ABSTRACT Three aspects of the relation between quantum theory and relativity are discussed in the context of the Bohm hidden-variable model. First, a version of the model that is consistent with Einstein-Lorentz locality is proposed and it is suggested how it might be tested through its disagreement with orthodox predictions in some circumstances. Next, the Bohm model is shown to remove the problem of information-loss when it is used in space-times that have closed time-like loops. Finally the issue of time in quantum cosmology is discussed, and it is suggested that there are here new features of the Bohm model. A simple mini-superspace example is treated to show how a universe that expands and then contracts can be obtained.

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1. Introduction.

I want to discuss some aspects of the relation, or perhaps it would be better to say conflict, between quantum theory and relativity. It seems clear that, in order to understand the nature of this conflict, we need a complete quantum theory. Attempts to suppress this need, by using the obscurities of the Copenhagen interpretation, for example, merely hide the conflict, and give neither motivation nor guidance towards its solution.

To my knowledge, there are only three methods of providing a "complete" quantum theory that are presently available. These involve either hidden variables, in particular as in the Bohm model\textsuperscript{1,2,3}, explicit collapse of the wavefunction, as in the work of Ghirardi, Rimini and Weber\textsuperscript{4}, and of Pearle\textsuperscript{5} (see ref.\textsuperscript{6} for a recent review and further references); or some form of many-worlds plus consciousness model, as proposed by Albert and Loewer\textsuperscript{7}, Lockwood\textsuperscript{8}, Stapp\textsuperscript{9} or, in a form which, naturally, I prefer, Squires\textsuperscript{10}. This last method inevitably takes us outside of what we conventionally (at present) regard as physics, so I shall ignore it (which is not to say that I do not believe it might be true). Mainly I shall concentrate on the Bohm model, since there the issues seem more clear (I guess because the concept of trajectories is one with which we are familiar - certainly more so than with stochastic background fields).

2. The Bohm model and Lorentz-invariance.

In the standard version of the (non-relativistic) Bohm model, the velocity of particle 1, at time \( t \), is given by the expression

\[
\dot{x}_1(t) = \frac{\nabla_{x_1} \Psi(x_1(t), x_2(t), \ldots)}{m_1 \Psi(x_1(t), x_2(t), \ldots)},
\]

where \( \Psi \) is the quantum wavefunction. The non-relativistic nature of this is apparent from the fact that the velocity of particle 1 at a given time is assumed to depend on the position of the other particles at the same time.

A simple (in principle) way to demonstrate the conflict in an experiment discussed in Hardy and Squires\textsuperscript{11}. This paper, based on earlier work by Hardy\textsuperscript{12}, also demonstrates that any hidden-variable model (at least within some class defined in the paper) must inevitably violate either Lorentz-invariance or quantum theory. It is important to recognise that this conflict has nothing to do with the fact that particle number cannot be conserved in Lorentz-invariant theories and that it is necessary to use field theory. Indeed, the experiment referred to can be performed with arbitrarily low velocities for all the particles involved, so the field-theoretic aspects should not be important.

Although pre-1905 physicists were, apparently, happy with an absolute concept of simultaneity, as required in eq.(1), and in the quantum understanding of the above experiment, it is hard for those of us trained in relativity to accept it. As we shall see below, the idea becomes even harder in more general space-times required in General Relativity. Hence it is natural to suggest that the conflict should be resolved by relaxing the requirement of perfect agreement with the predictions of standard quantum theory. The way to proceed then becomes rather obvious. The problem arises essentially because the wavefunction, used in calculating the velocities (or the quantum potential, if we write the theory as a Newton-like equation for acceleration), is defined in configuration space, i.e., it depends on the position of all the relevant particles. This, however, is normal in non-relativistic physics; it is true for example of the electrostatic potential of several charged particles. There, the conflict with relativity is resolved by going from electrostatics to electrodynamics, which is a fully relativistic theory. What this does, among other things, is to replace the "synchronous" positions by, so-called, retarded positions. I have therefore proposed\textsuperscript{13} to make a relativistic version of the Bohm model by the same device. Thus we replace eq.(1)

\[
\dot{x}_1(t) = \frac{\nabla_{x_1} \Psi(x_1(t), x_2(t), \ldots)}{m_1 \Psi(x_1(t), x_2(t), \ldots)},
\]

where

\[
t_2 = t - \frac{|x_1(t) - x_2(t)|}{c}.
\]

Of course, the formal similarity between the electrostatics case and quantum theory is probably misleading. For one thing the wavefunction usually has an explicit dependence on time, and it is not clear how to treat this. Nevertheless, there is no ambiguity if we apply eq.(2) to situations where the particles are far apart, compared to the quantum uncertainty in their positions, and each is described by a wavefunction that is evolving independently. Then\textsuperscript{13} we use the retarded time \( t_2 \) to describe the wavefunction associated with particle 2, etc.

In order to understand some of the effects of this we consider an EPR-like measurement of the spins of two correlated particles, e.g., in the singlet-state. First, we use the standard Bohm model, where it is important to realise that the measured value of the spin is determined by the hidden variables in the detector. (Examples of this are given in the work of the Portsmouth group\textsuperscript{14}. Exact calculations for a very simple case are given in Squires\textsuperscript{15} and in Squires and Mackman\textsuperscript{16}. Suppose, first, that there is only one detector, say, in the path of the L particle. Then eq. 1 shows that it will record +1/2 and -1/2 with equal probability, the actual value in a given case depending on the values of the detector-hidden-variables. Similar results hold if only an R detector is present. When both detectors are included, then the value that is recorded depends on the relative values of the hidden variables in the two detectors. The way the Bohm model is constructed guarantees that the resulting probabilities are correlated exactly as required by the predictions of quantum theory. This is all illustrated schematically in fig. 1.

To see how this changes if we use the retarded model, eq. 2, we consider the process depicted in fig.2. The left particle reaches the detector at time \( t_1 \), but it is not until the later time \( t_2 \) that the formula given in eq.2 can "know about" the presence of the right detector. Clearly then, in the time between \( t_1 \) and \( t_2 \), the left detector begins to record the two values with equal probability. The same
thing is true of the right detector. At time $t_2$, the information about the other detector becomes available, and now one of several things might happen, depending on the detector hidden variables. Consider for example the extreme case where the detectors are measuring spins in the same direction. Then, in half of the cases, the initial movement of the detectors will correspond to "wrong" results, i.e., both giving the same value of the spin in contrast to the perfect anticorrelation expected in the singlet state. Suppose, for example, these are $+$ values, which occur if the hidden variables of both detectors are in the regions of space that lead to such values. When evolution beyond $t_2$ is considered, the detector whose hidden variable is furthest from the boundary of the region will be unaffected, the other will in some cases reverse, to give the "correct" result ($-$), but in others it will not, and hence the wrong result will be the final reading of the detectors. The condition for the wrong results is that the initial hidden variables are sufficiently close to each other, how close depending on the time difference $t_2 - t_1$.

This is again illustrated schematically in fig.1, where the enormous number of hidden variables in each detector is represented by one variable for each detector. In this figure the dotted region is the one that gives "wrong" results, and the crucial issue is how large it is in relation to the region available for the hidden variables. We expect (see also ref.13) that the condition for agreement with quantum theory is that

$$\frac{T}{T_M} \ll 1,$$

where $T = t_2 - t_1$ is the time for a signal to travel from one detector to another, i.e.,

$$T = \frac{L}{c},$$

$L$ being the separation between the detectors, and $T_M$ is the time taken for the "measurement" to be complete. Clearly, without a more adequate model of the measurement apparatus it is difficult to estimate $T_M$, but if we say that macroscopic systems are unlikely to make significant responses to microscopic stimuli in times less than about $10^{-8}s$, then we only expect departures from quantum theory to occur with reasonable frequency if $L$ is of the order of kilometres. I believe there is a need here for a careful study of actual measurements in order to obtain reliable estimates of what effects might be expected. Without such a study it is, in my opinion, premature to claim that quantum non-locality has been experimentally established. (Very similar considerations can be made for explicit collapse models: see ref.12).

It is worth noting here that we could imagine relativistic models that are more effective in hiding departures from quantum theory (and from angular momentum conservation) than the retarded Bohm model considered here. We could, for example, suppose that the particles carry information about their spin direction, i.e., one will be positive in a particular hemisphere, the other negative in the same hemisphere. Then, regardless of any information coming from the other measurement, there will be no violation of strict anticorrelation. Then the disagreement with quantum theory would arise only because, for sufficiently large separation, the results would have to be consistent with Bell's inequality. Clearly evidence for this would be harder to find because it would depend on a statistical analysis whereas, in principle, just one result in violation of the anticorrelation would be sufficient to establish a disagreement.

3. The Bohm model and Information Loss.

Since Hawking demonstrated that black holes radiate a thermal spectrum, there has been growing interest in the apparent loss of information associated with parts of correlated wavefunctions disappearing down black holes. It seems that an initially pure quantum state turns into a mixed state. However, if the quantum state is supposed to be a statement about "what is", rather than about what we know about it, then this is hard to interpret in a way that makes sense. The statement that the universe is either in state $A$ or state $B$ surely means that it is actually in one or the other; we may be unsure about which, but the universe itself can have no such doubt!

The Bohm model is ideally suited to considering questions of this type since it has a clear deterministic ontology. We can ask whether the Bohm model continues to work in its usual determinate way even in the presence of black holes. To answer such a question requires a formulation of the model which is applicable to non-flat background spacetimes, and a student of mine (Steve Mackman) is working on this problem here in Durham. Even without this, however, we can discuss the application of the model in flat space-times which are topologically non-trivial, in particular which contain closed time-like loops. These allow information loss very similar to that which occurs in black holes.

As an example, we consider the two-dimensional space shown in fig. 3. This is made from an infinite $x,t$ plane with a cut along $z = 0$ from $t = -\infty$ to $t = t_1 > 0$ and from $t = t_2 > t_1$ to $t = +\infty$, and with the region $z < 0$ being rolled up by the identification of $t = t_1$ with $t = t_1$. We impose the boundary condition that the wavefunction is zero on the cut.

We take a two-particle wavefunction chosen by the initial condition that at $t = 0$ it has the form

$$\psi_0(x,y) = \alpha \phi_0^a(x) \phi_0^b(y) + \beta \phi_0^b(x) \phi_0^a(y),$$

where the states $\phi_0^a$ and $\chi_0^b$ are normalised to unity, and the latter are chosen to be orthogonal,

$$\langle \chi_0^a \phi_0^b \rangle = 0.$$

Explicitly we suppose that these states are wavepackets approximately localised around $y = a, y = b > a, x = a + c > b$ and $x = b + c$, as shown in fig.4. We assume that the states are solutions of the two-dimensional Dirac equation so that they move as indicated in the figure (see Squires15).

Consider now the trajectory of the $z$-particle. This is given by the usual Bohm expression

$$\dot{x} = \frac{\mathbf{p}_x \psi}{m \psi}$$
which becomes
\[ x = A \chi_a(y) \phi_0(x) + \beta \chi_a(y) \phi_1(x) \]
\[ = \frac{\alpha \chi_a(y) \phi_0(x) + \beta \chi_a(y) \phi_1(x)}{m \chi_a(y) \phi_0(x) + m \chi_a(y) \phi_1(x)}, \]  
(9)

an expression which clearly shows how the trajectory for the z-particle depends on the value of y. As long as the two \( \chi \) wave packets do not overlap, then the value of y ensures that the trajectory is determined entirely by either \( \phi_0 \) or \( \phi_1 \), exactly as would be the case for a pure state, rather than the pure state of eq. (6). This is no longer the case however when the \( \chi \) states overlap, even though the states remain orthogonal. This fact, that in the Bohm model there is a clear distinction between a mixed state and a pure state was made already in 1980 by John Bell. I am grateful to Shelly Goldstein for telling me about this work.

Now we suppose, just as shown in fig.4, that the two wave packets (\( \chi_a \) \( \chi_b \)) for the y-particle enter the “black-hole”. In this case it clearly makes no sense to use the correlated wavefunction at much larger times, in particular, the shaded region of the figure. It is not reasonable to suppose that anything happens to the y-particle as it moves along the tube: it can have any effect on the z-particle outside it. It is this that gives rise to the loss-of-information problem in “orthodox” quantum theory. Since we cannot use the state \( \Psi \) to describe the z-particle for large times, we have to use the mixture: either \( \phi_0 \) or \( \phi_1 \). In the Bohm model, however, there is a natural resolution of this problem. At all stages, the z-wavefunction that is used to calculate the z-trajectory is determined by the value of y, so at the moment when the y particle enters the black hole, with a consequent ending of the correlation, we have a unique wavefunction as a function of y. This wavefunction, evolved forward in time by the Dirac equation, determines the trajectory at all future times.

In fact it seems most natural here to define the time when the correlation ceases as being along the forward light-cone from the point \( x = 0, t = t_2 \) (see fig.4). Thus we use the Lorentz-invariant modification of the Bohm model, discussed above, to calculate the trajectory from the complete wavefunction throughout the unshaded region of the figure. On the boundary, i.e., along \( x = t - t_2 \), the z-wavefunction is taken to be
\[ \Psi(x) = \alpha \chi_a(y(t - |x - y|)) \phi_0(x) + \beta \chi_a(y(t - |x - y|)) \phi_1(x), \]  
(10)

with \( y \) being the position of the y-particle of course. This is a function of x along the boundary and as such it can be calculated for all future times using the Dirac equation. The trajectory followed by the particle outside the black hole is now no longer correlated with anything inside, and there is no loss of information. Thus here, as elsewhere, the Bohm model successfully removes any ambiguities in the application of quantum theory.

4. The Bohm model and time in quantum cosmology.

As is well known, the wavefunction of the universe in canonically quantised gravity is independent of time. Many workers have considered how this fact should be reconciled with a world of change. For a recent review and many other references see Isham. To some extent, the solutions proposed depend on the attitude adopted to the quantum measurement problem. For example, if we follow the explicit collapse route then there appears to be a natural way of introducing time, because although the collapsed wavefunction might be annihilated by the hamiltonian this will not normally be the case with the collapsed wavefunction (since in general the collapse function will not commute with the hamiltonian). Thus, “time” begins with the first collapse. In many-worlds quantum theory, on the other hand, there really is no such thing as external time in physics, and an endeavor is made to understand our experience of time by equating it to some other variable, i.e., something that plays the role of the hands of a clock. Again I believe this pushes the issue outside of present physics and into “consciousness”.

In the Bohm model there is, at first sight, a natural solution to the missing time problem because the Bohm formula does not necessarily give zero velocities when the wavefunction is constant. In other words, a constant wavefunction need not correspond to a situation where nothing is changing. There are, however, some difficulties with this. The first is conceptual. If we regard the Bohm equation as being the condition that preserves the property that an initial distribution of positions, agreeing with the quantum probability law, will still agree at all later times, then the natural solution for a constant wavefunction will clearly be that with zero velocities. The Bohm formula is just one possible way of satisfying this condition, and, at least for a constant wavefunction, it is not the simplest.

The second difficulty is more practical. Although non-zero velocities are possible, they are not easy to obtain, and the obvious (real) solutions of the Wheeler-DeWitt equation have zero velocities. For this reason, the Bohm model actually makes the problems arising from a constant wavefunction harder, not easier, to resolve. A model that does not actually have a radius of the universe, say, is not troubled by its rate of change, but such a thing actually has a value in the Bohm model so it had better not be a constant! Note also that there seems to be an important difference here between the Bohm model as used in the microscopic world, and as used in cosmology (see ref.20). In the former we are not unduly disturbed by the fact that the model gives zero velocities for particles in systems that are in energy eigenstates, e.g., for the electrons in a hydrogen atom. We know from the way the model is constructed that it will give the correct distribution for measured velocities. In other words, measurements of velocities are not “faithful”; the values that are observed being different to the pre-existing values given by the Bohm formula. In quantum cosmology, however, this “escape” is not possible: if everything has zero velocity, according to the Bohm expression, then there simply is no possibility that anything can change with time. There can, indeed, be no “measurements” even.

We can see the relevance of all this if we think about the Bohm description of reflection by potential barrier. This should be analogous to cosmological models in which an initially expanding universe recollapses after the radius has “bounced” off some potential barrier. We therefore consider a particle, moving in one dimension,
under the influence of a square barrier. This is described by the Schrödinger equation

$$\left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x)\right)\psi = E\psi,$$  \hskip 1cm (11)

where the barrier potential is given by

$$V(x) = 0, \ x < 0,$$
$$= V_1 > E, \ x > 0.$$

The standard method of solution, given in every quantum theory course, leads to the wavefunction

$$\Psi = Ae^{ikx} + Be^{-ikx},$$  \hskip 1cm (13)

in the region $x < 0$, where $k^2 = \frac{2mE}{\hbar^2}$. The reflection coefficient, $R$, is given by

$$R = \frac{K + ik}{K - ik},$$  \hskip 1cm (14)

where $K^2 = \frac{2m(V_1 - E)}{\hbar^2}$, so that of course $|R|^2 = 1$, corresponding to the fact that all particles are reflected. The “story” that accompanies this in typical quantum theory courses is that particles move from the left, hit the barrier, where, after penetrating for a short distance, they are reflected. This, however, is not what actually happens in the Bohm model, for which the usual formula gives

$$\dot{x} = \frac{(1 - |R|^2)k}{i\hbar^2 + R e^{-ikx}} \psi,$$  \hskip 1cm (15)

$$= 0.$$  \hskip 1cm (16)

In other words, the particles do not move. There is thus a big difference between the usual story and the actual behaviour in the Bohm model. This difference does not, of course, contradict the well-known agreement between the Bohm model and the predictions of orthodox quantum theory. This agreement refers to statistical predictions for position observations, and, since in the above situation we have a constant wavefunction, it is quite in order for the particles to be stationary.

Note that we can recover the usual story if we consider a time-dependent wavefunction with an incident wavepacket. This of course corresponds more closely to the actual situation in a laboratory experiment, but is not an option which is available for the cosmological case, at least not if we wish to maintain the Wheeler-deWitt equations.

Of, perhaps, more relevance is the fact that even with a stationary wavefunction it is possible to obtain non-zero velocities from the Bohm expression. To see this we write the solution, in $x < 0$, as

$$\psi = Ae^{ikx} + Be^{-ikx},$$  \hskip 1cm (17)

then the velocity will be given by

$$\dot{x} = \frac{|A|^2 - |B|^2k}{(|A|^2 + |B|^2 + 2RA^* Be^{-2ikx})m}.$$  \hskip 1cm (18)

The condition that $|A|^2 = |B|^2$ arises from the physical requirement that only the decreasing exponential exists in the “forbidden” region, $x > 0$. If we ignore this requirement, then the Bohm model has non-zero velocities. Of course, such a solution does not show any reflection by the barrier; the particles just move into (or out of) the infinite sink in the region $x > 0$. A model somewhat similar to this is suggested in the work of Kiefer, where it is claimed that “decoherence” may be responsible for effectively removing one of the exponential terms in the allowed region. It is not clear what effect this has on the wavefunction in the forbidden region, but any solution where this wavefunction increases exponentially is surely not relevant to physics. Nevertheless, this opens up the possibility that coupling to other systems will in fact allow the static wavefunction to have non-zero, and even reversing, velocities.

To see how these might occur, we here take a very simple “minisuperspace” model, which has at least some of the desired properties. We write the metric in the form

$$ds^2 = dt^2 - e^{2\lambda}d^2\Omega,$$  \hskip 1cm (19)

where the last term is the interval on a spatial hypersurface of constant curvature, $k$. In fact we take $k = 1$, corresponding to a closed universe. We also introduce a constant scalar field $\phi$. For the “Wheeler-deWitt” equation we take

$$\left(\frac{\partial^2}{\partial \phi^2} - \frac{\partial^2}{\partial \psi^2} + A(\alpha)\phi^2 - B(\alpha)\right)\Psi = 0,$$  \hskip 1cm (20)

with

$$A(\alpha) = A_0, \ \alpha < 0,$$
$$= A_1, \ \alpha > 0,$$  \hskip 1cm (21)

and

$$B(\alpha) = B_0, \ \alpha < 0,$$
$$= B_1, \ \alpha > 0.$$  \hskip 1cm (22)

In the region $\alpha < 0$ we write the solution of eq.20 as

$$\Psi = \sum \alpha e^{\alpha} \chi_n^{(0)}(\phi),$$  \hskip 1cm (23)

where the $\chi_n^{(0)}$ are harmonic oscillator bound states:

$$\chi_n^{(0)}(\phi) = (\pi \frac{1}{2^n n!})^{-\frac{1}{2}} H_n(\phi)e^{-\phi^2/2},$$  \hskip 1cm (24)
with
\[ z_0 = A_0^{\frac{1}{2}} \phi, \]
and where
\[ k_n^2 = (2n + 1)A_0^{\frac{1}{2}} - B_0. \]
The \( k_n \) are all real, corresponding to a classically allowed region, provided
\[ B_0 < A_0^{\frac{1}{2}}. \]

In restricting the solution to this form, in particular by using eq.(24), we have imposed the physical requirement that the wavefunction should be square-integrable in \( \phi \). We cannot impose a similar requirement on the \( \alpha \) dependence, but we shall require at least that it should not increase exponentially. Thus, in the region \( \alpha > 0 \) we write
\[ \Psi = \sum_n c_n e^{-\kappa_n \alpha} \chi_n^{(1)}(\phi), \]
where the \( \chi_n^{(1)} \) are defined by a similar equation to (24) but with \( z_0 \) replaced by \( z_1 \) defined by
\[ z_1 = A_1^{\frac{1}{2}} \phi. \]
Similarly the \( \kappa_n \) are defined by
\[ \kappa_n^2 = B_1 - (2n + 1)A_1^{\frac{1}{2}}. \]

In order that the region \( \alpha > 0 \) should be classically forbidden we require the \( \kappa_n \) to be real, which means that \( B_1 \) must be many times larger than \( A_1^{\frac{1}{2}} \). For sufficiently large \( n \), however, it is inevitable that the rhs of eq.(30) will change sign, so there will be some "leakage".

To be specific, we choose as the boundary condition:
\[ \alpha_n = \delta n_0, \]
which corresponds to associating the expansion term (with positive \( \alpha \)) with the ground state of the \( \phi \) field. Other modes become excited at the \( \alpha = 0 \) boundary. To prevent any leakage we truncate the expansions at the first non-trivial term, i.e., that with \( n = 2 \). The problem is then completely solved by imposing continuity of the wavefunction and its first derivative at the boundary.

For details we refer to Hind and Squires[31]. Here we note that if we ignore the \( \phi \) field it is not possible for the expansion to reverse unless the wavefunction is double-valued in \( \alpha \), whereas in these models it is not. This of course follows directly from the Bohm formula where the velocity (\( \dot{\alpha} \)) is a function only of the value of \( \alpha \) (for a wavefunction that does not have any explicit dependence on time). In the presence of the \( \phi \) field such a reversal is possible, because the velocity can now depend also on the value of \( \phi \).

Indeed we have a prediction: when the universe recollapses, it does so with different values for the fields (only one in our case). In other words there is not an effective reversal of the direction of time, in the sense of everything returning exactly to its initial state. This would appear to be contrary to the claim of Kiefer and Zeh[24], who of course do not explicitly use the Bohm model in their discussion.

In actual fact the velocity given in the above model tends to oscillate (the details depend on the starting points - see ref[23]). This occurs because of interference effects between the various terms in the expansion of eq.(23). It seems likely that if we were to include more complex states coupling to the \( \alpha \) variable then such oscillation would be suppressed, essentially because there would be less likelihood of the many hidden variables having values which allow both the two exponentials in eq.(23) to contribute. This would be analogous to the phenomena of decoherence[25], except that here it is not orthogonality of the states that is significant, but rather their limited spatial overlap.

Clearly much more work is required before reliable conclusions can be drawn from discussions of this nature. In our model the expansion is associated with low excitation of the \( \phi \) field, whereas in fact it is more likely that it is the de-excitation that causes the bounce (see, for example, eq.(30)). Also more complex situations than that involving only a single constant field need to be analysed. Nevertheless, the results show that there is a real role for the Bohm model in quantum cosmology. In this connection earlier work by Vink[26] and by Valentini[26] should be mentioned.

Figure Captions

Fig.1 A schematic representation of the effect of the L and R detector variables (\( d_L, d_R \)) on the measurement outcomes (\( L_\pm, R_\pm \)). The horizontal (vertical) divisions are relevant when only the L (R) detector is present. The line at 45\(^\circ\) gives the division with both detectors. The hatched areas show where the outcome from one detector is affected by the presence of the other. The doubly hatched area is where "wrong" results might occur in the retarded model.

Fig.2 The space-time diagram for an EPR-like event in which a correlated pair leaves \( x = 0 \) at \( t = 0 \) arriving at the detectors at \( t_1 \). In the retarded Bohm model each detector, during the time \( t_1 < t < t_2 \), behaves as though the other were absent.

Fig.3 Showing the topology of the two-dimensional space used to consider the problem of information loss.

Fig.4 Showing possible paths of two correlated particles. The shaded region is where the possible loss of information might occur.
References
