AC

BROOKHAVEN NATIONAL LABORATORY

April 1994

BNL-60387

SW 3423

**Implication of gallium results on the possibility of
observing day-night matter oscillations
at SNO, Super-Kamiokande, and Borexino**

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P00023674

ABSTRACT

Calculations are presented to determine what real time day-night effect would be observable in SNO, Super-Kamiokande, or Borexino for the Δm^2 , $\sin^2 2\theta$ space allowed by the present gallium, ^{37}Cl , and Kamiokande solar neutrino results. We show that the combination of possible day-night effects and the observation of overall neutrino detection rates in the upcoming experiments might allow discrimination between the allowed regions of mass and mixing parameters. Approximate analytical expressions for the real time MSW effect in the Earth are presented to clarify the nature of electron neutrino regeneration as a function of path length through the Earth. We point out that even for the allowed small $\sin^2 2\theta$ MSW solution, it might be possible to detect a day-night effect for neutrino trajectories through the core of the Earth.

Submitted to *Physical Review D*

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1. Introduction

The results of the two gallium solar neutrino experiments (GALLEX^{1,2} and SAGE³) when combined with the Homestake ³⁷Cl detector result⁴ and the Kamiokande water Cherenkov detector data⁵ appear to provide strong evidence that observed neutrino rates cannot be explained by the available solar models without the introduction of new physics, i.e. neutrino mass and mixing via the Mikheyev-Smirnov-Wolfenstein (MSW) mechanism. At the time of the first announcement of the observation of solar neutrinos by the GALLEX collaboration, members of the same collaboration asserted that the MSW mechanism provided a good fit to the combined GALLEX, Homestake, and Kamiokande results and fixed parameters at the 90% confidence level acceptance in two very confined ranges, which we will refer to as Region A (around $\Delta m^2 = 6 \times 10^{-6}$, $\sin^2 2\theta = .007$) and Region B (around $\Delta m^2 = 8 \times 10^{-6}$, $\sin^2 2\theta = .6$). There was also evident in Figure 1 of the GALLEX interpretation paper,² but not commented on by the authors, a third Region C (in the range $\Delta m^2 = 10^{-7}$, $\sin^2 2\theta = .7$) whose MSW solution fell within 2σ for each of the three experiments but whose overall confidence level was less favorable than the stated 90% of regions A and B.

In this paper we discuss the implications of the updated experimental results on the specific possibility of day-night matter oscillations in these three allowed regions of $\sin^2 2\theta$, Δm^2 space. The day-night effect is the MSW Earth effect: electron neutrinos that are converted into muon neutrinos in the Sun may then be regenerated back into electron neutrinos in the Earth. In earlier papers⁶⁻¹⁰ the day-night effect has been discussed in some detail, and we have presented⁷ calculations exhibiting the effect for Homestake, GALLEX, and SNO. Particular calculations of the day-night effect for the ⁷Be line neutrinos expected to be observed in Borexino have also been carried out.¹¹ In this paper we emphasize the particular advantages inherent in the real time detection of neutrinos in SNO,¹² Super-Kamiokande¹³ and Borexino¹⁴: knowing the time of day and year when each neutrino is detected allows correlation with the night-time path taken through the Earth to discriminate between previously allowed $\sin^2 2\theta$, Δm^2 solutions. Our calculations exhibit curves for the number of neutrinos detected as a function of trajectory through the Earth that are characteristic of the particular values of $\sin^2 2\theta$ and Δm^2 .

$R_i^{\text{MSW} \times \text{SM}}$ is the model rate; in the separate sets of calculations we have used the Bahcall–Pinsonneault standard solar model with diffusion and the standard solar model of Turck–Chiéze *et al.* As has been pointed out by Hata and Langacker,²⁰ assigning theoretical errors involves strong correlations in the theoretical uncertainties from experiment to experiment. We have chosen not to assign theoretical errors, but have used these independently calculated solar models. We regard the different outcomes as a reasonable measure of the underlying uncertainties. The $R_i^{\text{expt}}, \sigma_i$ are the most recent values of the GALLEX (79 ± 12 SNU),¹⁵ Kamiokande ($.5 \pm .07$ times the Bahcall–Ulrich SSM),²¹ and Homestake (^{37}Cl) ($2.32 \pm .23$ SNU)²² experiments. The notation has been written to emphasize that the χ^2 is a function of the MSW parameters and dependent on the choice of theoretical model.

Within the Bahcall–Pinsonneault model, the $\sin^2 2\theta$ – Δm^2 Region A corresponds to the lowest minimum by far; it centers around a minimum χ^2 of .3 at $\sin^2 2\theta = .0063$, $\Delta m^2 = 6.3 \times 10^{-6}$. Region B centers about 4.7, at $\sin^2 2\theta = .70$, $\Delta m^2 = 1.8 \times 10^{-5}$ and Region C at 8.0 at $\sin^2 2\theta = .77$, $\Delta m^2 = 1.3 \times 10^{-7}$. Translation of these χ^2 values to confidence levels requires some analysis of the underlying statistical theory. Since the χ^2 numbers do not involve a minimization procedure, there are no constraint requirements that subtract off any degrees of freedom. We, therefore, adopt the most liberal approach and consider confidence levels corresponding to the full 3–degrees–of–freedom. Then, the central $\chi^2 = .3$ of Region A signifies a 96% probability of a larger, worse value, CL=.96; the 4.7 of Region B corresponds to a CL=.20, while 8.0 of Region C corresponds to a CL=.05. A similar analysis based on the Turck–Chiéze model also strongly favors Region A, with a minimum χ^2 of 1.0 (CL=.80) at $\sin^2 2\theta = .005$, $\Delta m^2 = 5.6 \times 10^{-6}$, somewhat lower in both parameters than those obtained with Bahcall–Pinsonneault, Regions B and C have nearly equal minimum χ^2 , 7.2 (CL=.07) at $\sin^2 2\theta = .88$, $\Delta m^2 = 6.3 \times 10^{-5}$ and 7.5 (CL=.06) at $\sin^2 2\theta = .92$, $\Delta m^2 = 10^{-7}$.

It is clear that were we given a single choice, Region A would be strongly preferred due to its much lower χ^2 . Since the interpretation of multiple, disconnected regions is not within the purview of conventional statistical analyses, it is not obvious how seriously to take Regions B and C. However, we are accustomed by experience to believe that dismissal

of hypotheses at a conventional 2σ -deviation, $CL=.05$, is dangerous. We, therefore, shall analyze all three regions for their Earth effect.

In Figure 1 we display the contours for $\chi^2 = 9$ in each of the three regions for the Bahcall–Pinsonneault (solid curve) and Turck–Chière (dashed curve) standard solar models. According to our above interpretation, values of $\sin^2 2\theta$ and Δm^2 lying at the boundary of the contours are only acceptable at the $CL= 3\%$ for the appropriate SSM. Note that although the three well localized regions, A, B, C, are evident for both SSMs, there is limited overlap in the allowed values of $\sin^2 2\theta$ and Δm^2 between the two models. In our exploratory calculations based on Turck–Chière SSM we have only made use of the Turck–Chière rates for the various neutrino processes in the sun; for the density profiles we keep the Bahcall–Pinsonneault shapes. The $\chi^2 = 9$ curves generated by the Bahcall–Ulrich SSM are so close to those of Bahcall–Pinsonneault (nowhere more than about a line's width different) that we do not display them.

The spectra of fluxes prescribed by the solar models that form the basis of our analyses are given in the papers to which reference has already been made; a condensed summary table, prepared from those references, appears in reference 2. These spectra are distorted by the MSW mechanism, and the distortions are quite distinctly different in the three regions that have been noted above. To characterize these we describe the distortions at around the central points:

1. Region A – The greatest transformation occurs in the neutrino energy range from $\sim .6$ MeV to several MeV. Thus, the survival of the low energy pp neutrino flux ($0- .42$ MeV) is at $\sim 80\%$ level, while only a few percent of the ${}^7\text{Be}$ flux remains. The ${}^8\text{B}$ neutrino flux ($0- \sim 14$ MeV) has a wide energy range and a wide transformation response: $\sim 30\%$ at ~ 8.5 MeV, $\sim 45\%$ at ~ 14 MeV.
2. Region B – Since the mixing angle is large, the transformation response has a much more gradual energy dependence, and can be described as a gradual decrease in electron neutrino survival with increasing energy within the range of solar interests: pp flux at $\sim 55\%$; ${}^7\text{Be}$ flux at $\sim 35-50\%$, depending on the Δm^2 value; the higher end of the ${}^8\text{B}$ spectrum at $\sim 25\%$. All of these are to be understood as in the absence of an Earth effect.

3. Region C – The low values of Δm^2 , taken with the large mixing angle, prescribe a response that has a flat energy dependence that rises gradually in the region of the highest energies of the ^8B flux: a $\sim 30\%$ survival percentage, rising to $\sim 35\%$ at the top of the ^8B neutrino spectrum — again absent an Earth effect.

These specific values only crudely characterize the regions. As the neutrino mass and mixing parameters edge away from the respective centers, the transformation response can change appreciably.

3. A Qualitative Analysis of the Earth Effect

Although our calculations of the changes attributable to neutrino trajectories through the Earth were performed numerically, we will digress into consideration of available approximate analytical expressions for the earth effect to shed light on the numerical results.

Mikheyev and Smirnov¹⁰ have presented an expression for electron neutrino survival after passing through the Sun and the Earth which they point out is equivalent to an expression we had previously presented.⁶ It is valid when the phase between different mass states averages cross-terms to zero, as is the case for the very long path length and the neutrino parameters of interest here. We will adopt a trivially transformed version of the Mikheyev–Smirnov expression as most transparent in various limits. The probability P of a solar neutrino remaining an electron neutrino after passing through the Sun and Earth is given in the expression by

$$P = P_S + \frac{(1 - 2P_S)}{\cos 2\theta} (P_{2e} - \sin^2 \theta) \quad (1)$$

where P_S is the averaged survival rate in the Sun, P_{2e} is the probability of a transition from the mass eigenstate ν_2 to the flavor eigenstate ν_e in the earth and θ is the vacuum mixing angle. Mikheyev and Smirnov also recall that in the case of complete adiabatic conversion in the sun $P_S = \sin^2 \theta$, leading to a simplification of their expression in this case to

$$P = P_{2e}.$$

Although in our realistic calculations of the next section we use the Earth's density profile as given by the geo-physicists²³, in this section we caricature the density distribution as two constant density regions: mantle and core. If one makes the simplifying approximation of one constant density (very roughly valid in the mantle but outside the core of the Earth) then P_{2e} may be easily evaluated analytically. We obtain the expression

$$P_{2e} - \sin^2 \theta = \left(\frac{\ell_v}{\ell_0} \right) \sin^2 2\theta_m \sin^2 \frac{\pi X}{\ell_m} \quad (2)$$

with the usual notation²⁴

$$\ell_v = 4\pi p_\nu / \Delta m^2 = 2.48 \times 10^{-3} \frac{E(\text{MeV})}{\Delta m^2(\text{eV})^2} \text{ kilometers},$$

$$\ell_0 = 2\pi / \left(\sqrt{2} G_F N_e \right) = \frac{3.28 \times 10^4}{\rho (\text{g/cm}^3)} \text{ kilometers},$$

$$\ell_m = \frac{\ell_v}{\sqrt{1 + \left(\frac{\ell_v}{\ell_0} \right)^2 - 2 \frac{\ell_v}{\ell_0} \cos 2\theta}},$$

$$\sin^2 2\theta_m = \left(\frac{\ell_m}{\ell_v} \right)^2 \sin^2 2\theta;$$

ρ is the density (about 4.5 g/cm³ in the Earth's mantle and about 11 gm/cm³ in the core); the electron fraction is here assumed to be 1/2; X is the distance in kilometers of the neutrino path through the Earth, which may be simply given in terms of the angle from the horizon, ϕ , and the diameter of the Earth D , $X = D \sin \phi$ ($\phi < 57^\circ$ for trajectories through the mantle that do not cross the core). In fact we may make use of the constant density approximation in the Earth along with adiabatic conversion in the Sun as a valid approximate description of both regions B and C. We are interested in the difference, P_Δ , between the probability of electron neutrino survival for a night trajectory (Sun and Earth effect), P , and that for a daytime trajectory (Sun only), P_S .

$$P_\Delta = P - P_S$$

now takes the simple form

$$P_\Delta = \left(\frac{\ell_v}{\ell_0} \right) \sin^2 2\theta_m \sin^2 \frac{\pi X}{\ell_m}. \quad (3)$$

Note that this expression is equal to ℓ_v/ℓ_0 times the probability of flavor conversion of a neutrino created not in the Sun but at the surface of the Earth. The maximum amplitude of P_Δ occurs at $\ell_v = \ell_0$. In contrast, the transformation of neutrinos that originate at the surface of the Earth resonates according to the well known $\ell_v \cos 2\theta = \ell_0$. Equation (3) also makes very explicit the oscillatory dependence on the trajectory length through the Earth.

In what follows we assume just two active neutrino flavors, and write in terms of electron–muon neutrino transformation, each describable in terms of the Standard Models’ charge and neutral currents.

Perhaps the simplest application of Eq. (3) is to neutrinos of Region C for the Borexino¹⁴ detector. Recoil electrons in the 250–665 keV window (the region of Borexino sensitivity) are primarily due to the sharp–line, .86 MeV neutrinos from ^7Be in the Sun. The Borexino experiment at full design size consists of 100 tons of liquid scintillator placed in the Gran Sasso tunnel laboratory (latitude 42.4°). The effect of neutrino transformation appears in neutrino–electron scattering because the muon–neutrino electron scattering rate σ_μ in the scintillator is only about 21% of the electron–neutrino electron scattering rate σ_e . The scattering cross section σ is then given by

$$\sigma = \sigma_\mu + (\sigma_e - \sigma_\mu) P \quad (4)$$

where P is the electron neutrino survival fraction as above. The expected rates are, then, proportional to

$$.21 + .79(P_S + P_\Delta) \cong (.21 + .79 \sin^2 \theta) + .79 P_\Delta.$$

To typify Region C we choose as neutrino parameters

$$\Delta m^2 = 1.3 \times 10^{-7} (\text{eV})^2, \quad \sin^2 2\theta = .77;$$

then, $\ell_v = 1.6 \times 10^4 \text{ km}$, $\ell_0 = .73 \times 10^4 \text{ km}$, $\sin 2\theta_m = .46$ for those neutrino trajectories that are confined to the Earth’s mantle, $\phi < 57^\circ$. As can be seen $\ell_v/\ell_0 \sim 2.2$, $\sin 2\theta_m < \sin 2\theta$, and we are not too far from resonance. The results of this qualitative estimate are shown in Figure 2, together with numerical evaluation of the full transmission equations that also include the real density variations. The qualitative estimate clearly resembles the accurate calculations for the angles appropriate to mantle only; at $\phi > 57^\circ$ the greater densities of

the Earth's core enter importantly, as shown in the sharp change. Even though not exactly on resonance, there is clearly a sizeable day-night effect, with a swing almost as large as the day rate.

The day-night effect for neutrinos of Region C in the detectors sensitive only to higher energy neutrinos can be seen to be considerably smaller. Thus for the SNO or Super-Kamiokande detectors, assuming an average observed neutrino energy of the order of 10 MeV, with the same Δm^2 , $\sin^2 2\theta$, the much larger ℓ_v/ℓ_0 ratio dictates a P_Δ of less than 10% of the day rate. Thus, to explore a day-night variation for Region C neutrinos, the lower energy sensitivity of a Borexino is preferable. Were it possible to effectively separate night from day signals in the radiochemical detectors sensitive to the low energy neutrinos of the pp -solar cycle, $\ell_v/\ell_0 \sim 1$ and resonant behavior would be available.

Region B can be characterized by the parameters $\Delta m^2 = 10^{-5}(\text{eV})^2$, $\sin^2 2\theta = .7$. For such neutrinos at an energy of 10 MeV, transit through the Earth's mantle is described by $\ell_v/\ell_0 = .34$, $\sin^2 2\theta_m = .97$, $P_\Delta = .32 \sin^2(4.4\pi \sin \phi)$. There is then a very sizeable intensity oscillation, high/low ~ 1.8 , that is a rapid function of the angle that the Sun lies below the horizon, going from minimum at 0° to maximum at $\sim 6.5^\circ$ (the period around Sunset). The discernment of such a sizeable but rapid oscillation (in X or ϕ) would, of course, be possible only in a real-time detector large enough to insure good statistics; otherwise the necessary averaging in ϕ makes for a still respectable night/day ratio of ~ 1.4 . Low energy neutrinos, such as the .86 MeV of the ${}^7\text{Be}$ branch or the still lower pp branch would have only a very small Earth effect.

To analyze Region A we choose $\Delta m^2 = 6.3 \times 10^{-6}(\text{eV})^2$, $\sin^2 2\theta = .0063$, the parameters at the minimum χ^2 value. Since the neutrinos of Region A do not undergo the complete adiabatic conversion (appropriate to those of B and C), the constant density approximation for the Earth effect takes the form

$$P_\Delta = \frac{(1 - 2P_S)}{\cos 2\theta} \left(\frac{\ell_v}{\ell_0} \right) \sin^2 2\theta_m \sin^2 \pi X/\ell_m. \quad (5)$$

The value of P_S must be taken from the actual numerical calculations that describe the neutrino transit through the Sun; our choice of neutrino parameters corresponds to a $P_S = .35$. It is important to note the reduction produced by P_S values in the vicinity of $1/2$, and the interesting circumstance that the Earth effect can be positive or negative depending

on P_S less or greater than $1/2$; these effects follow, of course, directly from the fact that a 50–50% mix of ν_e and its transform ($P_S = 1/2$) will remain in that equilibrium, while a greater/lesser preponderance of ν_e ($P_S < 1/2 / > 1/2$) leads to regeneration/depletion.

The amplitude, $(\ell_v/\ell_0) \sin^2 2\theta_m$, is equal to $\cos^2 \theta$ at maximum, which occurs at the resonant neutrino energy such that $\ell_v = \ell_0$. For small mixing angles the amplitude takes the form

$$(\ell_v/\ell_0) \sin^2 2\theta_m \simeq \frac{1}{1 + \frac{1}{\ell_0 \ell_v} \left(\frac{\ell_0 - \ell_v}{\sin 2\theta} \right)^2} \simeq \frac{1}{1 + \frac{1}{E_\nu E_{\nu, \text{res}}} \left(\frac{E_\nu - E_{\nu, \text{res}}}{\sin 2\theta} \right)^2}$$

where $E_{\nu, \text{res}}$ is the resonant energy

$$E_{\nu, \text{res}} (\text{MeV}) = 1.3 \times 10^7 \frac{\Delta m^2 (\text{eV})^2}{\rho (\text{gm/cm}^3)}.$$

Therefore for small mixing angles the amplitude is small (proportional to $\sin^2 2\theta$) except at the narrow resonance (width proportional to $\sin 2\theta$), and the possibility of an Earth effect depends critically on a resonance in the matter encountered there. In the mantle $\ell_0 = 7.3 \times 10^3$ km, while the core value is $\ell_0 = 3.3 \times 10^3$ km. Resonance, $\ell_v = \ell_0$, would be achieved for the Δm^2 considered at $E_\nu = 7.5$ MeV in the core, and at an unavailable 18.5 MeV in the mantle. A plausible hope for an appreciable Earth effect must then be sought in measurements sensitive to the ^8B neutrino branch in the core. We analyze the ϕ dependence of an $E_\nu = 8.5$ MeV neutrino (a little off resonance but representative of the response of the SNO and Super-Kamiokande detectors). Our expression Eq. (5) is qualitatively usable if we ignore the effect of the mantle upon trajectories that cross through the core and take X as just the path length through the core and ρ as the core density, (about 11 gm/cm^3). Then for $\phi > 57^\circ$,

$$X = D \sqrt{r_{\text{core}}^2 - \cos^2 \phi},$$

where $r_{\text{core}} = R_{\text{core}}/R_{\text{Earth}} = .55$. For trajectories that do not cross the core we again assume a constant density mantle; Eq. (4) must be again used, with the neutrino scattering cross sections appropriate to the higher energy electrons detected ($\sigma_\mu \simeq \sigma_e/7$). Figure 3 shows the results. The constant density analytical solution (dashed curve) gives at least a qualitative approximation to the numerical calculation (solid line). It is clear from either curve that the day–night effect is essentially confined to those trajectories that go through

the Earth's core. Any realistic search for an Earth effect for Region A requires sensitivity to this Earth-Sun juxtaposition.

The results of these exploratory and qualitative calculations then steer us to the more complete spectrum weightings necessary to confront the oncoming experiments.

4. Expectations for the Borexino, SNO, Super-Kamiokande Detectors

Region A

Since Region A appears at this time to be the most statistically likely MSW solution, it is worth a particular effort to examine the possibilities for detection of an Earth effect, even though the effect is expected to be small. We have seen that it is necessary to concentrate on the high energy range of solar neutrinos, those from the ^8B branch. Further, one must concentrate on the $\phi > 57^\circ$ trajectories. Both SNO¹² and Super-Kamiokande¹³ are sensitive to this high-energy range. But the expected counting rate of the 22,000 ton Super-Kamiokande water Cherenkov detector makes it the more promising possibility: for a given trajectory through the Earth the day-night difference in counting rates is about twice as large in Super-Kamiokande as it is in the SNO 1000 ton heavy water detector. Furthermore Super-Kamiokande has the additional advantage of being closer to the equator, at north latitude 36.4° , compared to SNO at 46.5° . Therefore, during the course of the year about 7% of Super-Kamiokande events have gone through the core of the Earth compared to 3.5% of SNO events.

We have presented in Table 1 the expected number of excess counts per year in Super-Kamiokande for all those trajectories that go through the Earth's core, $\phi > 57^\circ$; the $(\Delta m^2, \sin^2 2\theta)$ values analyzed are limited to a sampling of those for which $\chi^2 < 2.5$. For the purpose of the calculations in this paper Super-Kamiokande is assumed to be sensitive to electrons of total energy greater than 5.8 MeV. $N_{\phi > 57^\circ}$ is the actual number of counts per year including the earth effect for $\phi > 57^\circ$. N_{noEarth} is the number of counts for the half year's counting (daytime). $\Delta N_{\phi > 57^\circ}$ is the excess of counts per year due to the Earth effect for $\phi > 57^\circ$; for our binning $\Delta N_{\phi > 57^\circ}$ corresponds to $N_{\phi > 57^\circ}$ minus 7.32% of a full

years counting with no Earth (or $.1464 \times N_{\text{noEarth}}$). Stated errors are statistical, calculated from the expected counting rates. The main limitation on the statistical errors comes from $N_{\phi > 57^\circ}$. The corresponding ϕ dependence is exhibited in Figure 4. At the minimum $\chi^2 = .3$ (relative to the Bahcall–Pinsonneault SSM) ($\Delta m^2 = 6.3 \times 10^{-6}$, $\sin^2 2\theta = .0063$) there is about a 2σ effect in one year of counting, and the effect is slightly larger for the neighboring second lowest χ^2 point. One might hope to measure an effect of this size by accumulating over several years. Other parameters corresponding to values of P_S closer to .5 and/or to smaller mixing angles (such as allowed by the Turck-Chièze SSM) clearly do not have a measurable day-night effect. In the limit of $P_S = .5$ of course there is no day-night effect. There is even one example of a negative day-night effect (which occurs when $P_S > .5$ as is evident from Eq. (5)).

The Borexino experiment would not show any appreciable day-night effect if the neutrino parameters are those of Region A.

Region B

Neutrinos whose MSW parameters lie in the vicinity of $\Delta m^2 = (1 - 5)10^{-5}(\text{eV})^2$, $\sin^2 2\theta = .7$, are expected to display day-night differences if their energy is high, E_ν in the several to many MeV range. Therefore, Borexino is out of consideration, and we are to consider both SNO and Super-Kamiokande.

The effects for these latter two detectors are presented in Figures 5,6 (SNO) and 7 (Super-Kamiokande) for $\sin^2 2\theta = .7$ and Δm^2 that range over $(.63 - 4)10^{-5}(\text{eV})^2$. The Earth effect is clearly very sizeable for neutrino parameters in the B region, of the order of a factor of 2 for the SNO detector, of order 3/2 for Super-Kamiokande. The ϕ dependence is qualitatively explained in terms of the fixed density, monoenergetic analytic forms examined in the previous section. The regular oscillations of the form $\sin^2(\pi X/\ell_m)$, are however, damped by the averaging over the part of the ^8B spectrum of neutrino energies to which these detectors are sensitive; this damping feature is more pronounced at the larger values of Δm^2 , as is clear in Figure 6. Also note the sharp break at $\phi = 57^\circ$, the angle beyond which a neutrino trajectory goes through both mantle and core. This, in turn demonstrates that the spectrum reach is such that both core and mantle resonances come into play.

It is important to recall at this point that the earlier work with the Super-Kamiokande's predecessor⁵ sought but did not find a day-night effect; their measurement,

$$\frac{\text{day} - \text{night}}{\text{day} + \text{night}} = -.08 \pm .11 (\text{stat}) \pm .03 (\text{syst}),$$

enabled them to eliminate an otherwise permissible MSW area that is roughly bounded by $1.5 \times 10^{-6}(\text{eV})^2 < \Delta m^2 < 6 \times 10^{-6}(\text{eV})^2$. The Super-Kamiokande detector with its greater than order of magnitude volume increase could, then, expand the tested MSW region. Judging by the count-rates alone, without attempting to estimate the systematic errors of background subtraction, sensitivity to the Earth's day-night effect would extend to a Δm^2 larger than twice the present upper bound.

A similar extension of the region of sensitivity to the Earth effect would be afforded by the SNO experiment, the lower count rate being compensated by the higher nighttime regeneration. Thus at $\Delta m^2 = 1.6 \times 10^{-5}(\text{eV})^2$, $\sin^2 2\theta = .7$ the difference in day-night counts accumulated over a year is expected to be $\sim 1130 - 875 = 255$ to which can be assigned a gross error of $\sqrt{1130 + 875} \sim 45$. More precise fits can be obtained by fitting to the ϕ dependence.

Region C

As we have noted the likelihood that this is nature's choice for the neutrino parameter appears to be quite small — given the recent GALLEX result, combined with the older Kamiokande finding and the accumulation of ^{37}Cl data. However, were it so, then we have seen from our simple qualitative analysis that the sensitivity of the Borexino detector to energies around those of the sharp ^7Be line (.86 MeV) is expected to demonstrate a sizeable day-night effect.

Figure 8 shows calculated day-night effect in Borexino at $\sin^2 2\theta = .77$, $\Delta m^2 = 1.2 \times 10^{-7}$ (solid curve) corresponding to the minimum χ^2 in Region C for Bahcall-Pinsonneault. For comparison, we have repeated the curve from Figure 1 with only the ^7Be source included (dotted line). Clearly most of the recoil electrons in the .250-665 MeV window are from ^7Be neutrinos. The short-dashed curve is the calculation at the minimum χ^2 of the Turck-Chièze SSM for Region C. The long-dashed curve that nearly falls on the dotted curve is calculated for the same values as the solid curve but with the Turck-Chièze SSM.

The inclusion of the few percent of neutrinos from higher energy neutrino branches other than ${}^7\text{Be}$ clearly has little effect on the ϕ -dependence. The magnitude of the day-night effect is a quite impressive ϕ -dependence, with a swing from high to low of a factor of 2, and an averaged night count 50% over that of the day.

The day-night effect is smaller at SNO or Super-Kamiokande in Region C than in Borexino, but not negligible. Figure 9 shows calculations using the Bahcall-Pinsonneault SSM for the two sets of mass and mixing parameters of Fig. 8. Swings from high to low are of the order of 10% or less, and average night counts of the order of several percents over day counts.

5. Summary of Conclusions

What special tests does the Earth effect give us beyond the other capabilities of the three oncoming real time experiments? Of course the possible neutral current determination in SNO would provide a qualitatively new piece of information, the neutrino flux independent of flavor. But the observation of an Earth effect, especially in its day-night oscillation form, would also provide the clear signature that the field of solar neutrino physics lacks so far.

Should Borexino with its sensitivity to the middle energy region, the ${}^7\text{Be}$.86 MeV neutrino, measure a sizeable day-night oscillation, then Region C would be established. The predicted averaged rate for Borexino is about 1/2 SSM for Region C; a similar rate would hold for Region B, but there would be a vanishingly small day-night effect. The Region A solution would have no day-night effect, corresponds to a very small contribution from ${}^7\text{Be}$ neutrinos, and thus a small rate in Borexino, about 1/4 SSM.

If a sizeable day-night oscillation is observed in the higher energy detectors, SNO and Super-Kamiokande, then Region B would be picked out. A small day-night effect in these detectors in the trajectories through the mantle would correspond to Region C. Such a small effect would not pinpoint the parameter region, but it would provide identification of the basic transformation mechanism.

Region A has the dominant position in the credibility hierarchy, and it prescribes only a small day–night effect in the higher energy detectors, essentially limited to trajectories through the core of the Earth. But the effect might be measureable, and might, then, provide the desired clear signature, the unambiguous confirmation of neutrino mass and mixing.

Acknowledgment

We would like to acknowledge useful conversations with Martin Deutsch and R.S. Raghavan. We would also like to thank John Bahcall for providing us with the data files of densities from the Bahcall–Pinsonneault calculations.

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Table Caption

Table 1: Counts for the Super-Kamiokande detector: total counts per year, $N_{\phi > 57^\circ}$, for trajectories through the core of the Earth; total counts per year without the Earth effect, $N_{\text{no Earth}}$ (half a year's counting with only MSW in the Sun); and total counts per year, $\Delta N_{\phi > 57^\circ}$ excess of the night-time over the day-time counts for trajectories through the core of the Earth. All points in the $\sin^2 2\theta$, Δm^2 mesh for which either the χ^2_{BP} based on Bahcall–Pinsonneault SSM, or the χ^2_{TC} based on the Turck–Chière SSM, is less than 2.5 are tabulated.

Table 1

$\sin^2 2\theta$	Δm^2 ($\times 10^{-6} eV^2$)	χ^2_{BP}	χ^2_{TC}	$N_{\phi > 57^\circ}$	$N_{\text{no Earth}}$	$\Delta N_{\phi > 57^\circ}$
.004	6.3	22.5	2.0	883	6139	-5 ± 32
.004	7.9	8.1	1.6	791	5436	4 ± 30
.005	5.0	20.4	1.8	888	6111	4 ± 32
.005	6.3	5.9	1.7	801	5418	17 ± 30
.005	7.9	.6	7.0	701	4692	22 ± 28
.0063	5.0	5.1	2.2	810	5387	30 ± 30
.0063	6.3	.3	8.3	728	4673	51 ± 29
.0079	5.0	.5	9.3	741	4642	69 ± 29

Figure Captions

- Fig. 1: $\chi^2 = 9$ contours calculated for MSW and the combined Homestake, Kamiokande, and GALLEX experiments. The solid χ^2 contours show the allowed regions in the Bahcall-Pinsonneault SSM; the dashed ones are for Turck-Chièze.
- Fig. 2: The dependence of the number of detected neutrinos at Borexino per 24 hours of counting at the angle the Sun lies below the horizon. The dashed curve is from Eq. 3 and the solid curve is from the full numerical calculation. Note the sharp break in the solid curve at 57° , beyond which the neutrino trajectory crosses the core
- Fig. 3: The dependence of the Super-Kamiokande counting rate on the angle the Sun lies below the horizon in for characteristic neutrino parameters in Region A. The dashed curve corresponds to the analytical approximation discussed towards the end of Section 3; for neutrino trajectories that do not cross the core we use densities of 4.5 gm/cm^3 , while for those that do cross the core we ignore the mantle entirely and use a core density of 11 gm/cm^3 . The solid curve corresponds to a full calculation.
- Fig. 4: Earth trajectory dependence of Super-Kamiokande for neutrinos of Region A: dashed lines — $\sin^2 2\theta = .004$, $\Delta m^2 = 6.3 \times 10^{-6}$ (top), 7.9×10^{-6} (middle); dotted lines — $\sin^2 2\theta = .005$, $\Delta m^2 = 5. \times 10^{-6}$ (top), 6.3×10^{-6} (middle), 7.9×10^{-6} (bottom); solid lines — $\sin^2 2\theta = .0063$, $\Delta m^2 = 5.3 \times 10^{-6}$ (middle), 6.3×10^{-6} (bottom); dot-dashed line — $\sin^2 2\theta = .0079$ $\Delta m^2 = 5. \times 10^{-6}$ (bottom).
- Fig. 5: Earth trajectory dependence for the SNO detector in Region B.
- Fig. 6: Earth trajectory dependence for the SNO detector in Region B; $\sin^2 2\theta = .7$: solid line — $\Delta m^2 = 1.6 \times 10^{-5}$, dot-dashed line — $\Delta m^2 = 2.5 \times 10^{-5}$, dotted line — $\Delta m^2 = 4. \times 10^{-5}$. The long dashed line is for $\Delta m^2 = 1.6 \times 10^{-5}$, $\sin^2 2\theta = .77$
- Fig. 7: Earth trajectory dependence for the Super-Kamiokande detector in Region B.

Fig. 8: Borexino lack of sensitivity to parameters in Region C. Curves are shown for the 250-665 keV electron window. The solid line is the numerical calculation at the minimum χ^2 of the Bahcall-Pinsonneault SSM with all sources in the sun included. The dotted line includes only the .86 MeV ^7Be neutrinos (the same as the numerical calculation of Fig. 1.) The short-dashed line is based on the Turck-Chièze SSM at its Region C minimum χ^2 ($\sin^2 2\theta = .92, \Delta m^2 = 10^{-7}$). The long dashed line is based on the Turck-Chièze SSM, but at the Bahcall-Pinsonneault minimum χ^2 ($\sin^2 2\theta = .77, \Delta m^2 = 1.2 \times 10^{-7}$).

Fig. 9: Earth trajectory dependence for the SNO detector in Region C.

Fig. 1

Fig.2

Fig. 3

Fig. 4

Fig. 5

Fig. 6

Super-Kamiokande

Fig. 7

Fig.8

Fig. 9