The Role of Vector Torsion in Conformally Induced Gravity  

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We have investigated the role of vector torsion in conformally induced gravity. It is found that the vector torsion background exactly cancels the contribution of the non-conformal coupling of the dilaton field in the equation of motion. Especially, in Robertson-Walker metric the effective potential of the conformally induced gravity with vector torsion background could not be an arbitrary form, but should be the quartic form in the dilaton field to give a consistent equation of motion. This constraint is originated from choosing the Robertson-Walker metric based on the assumption of homogeneity of the dilaton field.

Consequently, the vector torsion field should be purely quantum fluctuation and be completely integrated out in the path integral to give a consistent classical equation of motion and a successful symmetry breaking to the conformally induced gravity in Robertson-Walker metric. The quantum fluctuation of vector torsion can be important than that of metric if \(\sqrt{(1 + 6\xi)} \ll 1\) is satisfied.

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I. INTRODUCTION

It is considerable that gravity is characterized by a dimensionless coupling constant \(\xi\) and that the gravitational constant \(G_N\) is given by the inverse square of the vacuum expectation value of the dilaton field \([1,2]\). The weakness of gravity can be associated with spontaneous symmetry breaking at very high energy scale. The induced gravity action in Riemann space is given by

\[
S_{\text{eff}}(g, \phi) = \int d^4 x \sqrt{g} \left\{ -\frac{1}{2} \xi \phi^2 R(g) + \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V_{\text{eff}}(\phi) \right\}.
\]  (1)

The effective potential \(V_{\text{eff}}(\phi)\) for the dilaton field may be attributed to quantum fluctuations of the conformal factor or gauge fields coupled to the dilaton field \([3-6]\). In any case, it is assumed that the effective potential attains its minimum value when \(\phi = \sigma\), then the above induced gravity action is reduced to the well known Einstein-Hilbert action with the gravitational constant

\[
G_N = \frac{1}{8\pi\xi\sigma^2}.
\]  (2)

This symmetry breaking in induced gravity might be applied to the inflationary models \([7-9]\).

On the analogy of the \(SU(2) \times U(1)\) symmetry of the weak interactions, we can consider a continuous symmetry which is broken through a spontaneous symmetry breaking in the gravitational interactions. The most attractive symmetry is the conformal symmetry which rejects the Einstein-Hilbert action, but admits the induced gravity action (1) with the specific coupling \(\xi = -\frac{1}{3}\) and the bare quartic potential. We can write down a conformally invariant induced gravity action without introducing the torsion field. However, the spontaneous symmetry breaking mechanism does not work for the scalar field theory with \(\xi = -\frac{1}{3}\) in Robertson-Walker metric \([8]\).

The quantum effects of the conformal factor for the dilaton field in conformally non-invariant induced gravity with torsion background has considered in Ref. \([10-13]\). If we extend Riemann space to the minimal Riemann-Cartan space which has almost Riemannian structure with the additional vector torsion like a conformal gauge field \([14]\), the conformally induced gravity action can be written as follows \([15]\):

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\[ S_{\text{eff}}(g, S, \phi) = \int d^4x \sqrt{g} \left\{ -\frac{\xi}{2} R(\{\}) \phi^2 + \frac{1}{2} \partial_\alpha \phi \partial^\alpha \phi - \frac{1}{4} H_{\alpha\beta} H^{\alpha\beta} - \frac{1}{2} (1 + 6 \xi) S^a \partial_\alpha \phi^2 + \frac{1}{2} (1 + 6 \xi) S^a \phi^4 - V_{\text{eff}}(\phi) \right\}, \]

where \( S_\mu = \frac{1}{2} T^\mu_\nu \) is the vector torsion and \( H_{\mu\nu} = \partial_\mu S_\nu - \partial_\nu S_\mu \) is the conformal field strength. This action (3) is conformally invariant for arbitrary \( \xi \) with the bare quartic potential \( \frac{1}{4} \phi^4 \). However, the effective potential \( V_{\text{eff}}(\phi) \) radiatively corrected by the quantum fluctuations of the dilaton field and the vector torsion field breaks the conformal symmetry and gives a non-vanishing vacuum expectation value \( \sigma \) to the dilaton field. In the symmetry broken phase, the torsion vector field gets the mass \( \sqrt{(1 + 6 \xi)} \sigma \).

**II. EQUATIONS OF MOTION FOR CONFORMALLY INDUCED GRAVITY**

In this section, we analyze the equations of motion for the action Eq.(3). By varying the action, we obtain the following three equations:

\[ \phi = -\xi R(\{\}) \phi + (1 + 6 \xi) \phi (S^a S_\mu - \nabla_\mu S^a) - \frac{\partial V_{\text{eff}}(\phi)}{\partial \phi}, \]  

\[ \partial_\mu (\sqrt{g} H^a_\mu) = -(1 + 6 \xi) \sqrt{g} \left\{ \partial_\mu \phi + S_\mu \phi^2 \right\}, \]

\[ \xi \phi^2 G_{\mu\nu} = -(H_{\mu\alpha} H^\alpha_\nu - \frac{1}{4} g_{\mu\nu} H_{\alpha\beta} H^{\alpha\beta}) + (\partial_\mu \phi \partial_\nu \phi - \frac{1}{2} g_{\mu\nu} \partial_\alpha \phi \partial^\alpha \phi) + (1 + 6 \xi) \phi^2 (S_\mu S_\nu - \frac{1}{2} g_{\mu\nu} S^a S^a) + (1 + 6 \xi) (S_\mu \phi \partial_\nu \phi + S_\nu \phi \partial_\mu \phi - g_{\mu\nu} S^a \partial_\alpha \phi) + \xi \left\{ \nabla_\mu (\phi \partial_\nu \phi) + \nabla_\nu (\phi \partial_\mu \phi) - g_{\mu\nu} \phi^2 \right\} + g_{\mu\nu} V_{\text{eff}}(\phi). \]

Taking the divergence of Eq.(5), we obtain

\[ \nabla_\mu (S^a \phi^2) = -\frac{1}{2} \phi^2. \]

The trace of Einstein Eq.(6) is

\[ -\xi R(\{\}) \phi^2 = -\partial_\alpha \phi \partial^\alpha \phi - (1 + 6 \xi) (S^a \phi \partial_\alpha \phi + S_\alpha S^a \phi^2) - 3 \phi^2 + 4 V_{\text{eff}}(\phi). \]

From Eqs.(4) and (8), we have

\[ \phi \partial_\phi + \partial_\alpha \phi \partial^\alpha \phi + (1 + 6 \xi) \nabla_\alpha (S^a \phi^2) + 3 \phi^2 = 4 V_{\text{eff}}(\phi) - \phi \frac{\partial V_{\text{eff}}(\phi)}{\partial \phi}. \]

Using Eq.(7), we have a \( \xi \) independent equation of motion for the dilaton field from Eq.(9) as follows;

\[ \phi \phi + \partial_\alpha \phi \partial^\alpha \phi - \frac{1}{2} \phi^2 = 4 V_{\text{eff}}(\phi) - \phi \frac{\partial V_{\text{eff}}(\phi)}{\partial \phi}. \]

To investigate the symmetry-breaking for this theory, we examine this equation with a constant \( \phi = \sigma \). The Eq.(10) is reduced to

\[ \left( \frac{\partial V_{\text{eff}}}{\partial \phi} - 4 \frac{V_{\text{eff}}(\phi)}{\phi} \right) \bigg|_{\phi=\sigma} = 0. \]

This is the symmetry-breaking equation in the induced gravity, which is different from the usual one

\[ \frac{\partial V_{\text{eff}}}{\partial \phi} \bigg|_{\phi=\sigma} = 0 \]

in the scalar theory with Einstein-Hilbert action.

If we are in the symmetry broken phase with \( \phi = \sigma \) presently, the uniform background energy density \( V_{\text{eff}}(\sigma) \) acts like a cosmological constant in Einstein equation, and the planck mass is
Because the cosmological constant in present is very small, the constant part of \( V_{\text{eff}}(\sigma) \) and the regularization mass can be determine by the requirements,

\[
V_{\text{eff}}(\sigma) = 0 \quad \text{and} \quad \left. \frac{\partial V_{\text{eff}}(\phi)}{\partial \phi} \right|_{\phi=\sigma} = 0 ,
\]

in flat space-time. In this symmetry broken phase, the ratio of vector torsion mass \( M_s \) and planck mass \( M_p \) is

\[
\frac{M_s}{M_p} = \sqrt{\frac{(1 + 6\xi)}{8\pi\xi}} ,
\]

which is independent of the vacuum expectation value \( \sigma \) of the dilaton field. If \( \sqrt{(1 + 6\xi)} \) is small enough, we can consider an approximation such that we treat the metric as a classical background and the vector torsion as a quantum field [15].

We will assume that the dilaton field is spatially homogeneous and isotropic so that the space-time has the Robertson-Walker metric;

\[
ds^2 = dt^2 - R(t)^2 \left( \frac{dr^2}{1 - kr^2} + d\Omega^2 \right) ,
\]

where \( k = -1, 0 \) or \( 1 \) depending on the signature of the curvature. Especially, when \( k = 0 \) and \( R(t) = e^{Ht} \), the space is de Sitter space with the scalar curvature \( R = -12H^2 \). In Robertson-Walker metric, the left hand side of Eq.(10) vanishes and the equation gives the following constraint on the effective potential;

\[
\frac{4}{\phi} V_{\text{eff}}(\phi) = V'_{\text{eff}}(\phi) .
\]

This equation dictates that the effective potential should be quartic in the dilaton field, \( \frac{4}{\phi} \phi^4 \). Therefore, in Robertson-Walker metric the symmetry breaking via radiative correction does not work for the conformally induced gravity coupled with vector torsion background. This constraint comes from the assumption that the dilaton field is spatially homogeneous. Consequently, the vector torsion field should be purely quantum fluctuation and be completely integrated out in the path integral to give a consistent classical equation of motion and a successful symmetry breaking to the conformally induced gravity in Robertson-Walker metric. Actually, it was shown that the complete integration of the vector torsion without background gives a sensible phase transition leading to an inflationary phase through the conformal symmetry breaking [15]. However, if we apply this theory to non-homogeneous dilaton field, the constraint on the effective potential would not appear so that it is possible to consider non-zero torsion background.

### III. CONCLUSION

We have found that torsion background exactly cancels the non-conformal coupling of the dilaton field in the equation of motion. In Robertson-Walker metric this cancellation gives a constraint on the effective potential so that only the quartic form of effective potential is allowed. Therefore, the symmetry breaking with spatially homogeneous dilaton field is impossible in the conformally induced gravity with torsion background.

Consequently, if we want to have the homogeneous symmetry breaking in Robertson-Walker metric, we should treat the vector torsion as a purely quantum fluctuation which is completely integrated out in the path integral not to give the vector torsion background. The effective potential which is obtained by the complete integration of the vector torsion without background gives a phase transition through the conformal symmetry breaking [15]. The application of this effective potential to the phenomenology of the inflationary universe is considerable. One can investigate the role of vector torsion when matter fields are coupled to the conformally induced gravity.

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