduli of Section 7.1, the other to a (1,2) modulus, and the off-diagonal elements

$^6$The choice of subscript 3 rather than subscript 1 or 2 is clearly arbitrary.
are continuous Wilson line moduli \cite{13}. Parametrising $T$ in terms of the Pauli matrices,
\[ T = T_0 + T \sigma = \left( \begin{array}{cc} T_0 + T_3 & T_1 - iT_2 \\ T_1 + iT_2 & T_0 - T_3 \end{array} \right), \] (85)
gives\footnote{up to an irrelevant constant}
\[ K = - \ln \left[ (\text{Re } T_0)^2 - |\text{Re } T|^2 \right], \] (86)
which is of the form of Eq. (27). If $T_0$ is constant we get
\[ K = - \ln \left( 1 - |\text{Re } \phi|^2 \right). \] (87)
Then proceeding as in Section 5 also gives for one $\varphi$ field
\[ n = 1 - \frac{2}{N} = 1 - \frac{1}{30}, \] (88)
and
\[ R = \frac{3}{N^2} \left[ 1 + \left( \frac{V_{\text{Im } \varphi}}{V_{\text{Re } \varphi}} \right)^2 \right] \sim 10^{-3}. \] (89)

### 7.3 Fermionic four-dimensional string models

The Kähler potential of the untwisted sector of the revamped flipped SU(5) model \cite{15} is \cite{16}
\[ K = - \ln \left( 1 - |\Phi_1|^2 - |\Phi_{23}|^2 - |\Phi_{TT}|^2 - |h_1|^2 - |h_T|^2 + \frac{1}{4} \left| \Phi_1 + 2\Phi_{23} \Phi_{TT} + 2\Phi_1 h_T \right|^2 \right) \]
\[ - \ln \left( 1 - |\Phi_2|^2 - |\Phi_{31}|^2 - |\Phi_{TT}|^2 - |h_2|^2 - |h_T|^2 + \frac{1}{4} \left| \Phi_2 + 2\Phi_{31} \Phi_{TT} + 2\Phi_2 h_T \right|^2 \right) \]
\[ - \ln \left( 1 - |\Phi_4|^2 - |\Phi_5|^2 - |\Phi_{12}|^2 - |\Phi_{TT}|^2 - |h_3|^2 - |h_T|^2 + \frac{1}{4} \left| \Phi_4 + \Phi_5 + \Phi_{12} + 2\Phi_{12} \Phi_{TT} + 2h_3 h_T \right|^2 \right), \] (90)
all three parts of which are of the form of Eq. (67).\footnote{up to trivial redefinitions} Also, if we divide the fields as follows
\[ \Phi_4 \text{ and } \Phi_5 \in \varphi, \] (91)
\[ \Phi_3, \Phi_{12}, \Phi_{TT}, h_3 \text{ and } h_T \in \psi, \] (92)
\[ \Phi_1, \Phi_2, \Phi_{23}, \Phi_{TT}, \Phi_{31}, \Phi_{TT}, h_1, h_T, h_2 \text{ and } h_T \in \chi \subset \varphi, \] (93)
then the target space duality symmetries \cite{16,17} contain the R-parity Eq. (23).
7.4 Calabi-Yau compactifications

Here I give the Calabi-Yau manifold discussed in Section 4.3 of [18] as another example of a compactification of superstrings that can give Kähler potentials of the form discussed in Section 5. At a particular point in the moduli space of the above mentioned Calabi-Yau manifold, the low-energy gauge group includes an extra $U(1)^4$ factor, a subgroup of which may be preserved on subspaces of the moduli space that pass through that point. The Kähler potential on a subspace of the moduli space that preserves a $U(1)^3$ subgroup of the extra $U(1)^4$ gauge symmetry is [18]

$$K = -\ln \left( 1 - |N|^2 - |C|^2 \right) + O \left( C^2, C^3 \right),$$

where $N = (N_1, N_2)$ are the neutral (1,2) moduli that span the subspace, and $C$ is a vector whose components are 63 of the 99 charged (1,2) moduli and their associated matter fields. Also the Kähler potential on a subspace of the moduli space that preserves a $U(1)^2$ subgroup of the extra $U(1)^4$ gauge symmetry is [18]

$$K = -\ln \left( 1 - |N|^2 \right),$$

where $N$ is a vector whose components are the twelve neutral (1,2) moduli that span the subspace.

8 Conclusions

A global supersymmetric model of inflation (see for example [7, 19]) will not work in a general supergravity theory because the higher order, non-renormalisable supergravity corrections destroy the flatness of the inflaton’s potential. In this paper I have derived a form for the Kähler potential which eliminates these corrections if $W = W_\varphi = \psi = 0$ during inflation (where $W$ is the superpotential, the inflaton $\varphi$ and $W_\psi \neq 0$). It is encouraging that Kähler potentials of this form often occur in superstrings and that the target space duality symmetries of superstrings often contain R-parities which could make $W = W_\varphi = 0$ automatic for $\psi = 0$.

Also, I have shown that Kähler potentials of this form may give rise to inflation even if the corresponding global supersymmetric theory does not. The simplest examples of this new model of inflation give a spectral index $1 - n = 1/30$ for the density perturbations and negligible gravitational waves, though more complicated examples lose this predictive power. The question of whether this new model of inflation can be realised in superstrings is currently under investigation.

$^1$with respect to the unbroken $U(1)^3$
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