BLACK-HOLE-WAVE DUALITY
IN STRING THEORY

Eric Bergshoeff\textsuperscript{a1}, Renata Kallosh\textsuperscript{b2} and Tomás Ortín\textsuperscript{c3}

\textsuperscript{a} Institute for Theoretical Physics, University of Groningen
Nijenborgh 4, 9747 AG Groningen, The Netherlands
\textsuperscript{b} Department of Physics, Stanford University
Stanford CA 94305, USA
\textsuperscript{c} Department of Physics, Queen Mary and Westfield College,
Mile End Road, London E1 4NS, U.K.

ABSTRACT

Extreme 4-dimensional dilaton black holes embedded into 10-dimensional geometry are shown to be dual to the gravitational waves in string theory. The corresponding gravitational waves are the generalization of pp-fronted waves, called supersymmetric string waves. They are given by Brinkmann metric and the two-form field, without a dilaton. The non-diagonal part of the metric of the dual partner of the wave together with the two-form field correspond to the vector field in 4-dimensional geometry of the charged extreme black holes.

\textsuperscript{1}E-mail address: bergshoe@th.rug.nl
\textsuperscript{2}E-mail address: kallosh@physics.stanford.edu
\textsuperscript{3}E-mail address: ortin@qmchep.cern.ch
1. In this paper we continue our investigation of black holes and gravitational waves. From the point of view of General Relativity those are very different geometries. However the string theory brings in a completely different concept of "equivalent" background geometries. It was understood some time ago [1] that the pp-waves are dual to the fundamental strings [2]. The corresponding duality transformation, which is known as sigma-model duality transformation [3], has a very particular property: it changes the value of the dilaton field $e^{2\phi}$ by the $g_{xx}$-component of the metric, where $x$ is some direction on which all fields are independent. In our recent paper [4] we have established an analogous dual relation between more general solutions of the effective equations of the critical ($d = 10$) superstring theories. The purpose of this letter is to show that some solutions, which can be obtained by dual rotation from a particular case of supersymmetric string waves (SSW) [5] are identified as supersymmetric extreme charged dilaton black holes upon Kaluza-Klein dimensional reduction to $d = 4$.

The first known to us example of a deep relation between gravitational waves and black-hole type solutions was given by Gibbons [6]. He has observed that the 5-dimensional pp-waves upon Kaluza-Klein dimensional reduction to $d = 4$ are equivalent to a singular limit of electrically charged black hole. Those black holes have the scalar field $g_{55} = \sigma$ coupling to the vector field of the form $e^{2\sqrt{g}\phi} F^2$. The electric solutions are related via electric-magnetic duality to monopoles. Thus this example has shown the dual relation between gravitational waves and monopoles.

We will use the sigma-model duality of the string theory and relate solutions of 4-dimensional and 10-dimensional effective actions of string theory. We will limit ourselves by keeping only one scalar field, the fundamental dilaton. The method which we develop here may give many other interesting relations for the class of solutions which will include more fields of the string theory.

2. We consider the zero slope limit of the effective string action. This limit corresponds to 10-dimensional $N = 1$ supergravity. The Yang-Mills multiplet will appear in the first order of $\alpha'$ string corrections. The SSW [3] in $d = 10$ are given by the Brinkmann metric [7] and the following 2-form

$$
\begin{align*}
 ds^2 &= 2d\tilde{u}d\tilde{v} + 2A_M\tilde{x}^M d\tilde{u} - \sum_{i=8}^{i=8} d\tilde{x}^i d\tilde{x}^i, \\
 B &= 2A_M\tilde{x}^M \wedge d\tilde{u}, \quad A_\nu = 0,
\end{align*}
$$

where $i = 1, \ldots, 8$, $M = 0, 1, \ldots, 8, 9$ and we are using the following notation for the 10-dimensional coordinates $x^M = \{\tilde{u}, \tilde{v}, \tilde{x}^i\}$. We have put the tilde over the 10-dimensional coordinates, since we will have to compare the original 10-dimensional configuration with the 4-dimensional one, embedded into the 10-dimensional space. A rather non-trivial iden-
tification of coordinates, describing these solutions will be required later.

The equations that \( A_u(\hat{x}^i) \) and \( A_i(\hat{x}^j) \) have to satisfy are:
\[
\triangle A_u = 0, \quad \triangle \partial^i A^{\hat{j}} = 0 ,
\]
where the Laplacian is taken over the transverse directions only.

Sigma-model duality transformation [3] defines the changes in the metric, 2-form field \( B_{\mu\nu} \) and in the dilaton \( e^{\phi/2} \):
\[
\begin{align*}
g'_{xx} &= 1/g_{xx} , \quad g'_{x\alpha} = B_{x\alpha}/g_{xx} , \\
g'_{\alpha\beta} &= g_{\alpha\beta} - (g_{x\alpha}g_{x\beta} - B_{x\alpha}B_{x\beta})/g_{xx} , \\
B'_{x\alpha} &= g_{x\alpha}/g_{xx} , \quad B'_{\alpha\beta} = B_{\alpha\beta} + 2g_{x\alpha}B_{x\beta}/g_{xx} , \\
\phi' &= \phi - \frac{1}{2} \log |g_{xx}| .
\end{align*}
\]
This transformation is defined for configurations with a non-null Killing vector in the \( x \)-direction. The string theory considers such configurations as equivalent under the condition that the \( x \)-direction is compact.

A straightforward application of the sigma-model duality transformations given in (3) on the SSW solution given in eq. (1) leads to the following new supersymmetric solution of the zero slope limit equations of motion:
\[
\begin{align*}
ds^2 &= 2e^{2\phi} \{ d\hat{u}d\hat{v} + A_id\hat{u}d\hat{x}^i \} - \sum_{i=1}^{i=8} d\hat{x}^i d\hat{x}^i , \\
B &= -2e^{2\phi} \{ A_u d\hat{u} \wedge d\hat{v} + A_i d\hat{u} \wedge d\hat{x}^i \} , \\
e^{-2\phi} &= 1 - A_u ,
\end{align*}
\]
where as before, the functions \( A_M = \{ A_u = A_u(\hat{x}^i), A_v = 0, A_i = A_i(\hat{x}^j) \} \) satisfy equations (2). We called this solution generalized fundamental strings [4], since at that time we had in mind only a subclass of these solutions which depend on the 8-dimensional transverse coordinates and has an interpretation of a macroscopic string. However, if we assume that the functions \( A_M \) do not depend on all 8 transverse coordinates, but only on part of them, the name given to this class of solutions is not appropriate anymore. To avoid possible confusion we would call the generic solutions given in eq. (4) the dual partner of the SSW.\footnote{Indeed, we are going to show that the dual partner of the SSW corresponds, in particular, to the lifted black holes, when the functions \( A_M \) depend only on the coordinates of the 3-dimensional space. Therefore we want to stress that the dual partner of the SSW include more general configurations than strings.}
We can make the following particular choice of the vector function $A_M$. First of all these functions will depend only on 3 of the transverse coordinates, $\tilde{x}^1, \tilde{x}^2, \tilde{x}^3$, which will eventually correspond to our 3-dimensional space. Secondly, we choose one of $A_i$ e. g. $A_4$ to be related to $A_u$.

$$A_u = -\frac{\mu}{\rho}, \quad A_4 = \xi A_u, \quad A_1 = A_2 = A_3 = A_5 = \ldots = A_8 = 0 \quad (5)$$

where $\rho^2 = \sum_{i=1}^{3} \tilde{x}^i \tilde{x}^i \equiv \tilde{\rho}^2$ and $\mu$ is a constant. We will specify the constant $\xi$ later. Note that eqs. (2) are solved outside $\rho = 0$. We get

$$ds^2 = 2\epsilon^{2\phi} \{d\tilde{u}d\tilde{v} + \xi(1 - e^{-2\phi})d\tilde{x}^4d\tilde{u}\} - \sum_{i=1}^{8} d\tilde{x}^i d\tilde{x}^i,$$

$$B = -2\epsilon^{2\phi}(1 - e^{-2\phi}) \{d\tilde{u} \wedge d\tilde{v} + \xi d\tilde{u} \wedge d\tilde{x}^4\},$$

$$e^{-2\phi} = 1 + \frac{\mu}{\rho}. \quad (6)$$

We perform the coordinate change

$$\tilde{x} = \tilde{x}^4 + \xi \tilde{u}, \quad \tilde{v} = \tilde{v} + \xi \tilde{x}^4. \quad (7)$$

We also shift $B$ on a constant value, since equations of motion depend on $H = dB$ only.

The dual wave solution (6) takes the form

$$ds^2 = 2\epsilon^{2\phi} d\tilde{u} d\tilde{v} + \xi^2 d\tilde{u}^2 - d\tilde{x}^2 - \sum_{i=1}^{3} d\tilde{x}^i d\tilde{x}^i - \sum_{i=5}^{8} d\tilde{x}^i d\tilde{x}^i,$$

$$B = -2\epsilon^{2\phi} d\tilde{u} \wedge d\tilde{v},$$

$$e^{-2\phi} = 1 + \frac{\mu}{\rho}. \quad (8)$$

When $\xi^2 = -1$ we have

$$ds^2 = 2\epsilon^{2\phi} d\tilde{u} d\tilde{v} - d\tilde{u}^2 - d\tilde{x}^2 - \sum_{i=1}^{3} d\tilde{x}^i d\tilde{x}^i - \sum_{i=5}^{8} d\tilde{x}^i d\tilde{x}^i,$$

$$B = -2\epsilon^{2\phi} d\tilde{u} \wedge d\tilde{v},$$

$$e^{-2\phi} = 1 + \frac{\mu}{\rho}. \quad (9)$$

---

5In order to solve the equations (2) everywhere, it is understood that a source term at $\rho = 0$, representing an unknown object, perhaps a six-brane, has to be added to these equations. We hope that this point can be worked out in an analogy with the combined action for the macroscopic fundamental string, where the source term comes from the sigma-model action, see eqs. (3.1) - (3.3) in [2].
We can identify this particular dual partner of the SSW solution with the uplifted dilaton black hole if we make the following identification of coordinates

\[
\begin{align*}
t &= \tilde{t} = \tilde{v} + \xi \tilde{x}^4, \\
x^4 &= \tilde{u}, \\
x^9 &= \tilde{x} = \tilde{x}^4 + \xi \tilde{u}, \\
x^{1,2,3,\ldots,8} &= \tilde{x}^{1,2,3,\ldots,8}.
\end{align*}
\]

Our dual wave becomes

\[
\begin{align*}
ds^2 &= 2e^{2\phi} dt dx^4 - \sum_{i=1}^{9} dx^i dx^i - d\tilde{x}^2, \\
B &= -2e^{2\phi} dx^4 \wedge dt, \\
e^{-2\phi} &= 1 + \frac{\mu}{\rho}.
\end{align*}
\]

This is an extreme electrically charged 4-dimensional black hole, which is embedded into 10-dimensional geometry in stringy frame, as we are going to explain in the next section.

3. The embedding of the 4-dimensional bosonic solutions of the effective superstring action into 10-dimensional geometry is not unique, in general. There are different ways to identify the vector field of the charged black hole in 4 with the non-diagonal component of the metric in the extra dimensions as well as with the 2-form field. Also the identification of the 4-dimensional dilaton with the fundamental 10-dimensional dilaton and/or with some components of the metric in the extra dimension is possible.

However the identification of the 4-dimensional solution with the 10-dimensional one becomes unique under the conditions that the supersymmetric embedding for both solutions is identified. Dimensional reduction of \(N = 1\) supergravity down to \(d = 4\) has been studied by Chamseddine [8] in canonical geometry. We are working in stringy metric and also in slightly different notation. In a subsequent publication we will present a detailed derivation of the compactification of the bosonic part of the effective action of the 10-dimensional string theory which is consistent with supersymmetry [9]. Here we are interested in the relation between the extreme dilaton charged black holes, which have unbroken supersymmetry [11] when imbedded into \(d = 4, N = 4\) supergravity\(^7\) and the corresponding 10-dimensional supersymmetric configuration.

---

\(^6\)We are grateful to E. Witten for attracting our attention to this problem.

\(^7\)We do not know at present, whether the embedding of these black holes into other theories, including the Abelian part of Yang-Mills multiplet, will also correspond to some unbroken supersymmetries.
bosonic part of the action is

\[ S = \frac{1}{2} \int d^{10}x e^{-2\phi} \sqrt{-g} \left[ -R + 4(\partial \phi)^2 - \frac{3}{4} H^2 \right], \] (12)

where the 10-dimensional fields are the metric, the axion and the dilaton.

We want to make connection with the bosonic part of \( N = 4, d = 4 \) action. In this particular case we are interested in compactifying 6 space-like coordinates. All fields are assumed to be independent of six compactified dimensions. According to Chamseddine [8] dimensional reduction of \( N = 1, d = 10 \) supergravity to \( d = 4 \) gives \( N = 4 \) supergravity coupled to 6 matter multiplets. We are interested here only in dimensional reduction to \( N = 4 \) supergravity without matter multiplets.

Let us first reduce from \( d = 10 \) to \( d = 5 \) by trivial dimensional reduction, when we do not keep the non-diagonal components of the metric and 2-form field. We denote the 10-dimensional fields by un upper index \( ^{(10)} \) and the 5-dimensional fields by a hat. The 10-dimensional indices are capital letters \( M, N = 0, \ldots, 9 \), the 5-dimensional indices will carry a hat \( \hat{\mu}, \hat{\nu} = 0, \ldots, 4 \), and the compactified dimensions will be denoted by capital \( I \)'s and \( J \)'s, \( I, J = 5, \ldots, 9 \). We take the \( d = 10 \) fields to be related to the \( d = 5 \) ones by

\[
\begin{align*}
\hat{g}_{\hat{\mu}\hat{\nu}}^{(10)} &= \hat{g}_{\hat{\nu}\hat{\mu}} \\
\hat{g}_{\hat{I}\hat{J}}^{(10)} &= 0 \\
g_{\mu\nu}^{(10)} &= \eta_{\mu\nu} = -\delta_{\mu\nu} \\
B_{\mu\nu}^{(10)} &= \hat{B}_{\hat{\mu}\hat{\nu}} \\
B_{IJ}^{(10)} &= 0 \\
\phi^{(10)} &= \hat{\phi}.
\end{align*}
\] (13)

We get

\[ S = \frac{1}{2} \int d^{5}x e^{-2\hat{\phi}} \sqrt{-\hat{g}} \left[ -\hat{R} + 4(\partial \hat{\phi})^2 - \frac{3}{4} \hat{H}^2 \right]. \] (14)

As a second step we reduce from \( d = 5 \) to \( d = 4 \), keeping the non-diagonal components of the metric and 2-form field. Since we are interested also in supersymmetry, we will work with the 5-beins at this stage. The 4-dimensional indices do not carry a hat. We parametrize the 5-beins as follows

\[
\begin{align*}
(\hat{e}_{\hat{\mu}}^a) &= \begin{pmatrix} e_{\mu}^a & A_{\mu} \\ 0 & 1 \end{pmatrix}, & (\hat{e}_{\hat{a}}^\mu) &= \begin{pmatrix} e_\mu^a & -A_a \\ 0 & 1 \end{pmatrix},
\end{align*}
\] (15)
where $A_a = e^a \mu A_\mu$. With this parametrization, the 5-dimensional fields decompose as follows

\[
\begin{align*}
\hat{g}_{44} &= \hat{g}_{44} = -1, \\
\hat{g}_{4\mu} &= -A_\mu, \\
\hat{g}_{\mu\nu} &= g_{\mu\nu} - A_\mu A_\nu, \\
\hat{B}_{4\mu} &= B_\mu, \\
\hat{B}_{\mu\nu} &= B_{\mu\nu} + A_{[\mu} B_{\nu]}, \\
\hat{\phi} &= \phi,
\end{align*}
\]

where $\{g_{\mu\nu}, B_{\mu\nu}, \phi, A_\mu, B_\mu\}$ are the 4-dimensional fields.

The 4-dimensional action for the 4-dimensional fields becomes

\[
S = \frac{1}{2} \int d^4 x e^{-2\phi} \sqrt{-g} \left[ -R + 4(\partial \phi)^2 - \frac{3}{4} H^2 + \frac{1}{4} F^2(A) + \frac{1}{4} F^2(B) \right],
\]

where

\[
\begin{align*}
F_{\mu\nu}(A) &= 2\partial_{[\mu} A_{\nu]}, \\
F_{\mu\nu}(B) &= 2\partial_{[\mu} B_{\nu]}, \\
H_{\mu\nu\rho} &= \partial_{[\mu} B_{\nu\rho]} + \frac{1}{2} \{ A_{[\mu} F_{\nu\rho]}(B) + B_{[\mu} F_{\nu\rho]}(A) \}.
\end{align*}
\]

Now, we study the dimensional reduction of gravitino. We are specifically interested in the supersymmetry transformation rule of gravitino in $d = 4$ supergravity without matter. This leads to identification of the matter vector fields $D_\mu$ and the supergravity vector fields $V_\mu$.

\[
\begin{align*}
D_\mu &= \frac{1}{2} (A_\mu - B_\mu), \\
V_\mu &= \frac{1}{2} (A_\mu + B_\mu),
\end{align*}
\]

respectively. Now we want to truncate the theory keeping only the supergravity vector field $V_\mu$. We have then

\[
V_\mu = A_\mu = B_\mu, \quad D_\mu = 0.
\]

The truncated action is\(^8\)

\[
S = \frac{1}{2} \int d^4 x e^{-2\phi} \sqrt{-g} \left[ -R + 4(\partial \phi)^2 - \frac{3}{4} H^2 + \frac{1}{2} F^2(V) \right],
\]

\(^8\)This action, which came from the 10-dimensional theory is slightly different from the corresponding 4-dimensional action in our previous papers, e.g. in [11], due to the difference in notation. The detailed explanation of this difference will be given in [9].
where
\[
F_{\mu\nu}(V) = 2\partial_{[\mu}V_{\nu]} , \\
H_{\mu\nu\rho} = \partial_{[\mu}B_{\nu\rho]} + V_{[\mu}F_{\nu\rho]}(V) .
\] (22)

The embedding of the 4-dimensional fields in this action in \( d = 10 \) is the following:
\[
g^{(10)}_{\mu\nu} = 2_\mu - V_\mu V_\nu , \\
g^{(10)}_{44} = -V_\nu , \\
g^{(10)}_{44} = -1 , \\
\eta_{IJ} = -\delta_{IJ} , \\
B^{(10)}_{\mu\nu} = B_{\mu\nu} , \\
B^{(10)}_{44} = V_\nu , \\
\phi^{(10)} = \phi .
\] (23)

This formulae can be used to uplift any \( U(1) \) 4-dimensional field configurations, including dilaton and axion, to a 10-dimensional field configurations in a way consistent with supersymmetry.

The conclusion of this supersymmetric dimensional reduction is the following.

i) The dilaton of the supersymmetric 4-dimensional extreme black holes is identified as a fundamental dilaton of string theory (and not one of the modulus fields).

ii) Dimensional reduction of \( d = 10 \) supergravity to \( d = 4 \) gives \( N = 4 \) supergravity without 6 matter multiplets under condition that \( g_{4\mu} = -B_{4\mu} \). Therefore the vector field of the 4-dimensional configuration is actually a non-diagonal component of the metric in the extra dimension as well as the 2-form field. This works in our case since we have according to (11)
\[
g^{(10)}_{44} = -B^{(10)}_{44} = -V_i = e^{2\phi} .
\] (24)

4. We will use the formulae from the section above to uplift the dilaton black hole with one vector field. The electrically charged extreme 4d black hole is given by [10] 9.
\[
ds^2_{str} = e^{4\phi} dt^2 - d\vec{x}^2 , \\
V = -e^{2\phi} dt , \\
B = 0 , \\
e^{-2\phi} = 1 + \frac{2M}{\rho} .
\] (25)

9There is a difference of a \( 1/\sqrt{2} \) factor in the vector field with respect to the one given in [11].
The uplifted configuration, according to eq. (23) is:

\[
\begin{align*}
\frac{ds^2}{2} &= 2e^{2\phi} dt dx^4 - d\tilde{x}^2 - (dx^4)^2 - dx^I dx^I, \\
B^{(10)} &= B_{MN} dx^M \wedge dx^N = -2e^{2\phi} dx^4 \wedge dt, \\
\phi^{(10)} &= \phi.
\end{align*}
\]

(26)

Let us choose the parameter \( \mu \) in the dual partner to the wave, given in eq. (9) equal to the double mass of the black hole.

\[
\mu = 2M.
\]

(27)

This makes the uplifted black hole (26) identical to the dual partner to the wave, given in eq. (9).

For better understanding of black-hole-wave relation it is useful to do the following. By adding and subtracting from the metric the term \( e^{4\phi} dt^2 \) we can rewrite the dual wave in \( d = 10 \), given in eq. (11) as follows.

\[
\begin{align*}
\frac{ds^2}{2} &= e^{4\phi} dt^2 - d\tilde{x}^2 - (dx^4 - e^{2\phi} dt)^2 - dx^I dx^I, \\
B &= -2e^{2\phi} dx^4 \wedge dt, \\
e^{-2\phi} &= 1 + \frac{2M}{\rho}.
\end{align*}
\]

(28)

Now it is easy to recognize in the first 2 terms in the metric the 4-dimensional metric and in the third term the non-diagonal component of the 10-dimensional metric which together with the non-diagonal component of the 2-form plays the role of the vector field in the 4-dimensional geometry.

5. The case \( \xi^2 = -1 \) which gives the 4-dimensional black hole in Minkowski space with the signature (1, 3) times the compact 6-dimensional space with the signature (0, 6) corresponds to a complex 10-dimensional wave in the space with the signature (1, 9).

\[
\begin{align*}
\frac{ds^2}{2} &= 2d\tilde{u}d\tilde{v} - \frac{4M}{\rho} d\tilde{u} (d\tilde{u} - id\tilde{x}^4) - \sum_{i=1}^{8} d\tilde{x}^i d\tilde{x}^i, \\
B &= -i \frac{4M}{\rho} d\tilde{u} \wedge d\tilde{x}^4.
\end{align*}
\]

(29)

By performing a rotation \( i\tilde{x}^4 = \tilde{\tau} \) one can get

\[
\begin{align*}
\frac{ds^2}{2} &= 2d\tilde{u}d\tilde{v} - \frac{4M}{\rho} d\tilde{u} (d\tilde{u} - d\tilde{\tau}) + d\tilde{\tau}^2 - \sum_{i=1}^{7} d\tilde{x}^i d\tilde{x}^i, \\
B &= - \frac{4M}{\rho} d\tilde{u} \wedge d\tilde{\tau}.
\end{align*}
\]

(30)
This makes the wave real but with the signature of the space (2, 8).

Thus we may conclude that string theory considers as dual partners the extreme 4d electrically charged dilaton black hole embedded into 10-dimensional geometry, as given in eq. (28) or (26), and Brinkmann-type 10-dimensional wave (29), (30).

If we choose $\xi^2 = 1$ case we get the stringy equivalence between Brinkmann-type 10-dimensional wave

$$
\begin{align*}
\frac{ds^2}{\rho} &= 2d\bar{u}d\bar{v} - \frac{4M}{\rho} d\bar{u}(d\bar{u} - d\bar{x}^4) - \sum_{i=1}^{8} d\bar{x}^i d\bar{x}^i , \\
B &= -\frac{4M}{\rho} d\bar{u} \wedge d\bar{x}^4 ,
\end{align*}
$$

and lifted Euclidean 4-dimensional electrically charged dilaton black hole with the signature (0,4) and the 6-dimensional space has the signature (1, 5),

$$
\begin{align*}
\frac{ds^2}{\rho} &= -e^{4\phi} dt^2 - d\bar{x}^2 + (dx^4 + e^{-2\phi} dt)^2 - dx^I dx^I , \\
B &= -2e^{-2\phi} dx^4 \wedge dt , \\
e^{-2\phi} &= 1 + \frac{2M}{\rho} .
\end{align*}
$$

With such choice of the signature the gravitational wave does not have imaginary components. However, the fact that the metric as well as the 2-form field of the gravitational wave in $d = 10$ have an imaginary component to be dual to the lifted black hole in Minkowski space is strange. Note that this is necessary only if one insists that the $d = 10$ space as well as the $d = 4$ space are both Minkowski spaces. One can avoid imaginary components by allowing the changes in the signature of the space-time when performing duality and dimensional reduction as explained above. Still this remains a puzzle.

Could we actually consider the dual relation between waves and lifted black holes as something more than pure algebraic curiosity? We believe that the answer to this question is “yes”. The dual relation displayed above was established at the zero slope limit of the effective action of the superstring theory. The issue of $\alpha'$-corrections in string theory has been studied extensively for the waves [5], [4]. The pp-waves have the best known properties of absence of such quantum corrections [12]. The SSW are known to have special property of the absence of $\alpha'$-corrections under the condition that the non-abelian Yang-Mills fields is added to the configuration which at the zero slope limit $\alpha' = 0$ consists only of the metric and 2-form [5]. It was explained in [4] that the importance of sigma-model duality between supersymmetric configurations is in the fact that the structure of $\alpha'$ corrections is under control for the dual solution if it was under control for the original solution. In this way we have found that the nice properties of the pp-waves[12] are carried over to the fundamental
string solutions. The present investigation shows that the electrically charged extreme black hole embedded into 10-dimensional geometry may require to be supplemented by some non-abelian Yang-Mills configuration to avoid the possible $\alpha'$-corrections. In this respect we would like to stress that the study of the properties of quantum corrections established via duality may become a powerful mechanism of the investigation of quantum theory despite the strange imaginary factors in the waves, which are dual partners of the uplifted black holes.

At the very minimal level one can consider the method developed above, which consists of stringy duality combined with Kaluza-Klein dimensional reduction, as the solution generating method. This method has the advantage of generating new supersymmetric solutions from the original ones. If we did not know that extreme 4-dimensional black holes are supersymmetric, we would discover this via the supersymmetric properties of 10-dimensional gravitational waves. We hope to derive more general 4-dimensional supersymmetric solutions starting from our 10-dimensional supersymmetric waves and to explore generic relation between supersymmetry and duality [9].

We are grateful to G. W. Gibbons, G. T. Horowitz, J. H. Schwarz and A. Tseytlin for the most fruitful discussions. We are extremely grateful to the organizers of the programme “Geometry and Gravity” at the Newton Institute, Cambridge, which has allowed us to come and work together. One of us (T.O.) would like to thank Groningen University for the hospitality. The work of E.B. and R.K. was partially supported by a NATO Collaborative Research Grant. The work of E.B. has been made possible by a fellowship of the Royal Netherlands Academy of Arts and Sciences (KNAW). The work of R. K. was supported by NSF grant PHY-8612280 and the work of T. O. was supported by European Communities Human Capital and Mobility programme grant.

References


