Predictions from Quantum Cosmology

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The world view suggested by quantum cosmology is that eternally inflating universes with all possible values of the fundamental constants are spontaneously created out of nothing. I explore the consequences of the assumption that we are a “typical” civilization living in this metauniverse. The conclusions include inflation with an extremely flat potential and low thermalization temperature, structure formation by topological defects, and an appreciable cosmological constant.

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Why do the fundamental constants of Nature take the particular values that they are observed to have in our universe? It certainly appears that the constants have not been selected at random. Assuming that the particle masses are bounded by the Planck mass $m_p$ and the coupling constants are $\lesssim 1$, one expects that a random selection would give all masses $\sim m_p$ and all couplings $\sim 1$. The cosmological constant would then be $\Lambda \sim m_p^2$ and the corresponding vacuum energy $\rho_v \sim m_p^4$. In contrast, some of the particle masses are more than 20 orders of magnitude below $m_p$, and the actual value of $\rho_v$ is $\sim 10^{-120} m_p^4$.

(I use the system of units in which $\hbar = c = 1$.)

It has been argued [1] that the values of the fundamental constants are, to a large degree, determined by anthropic considerations: these values should be consistent with the existence of conscious observers who can wonder about them [2]. If one assumes that the production of heavy elements in stars and their dispersion in supernova explosions are essential for the evolution of life, then one finds that this Anthropic Principle imposes surprisingly stringent constraints on the electron, proton and neutron masses ($m_e$, $m_p$, and $m_n$), the $W$-boson mass $m_W$, and the fine structure constant $\alpha$. An anthropic bound on the cosmological constant can be obtained by requiring that gravitationally bound systems are formed before the universe is dominated by the vacuum energy.

I should also mention the popular view that there exists a unique logically consistent Theory of Everything and that all constants can in principle be determined from that theory. The problem, however, is that the constants we observe depend not only on the fundamental Lagrangian, but also on the vacuum state, which is likely not to be unique. For example, in higher-dimensional theories, like superstring theory, the constants in the four-dimensional world depend on the way in which the extra dimensions are compactified. Moreover, Coleman has argued [3] that all constants appearing in sub-Planckian physics become totally undetermined due to Planck-scale wormholes connecting distant regions of spacetime.

Finally, it has been suggested that the explanation for the values of some constants can be found in quantum cosmology. The wave function of the universe gives a probability distribution for the constants which can be peaked at some particular values [4]. Wormhole effects can also contribute an important factor to the probability [5]. Smolin [6] has argued that new expanding regions of the universe may be created as a result of gravitational collapse due to quantum gravity effects. Assuming that the constants in these “daughter” regions deviate slightly from their values in the “mother” region, he conjectured that the observed values of the constants are determined by “natural selection” for the values that maximize the production of black holes. Some problems with this conjecture have been pointed out in Ref. [7].

In this paper I would like to suggest a different approach to determining the fundamental constants. This approach is not entirely new and has elements of both anthropic principle and quantum cosmology. However, to my knowledge, it has not been clearly formulated and its implications have not been systematically explored. My approach is based on the picture of the universe suggested by quantum cosmology and by the inflationary scenario. In this picture, small closed universes spontaneously nucleate out of nothing,
where “nothing” refers to the absence of not only matter, but also of space and time [8]. All universes in this metauniverse are disconnected from one another and generally have different values of the fundamental constants.

After nucleation, the universes enter a state of eternal inflation [9,10]. The inflationary expansion is driven by the potential energy of a scalar field \( \varphi \), while the field slowly “rolls down” its potential \( V(\varphi) \). When \( \varphi \) reaches the minimum of the potential, its energy thermalizes, and inflation is followed by the usual radiation - dominated expansion. The evolution of \( \varphi \) is influenced by quantum fluctuations, and as a result thermalization does not occur simultaneously in different parts of the universe. In many models it can be shown that at any time there are parts of the universe that are still inflating. Thus, (almost) all universes in the metauniverse have a beginning, but have no end.

We are one of the infinite number of civilizations living in thermalized regions of the metauniverse. Although it may be tempting to believe that our civilization is very special, the history of cosmology demonstrates that the assumption of being average is often a fruitful hypothesis. I call this assumption the Principle of Mediocrity. We shall see that, compared to the traditional point of view, this principle gives a rather different perspective on what is natural and what is not.

The Principle of Mediocrity suggests that we think of ourselves as a civilization randomly picked in the metauniverse. Denoting the fundamental constants by \( a_i \), let us consider the product

\[
P(a; \Delta a) = Z^{-1} \mathcal{P}(a_i) \mathcal{N}(a_i) \prod_i \Delta a_i.
\]

Here, \( \mathcal{P}(a_i) \prod da_i \) is the probability of nucleation for an inflating universe with a given set of \( a_i \) in the intervals \( da_i \), \( \mathcal{N}(a) \) is the number of civilizations in such a universe [11], \( Z \) is a normalization constant,

\[
Z = \int \mathcal{P}(a_i) \mathcal{N}(a_i) \prod_i da_i,
\]

and \( \Delta a_i \) is the allowed range of \( a_i \). (For example, if we want to explain the value of \( a_i \) by order of magnitude, we should set \( \Delta a_i \sim a_i \).) The quantity \( \mathcal{P}(a; \Delta a) \) will be interpreted as an \textit{a priori} probability for \( a_i \) to take values in the prescribed intervals \( \Delta a_i \).

In an eternally inflating universe, the number \( \mathcal{N}(a) \) is, of course, infinite and has to be regulated. The simplest way to do this is to turn inflation off at some time \( t = \tau \) after nucleation and consider the asymptotic behavior at \( \tau \to \infty \). The time variable \( t \) can be defined as the proper time on the congruence of geodesics orthogonal to the initial hypersurface at the moment of nucleation [12]. Since geodesics tend to diverge during inflation, one can expect that the proper time gauge is well defined and that, with an appropriate coarse-graining, it can be continued well into the thermalized region. (We only need to continue it for as long as there are any surviving civilizations, e.g., until the stars die out or until protons decay.)

Very roughly, we can write
\[ N(\alpha) \sim \Omega(\alpha) \nu_{\text{civ}}(\alpha), \]

where \( \Omega(\alpha) \) is the spacetime volume in which conditions are suitable for life and \( \nu_{\text{civ}}(\alpha) \) is the average number of civilizations originating per unit spacetime volume. Even if we knew \( \nu_{\text{civ}}(\alpha) \) and the conditions that define \( \Omega(\alpha) \), a calculation of \( N(\alpha) \) would be a non-trivial problem. It could be approached using the stochastic methods developed in Refs. [9,13,14]. Without attempting such a calculation in this paper, I will explore the consequences of a simple observation that \( N(\alpha) \) grows if we increase the available volume where life can originate.

The inflating part of the universe can be divided into a quantum region, where the dynamics of the inflaton field \( \varphi \) is dominated by quantum fluctuations, and slow rollover region, where the evolution is essentially deterministic. The volume of the quantum region grows as \( \exp(dH_st) \), where \( H_s \) is the highest rate of expansion (I assume that \( H_s < \infty \), see below), \( d < 3 \), and the actual value of \( d \) depends on the downward slope of the potential \( V(\varphi) \) [15]. When the slope is decreased, \( d \) grows and approaches \( d_{\text{max}} = 3 \) in the limit of an absolutely flat potential, \( V = \text{const} \). The expansion factor during the slow rollover regime is also maximized by making the potential maximally flat: the field \( \varphi \) takes longer to roll down to \( V = 0 \) for a flatter potential. From this we conclude that the values of the fundamental constants should be such as to maximize \( H_s \) and to minimize the slope of \( V(\varphi) \) [16].

In Einstein’s gravity, the inflationary expansion rate is given by

\[ H^2 = \frac{8\pi}{3} \frac{V(\varphi)}{m_p^2} \]

and can be arbitrarily high if \( V(\varphi) \) is unbounded from above. However, when \( V(\varphi) \) reaches Planckian values, Einstein’s equations are likely to get modified, and Eq. (3) may no longer be valid. One expects this to happen in higher-dimensional and dilatonic gravity theories. It can be shown [17] that a more conservative approach based on semiclassical quantum gravity [18] gives an upper bound on the rate of inflation, \( H < H_{\text{max}} \sim m_p \). Here, we shall assume that, for one reason or another, the expansion rate is bounded by some \( H_{\text{max}} \) [19]. Then we expect \( H_s = H_{\text{max}} \).

The cosmological literature abounds with remarks on the “unnaturally” flat potentials required by inflationary scenarios. The slope of the potential is severely constrained by the isotropy of the cosmic microwave background and by the corresponding bounds on the amplitude of density fluctuations generated during inflation. With the Principle of Mediocrity, the situation is reversed: flat is natural! Instead of asking why \( V(\varphi) \) is so flat, one should now ask why it is not flatter.

Let us now consider the role of other factors in \( P(\alpha; \Delta \alpha) \). The calculation of \( P_{\text{nucl}}(\alpha) \) is a matter of some controversy. The result depends on one’s choice of boundary conditions for the wave function of the universe (see, e.g., [8,20]). Here we shall adopt the tunneling boundary condition. Then the semiclassical nucleation probability is proportional to \( \exp(-|S|) \), where \( S \) is the Euclidean action of the corresponding instanton. In Einstein’s gravity, \( |S| = \pi m_p^2/H_s^2 \), where \( H_s \) is the highest inflation rate allowed by the potential
$V(\varphi)$ ($H_*$ is given by Eq. (3) with the maximal value of $V$). A higher probability is obtained for greater values of $H_*$. Hence, $\mathcal{P}_{\text{nucl}}(\alpha)$ works in the same direction as the volume factor in $\mathcal{N}(\alpha)$: it tends to maximize the expansion rate, pushing it towards $H_{\text{max}}$. With $H_* = H_{\text{max}} \sim m_p$, $|S| \sim 1$, and the exponential factor in $\mathcal{P}_{\text{nucl}}(\alpha)$ is $\sim 1$. In this regime the dependence of $\mathcal{P}_{\text{nucl}}$ on $\alpha$ is only through the pre-factor and is not expected to be very sensitive.

An important role in constraining the values of $\alpha_i$ is played by the “human factor”, $\nu_{\text{ck}}(\alpha)$. We do not know what other forms of life are possible, but the Principle of Mediocrity favors the hypothesis that our form is the most common in the metauniverse. The conditions required for life of our type to exist (the low-energy physics based on the symmetry group $SU(3) \times SU(2) \times U(1)$, the existence of stars and planets, supernova explosions) may then fix, by order of magnitude, the values of $\epsilon^2$, $m_e$, $m_{\text{pr}}$ and $m_W$, as discussed in Ref. [1]. Anthropic considerations also impose a bound on the allowed flatness of the inflaton potential $V(\varphi)$. If the potential is too flat, then the thermalization temperature after inflation is too low for baryogenesis. The lowest temperature at which baryogenesis can still occur is set by the electroweak scale, $T_{\text{min}} \sim m_W$. Hence, we expect the universe to thermalize at $T \sim m_W$. (Specific constraints on $V(\varphi)$ depend on the couplings of $\varphi$ to other fields and can be easily obtained in specific models.) [21]

Super-flat potentials required by the Principle of Mediocrity give rise to density fluctuations which are many orders of magnitude below the strength needed for structure formation. This means that the observed structures must have been seeded by some other mechanism. The only alternative mechanism suggested so far is based on topological defects: strings, global monopoles and textures, which could be formed at a symmetry breaking phase transition [22]. The required symmetry breaking scale for the defects is $\eta \sim 10^{16}\text{GeV}$. With “natural” (in the traditional sense) values of the couplings, the transition temperature is $T_c \sim \eta$, which is much higher than the thermalization temperature ($T_{\text{th}} \sim m_W$), and no defects are formed after inflation. It is possible for the phase transition to occur during inflation, but the resulting defects are inflated away, unless the transition is sufficiently close to the end of inflation. To arrange this requires some fine-tuning of the parameters and leads to a decrease in the last factor in Eq. (1). However, the alternative is to have thermalization at a much higher temperature and to cut down on the amount of inflation. Since the dependence of the volume factor on the duration of inflation is exponential, we expect that the gain in the volume will more than compensate for the decrease in $\Delta \alpha_i$ due to the fine-tuning. We note also that in some supersymmetric models the critical temperature of superheavy string formation can “naturally” be as low as $m_W$ [23].

The symmetry breaking scale $\eta \sim 10^{16}\text{GeV}$ for the defects is suggested by observations, but we have not explained why this particular scale has been selected. An interesting possibility is suggested by Smolin [6] who argues that the density of matter in galaxies is such that the rate of star formation is maximized. Smolin looked for the highest rate of star formation to maximize the number of black holes, but the same thing may be needed to maximize the number of civilizations. The galactic density is determined by the time when galaxies form, which depends on the amplitude of density fluctuations, which is in
turn determined by the symmetry breaking scale $\eta$. Hence, $\eta$ may be fixed by maximizing the number of stars.

If something like Smolin's condition fixes the amplitude of density fluctuations and the epoch of active galaxy formation, then an upper bound on the cosmological constant can be obtained by requiring that it does not disrupt galaxy formation until the end of that epoch. The growth of density fluctuations in a flat universe with $\Lambda > 0$ effectively stops at a redshift [24] $1 + z \sim (1 - \Omega_\Lambda)^{-1/3}$, where $\Omega_\Lambda = \rho_v/\rho_c$ and $\rho_c$ is the critical density. Requiring that this happens at $z < 1$ gives $\Omega_\Lambda < 0.9$. The actual value of $\Lambda$ is likely to be comparable to this upper bound [25]. Thus, according to the Principle of Mediocrity, the most promising model of structure formation is a flat universe with a non-negligible $\Omega_\Lambda$ and density fluctuations seeded by topological defects.

Some of the parameters that we regard as constants may in fact be variable on sufficiently large scales. Such parameters may be represented by very weakly coupled scalar fields whose dynamics is dominated by quantum fluctuations during inflation. The fields are then approximately constant inside thermalized regions, but can vary from one thermalized region to another. One model of this sort with a Brans-Dicke field was discussed by Garcia-Bellido et. al [14]. Another example is the moduli fields in superstring theories. With a sufficiently large number of variable "constants", conclusions similar to those outlined above can be obtained even without invoking multiple universes. The probability distribution $P(\alpha; \Delta \alpha)$ can then be determined by solving a Smoluchowski equation, as discussed in Refs. [9,13,14].

Finally, it should be emphasized that predictions of the Principle of Mediocrity are not guaranteed to be correct. After all, our civilization may be special in some respects. Moreover, if the number of independent $\alpha_i$ is large, then one can expect significant deviations from the most probable values in some of the $\alpha_i$, even for a randomly chosen civilization. However, the predictions are expected to be statistically correct. That is, with a sufficiently large number of predictions, only few of them are likely to be wrong.

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References


[2.] It should be noted that the Anthropic Principle, as originally formulated by Carter, is more than a trivial consistency condition. It is the requirement that anthropic constraints should be taken into account when evaluating the plausibility of various hypotheses about the physical world.


[5.] S. Coleman, Nucl. Phys. B310, 643 (1988). In this paper Coleman obtained a probability distribution for $\rho_v$ with an extremely sharp peak at $\rho_v = 0$. However, his derivation was based on Euclidean quantum gravity, which has serious problems. For a discussion of the problems, see W. Fischler et. al., Nucl. Phys. B327, 157 (1989).


[11.] We may wish to assign a weight to each civilization, depending on its lifetime and/or on the number of individuals. This would not change the conclusions of the present paper.

[12.] There is no guarantee that other choices of a time variable $t$ will give identical results. In fact, Linde et. al. [14] who studied a related problem of finding the probability distribution for the inflaton and other fields in an eternally inflating universe, have shown that for some choices of $t$ the results can be very different. This issue requires further investigation.


It can be shown [M. Aryal and A. Vilenkin, Phys. Lett. B199, 351 (1987)] that $d$ is the fractal dimension of the region where the expansion rate is $H \approx H_*$. 

By decreasing the slope of $V(\varphi)$ in the quantum regime, we reduce the rate of formation of new thermalized regions per unit volume. However, the loss in the rate is only linear in $(3 - d)$, while the gain in the volume is exponential. Note also that, although the radii of thermalized regions grow at the rate of the inflationary expansion outside, the interior is expanding much slower, and the volume grows slower than $R^3$ (in proper time gauge). As a result, the dominant contribution to the total thermalized volume always comes from newly thermalized regions.

Including the vacuum contributions of matter fields to the expectation value of $T_{\mu\nu}$ and assuming slow rollover conditions, $\dot{\varphi}^2 \ll V(\varphi)$ and $\ddot{H} \ll H^2$, the evolution equation (3) is replaced by

$$H^2 = \frac{8\pi}{3m_p^2} V(\varphi) + \frac{H^4}{H_0^2} - \frac{6}{M^2} H^2 \dot{H}.$$ 

Here, $H_0 \sim m_p/\sqrt{N}$, $N$ is the number of matter fields with masses $m \ll H$, and $M$ can be adjusted to any value by a finite renormalization of the quadratic in curvature term in the gravitational Lagrangian. Physically reasonable models are obtained for $H_0^2 > 0$, $M^2 > 0$ (for details see Ref. [18]). Classical inflationary solutions must have $\dot{H} < 0$. This gives a quadratic inequality for $H^2$, which can be satisfied only if $V(\varphi) \leq 3H_0^2 m_p^2/32\pi$. The expansion rate cannot exceed $H_0$. A detailed discussion of this issue will be given elsewhere.


Linde et. al. [14] have argued that the inflationary expansion rate is bounded by $H_{\text{max}} \sim m_p$, because at this rate quantum fluctuations in the energy-momentum tensor $T_{\varphi}^{\mu\nu}$ of the inflaton field $\varphi$ become comparable to $T_{\varphi}^{\mu\nu}$ itself, and the vacuum form of $T_{\varphi}^{\mu\nu} \propto g^{\mu\nu}$ is destroyed. I disagree with this argument. At $H \sim m_p$, quantum fluctuations in $T^{\mu\nu}$ for all fields with $m \ll m_p$ have comparable magnitude. The average total energy-momentum tensor is $\langle T^{\mu\nu} \rangle \propto g^{\mu\nu}$, and its relative fluctuation is $\sim N^{-1/2}$, where $N$ is the number of fields with $m \ll m_p$. Since $N \gtrsim 100$, the vacuum form of $T^{\mu\nu}$ holds with a good accuracy.


I assume that the number of independent $\alpha_i$ is sufficiently large, so that fixing the constants of the low-energy physics does not interfere with the adjustment of the inflaton potential $V(\varphi)$.

For a review of topological defects, see A. Vilenkin and E.P.S. Shellard, “Cosmic Strings and Other Topological Defects” (Cambridge University Press, Cambridge, 1994).
Here I assume that \( p_{\text{nuc}}(\alpha) \) is a smooth function of \( \Lambda \) in the range of interest, and in particular that it does not have a Coleman-type [5] peak at \( \Lambda = 0 \). Negative values of \( \Lambda \) are bounded by requiring that our part of the universe does not recollapse while the stars are still shining, new civilizations are being formed, and the old ones are not yet extinct. This probably gives a bound comparable to that for positive \( \Lambda \) (by absolute value).