LYCEN/9404/
Février 1994

su 9423

Soft big-bang model induced by non-minimal coupling

M. Novello

*Centro Brasileiro de Pesquisas Fisicas, R.Xavier Sigaud 150,
Urca, 22290 Rio de Janeiro, Brasil*

E. Elbaz

*Institut de Physique Nucléaire de Lyon, IN2P3-CNRS et Université Claude Bernard
43 boulevard du 11 novembre 1918, F-69622 Villeurbanne Cedex, France*

Abstract

A non-minimal coupling $R\varphi^2$ leads to a geometrical structural change in the evolution of the Universe which can be interpreted in terms of a quantum tunneling process. The origin of the Universe admits thus two interpretations: either as a classical change of the Riemannian nature of spacetime or as a quantum process. The main consequence of this is a nonsingular Soft Big Bang. The same kind of non-minimal coupling RF^2 of the Electromagnetic field with Gravity induces non-conservation of the photon number and of the total entropy.

Submitted to Il Nuovo Cimento B

1 INTRODUCTION

Two recent developments in Cosmology may lead to an important change in our understanding of the Universe. It concerns the entropy problem that deals with the $\frac{n_\gamma}{n_B}$ ratio of the number of photons per baryon as a constant inprinted in our Universe since the beginning; it also concerns the question of the origin of the global spacetime geometry, that is, the cosmological singularity problem. We give here an insight on these developments.

2 THE STRUCTURE OF THE SPACETIME

There are two ways to arrive at the spacetime structure geometry. Either by a priori hypothesis (like in Newton's or Einstein's gravitational theories) or by some sort of dynamical process.

Einstein-Palatini gave a very beautiful trick to deal with the second alternative as to obtain a Riemannian structure by considering that in the action $S = \int \sqrt{-g} R d_4x$ both, the metric $g_{\mu\nu}$ and the affine connection $\Gamma^\mu_{\nu\lambda}$ should be treated as independent variables in the corresponding variational principle.

Although this is true even in the non-vacuum case in which matter is present, it ceases to be valid if matter is coupled non-minimally to the geometry [2]. This dependance of the structure of spacetime on the way matter couples to gravity seems not to have been observed before. For example, we can give the usual conformal coupling of a scalar field to gravity introduced in order to restore the broken conformal invariance of a massless scalar field in the presence of gravity, through the interacting action $S_{int} = \frac{1}{6} \int \sqrt{-g} R \varphi^2 d_4x$. The use of the Einstein-Palatini's trick thus reveals an unexpected non-Riemannian structure of the geometry. Indeed, it follows that the variation of the connection yields

$$g_{\mu\nu;\lambda} = \omega_\lambda g_{\mu\nu}. \quad (1)$$

where $\omega_\lambda = -\frac{1}{6}(\ln \varphi^2)_{,\lambda}$. Such a relation implies that the structure of the spacetime is a Weyl Integrable Space Time (WIST) geometry in which the famous Einstein's second clock problem is eliminated.

In recent times the scalar field has been extensively used by astro-particle physicists to solve some traditional problems of the old (hot Big-Bang) Standard Cosmological Model like, for instance, in the inflationary scenario. How-

ever all these models do not consider the singularity problem as relevant. In [1] this problem is solved in a very appealing way.

3 THE MODEL

In the framework of homogeneous and isotropic FRW geometry $ds^2 = dt^2 - a^2(t)d\sigma^2$ the main features ¹ of the model comes from a particular solution of the theory provided with the action

$$S_c = \int \sqrt{-g}(R + \epsilon \omega_{;\lambda}^{\lambda})d_4x.$$

The model² deals with Minkowski (Riemannian) structure in the infinite past and its instability (controlled by the above dynamics) leads to a collapse. The corresponding infinite future structure comes back to a similar Minkowski geometry. The analytic continuation connecting these two asymptotic regimes is a FRW-like Universe that collapses to a minimum radius a_0 and undergoes a subsequent expansion. The largest modification of the Riemannian structure of the spacetime occurs precisely at the turning point in which the minimum radius of the Universe is reached. This can be seen by a direct inspection of the form of the wist function $\dot{\omega} = \frac{\text{constant}}{a^3}$ (figure 1).

The evolution equation of the radius of the Universe takes the form

$$\dot{a}^2 == 1 - \left(\frac{a_0}{a}\right)^4$$

where a_0 is the minimum allowable radius. The value a_0 is a free parameter of the model. Once a_0 may take a very small value, different from zero, we can thus consider this as a Soft Big Bang model³.

¹We restrict our comments here only to the isotropic case. We note however that we can go beyond this and not assuming a priori the isotropic structure. We leave this point to a forthcoming paper in which the anisotropy stage can be treated in terms of a liquid-crystal state of the cosmical fluid.

²We stress that the covariant derivative is to be taken in the affine wist space and thus the second term cannot be reduced to a surface integral. The quantity ϵ is a non-dimensional parameter. This action can be reduced to the previous one S_{int} and the wist structure is then obtained through Einstein-Palatini's treatment.

³In case this quantity takes the extremum possible value of any classical process, e.g. the Planck length, this interpretation should be analysed in a quantum procedure.

It does not make sense to try to obtain its actual value by any other means than in a phenomenological way. Although the treatment here is purely classical, the fundamental equation for $a(t)$ is precisely the same that occurs in the Euclideanization procedure in Quantum Cosmology treatment, for example for baby-universes [4]. The non-singular behaviour of the Universe thus admits two equivalent interpretations, either as a structural modification of the geometry (the wist phase) or as a quantum tunneling process (figure 2). This suggest that quantum processes can be simulated by a Weylization modification of the structure of the spacetime. This supports Santamato's [3] idea of association of stochastic quantum process as a modification of the length of the rules throughout spacetime.

figure 1

figure 2

The standard difficulties of any eternal universe, i.e., the stabilization of the matter creation in order to avoid an infinite amount of energy, is solved in the wist approach in the following way.⁴ The interaction of matter with the wist is provided by the generalized rule $T_{\nu}^{\mu\nu} = 0$ in which the covariant derivative should be taken in the wist background (we neglect here the back influence of matter on geometry). Taking into account the previous solution of $\omega(t)$ it follows then that as the Universe collapses any matter vacuum fluctuation is exponentially suppressed while in the expansion phase, matter (and entropy) fluctuations are exponentially amplified.⁵ When this mecha-

⁴This solution should be expected, since in the equivalent quantum interpretation, the infinite lapse of time is just replaced by a tunnelling effect from a perfectly well behaved non-singular state.

⁵We note that an eventual baryon excess occurring at the start of the expansion era will

nism of pumping energy from the ω -field to matter creation (say, baryons and photons) stops, then, one should expect, as in the conventional model, that the resulting $\frac{n_\gamma}{n_B}$ ratio becomes constant throughout the remaining riemannian FRW epoch. However this ratio may not be constant as explained in the next section. Indeed one could ask what should be the consequences of an equivalent non-minimal coupling of the electromagnetic field with gravitation of the same form of the non-minimal coupling of the scalar field with gravitation as presented above.

4 PHOTON CHEMICAL POTENTIAL

Much of our knowledge of the Universe comes from photons. The basic properties of the behaviour of photons in curved spacetime known since the early days of General Relativity can be described by three statements:

- Photons propagate through null geodesics, $K^\mu{}_{;\nu} K^\nu = 0$, which explains the observed cosmological red-shift.
- The polarization of the electromagnetic waves in an optical path, that is $\epsilon^\mu{}_{;\lambda} K^\lambda = 0$, is conserved.
- The total number of photons is conserved. This can be settled by $\dot{n}_\gamma + 3Hn_\gamma = 0$, which shows that all modification in the photon number density in a given volume comes from Hubble expansion factor H .

This last statement is equivalent to the extension of the null value of the chemical potential of the photon in the flat Minkowski geometry to curved spacetime.

Among the above mentioned photon properties in curved spacetimes the third one (conservation of the total number of photon N_γ) should be singled out, because of its phenomenological status. Indeed, the fact that photons propagate through null geodesics has been firmly established by results from different observations, such as, for instance, the red-shift due to gravitational

be amplified as well. This should be a possible candidate for the generation mechanism of the non-symmetric baryonic content of our Universe: the random initial fluctuation of a given species becomes dominant by the resonant response of the background geometry. When the energy of the created baryons becomes higher than the energy contained in the ω -field then the above model is identical to standard FRW geometry.

fields. However the third property is only a theoretical consequence of the minimal coupling principle of the photon with the gravitational field. The question here is the following. What can we deduce for this coupling from direct observations? The answer is that the general Lagrangian of the interaction of the Electromagnetic and Gravitational fields is provided by the form

$$L = \sqrt{-g}(-\frac{1}{4} + \chi R)F^2$$

in which $F^2 \equiv F_{\mu\nu}F_{\alpha\beta}g^{\mu\alpha}g^{\nu\beta}$ and χ is a constant. The only difference between the standard minimal coupling concerns the third statement. When χ is not null the total number of photons becomes a non-conserved quantity. The most striking consequence is that $\frac{n_\gamma}{n_B}$ ratio is no more a time-independent number. Its present 10^9 value, does not reveal any particular intrinsic property of the Universe but merely a fortuitous circumstantial dependence on human presence of our epoch. It should not be taken as a universal constant and thus as a fundamental quantity. Thus, it is not a true cosmological problem to provide a theoretical justification for this number, but just a matter of today's astronomical observations.

The simplest way to deal with such a non-conservation of the total photon number is to take a non-zero value for its chemical potential μ . This means that μ in curved spacetime is given by

$$\mu = \mu_0 + \Delta\mu$$

where μ_0 is the flat space value, that vanishes for the photon, and $\Delta\mu$ depends on the curvature. In order to get an estimation of such a situation let us take a simple model in which $\Delta\mu$ is developed in a series on the expansion Hubble parameter. Since the photon is a boson and we are living in an expanding Universe we must set [2] to first order $\Delta\mu = -b^2H$.

Near thermodynamical equilibrium the entropy density is thus given by $s = \frac{S}{N} = \frac{1}{T}(\frac{4}{3}\rho v + b^2H)$ and to-day b^2H_0 is much less than ρv . Then the observed black-body background spectrum is not preserved throughout the history of the Universe, even after the decoupling era, contrary to the case in which electromagnetic and gravitational fields are minimally coupled. It is clear that this does not affect the isotropy of the radiation.

5 FINAL COMMENTS

The aims of the present work is to synthesize some cosmological consequences of non-minimal coupling of fields (scalar and vector) to gravity. We have limited ourselves here to two main features concerning the global geometrical structure and a mechanism allowing creation of photons by the expansion of the Universe. The same kind of non-minimal coupling could provide other important achievements like the creation of the observed galactic magnetic fields [5], [6].

Acknowledgements One of us (MN) would like to thank the Rhone-Alpes Region for the grant and the Universite de Lyon (IN2P3) for the hospitality.

References

- [1] M Novello, L A R Oliveira, J M Salim and E Elbaz. z Int.J. Modern Physics D, vol 1, 641 (1993).
- [2] M Novello, L A R Oliveira and J M Salim.
Class Quantum Gravity, 7, 51 (1990)
- [3] E.Santamato.
Phys Rev D 29, 216 (1984).
- [4] Quantum Cosmology and Baby Universes, World Scientific, vol 7(1991).
- [5] A Dolgov.
Phys Rev D 48,6, 2499 (1993).
- [6] M S Turner and L M Widrow.
Phys Rev D 37, 2743 (1988)