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MAN HEP 93_11
SW 9424

manchester
particle physics

MAN/HEP/93/11
20th December 1993

The longitudinal coordinate from 'Double Diamond' tapered cathode pad readout

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Abstract

The 'Double Diamond' cathode pad readout system, used in the OPAL muon detector, provides an elegant way of obtaining two dimensional readout from a single wire, with a precision in the wire direction of 2 *mm* over 10 *m*. In doing this 3 measurements of the longitudinal coordinate are provided, with different precisions and ambiguities. This note describes how the 3 measurements must be combined to give the best value.

Submitted To Nuclear Instruments and Methods

The ‘Double Diamond’ cathode pad readout system is used in the OPAL barrel muon detector [1,2,3]. A single-wire drift chamber, 60 *cm* wide and 10 *m* long, gives a conventional position measurement from drift time with resolution 1-2 *mm* over the 30 *cm* drift distance, and also a longitudinal measurement of similar precision over the 10 *m* length. Two dimensional readout from a single wire in large-area inexpensive chambers, with a resolution of the order of millimetres, makes them well suited to the purpose of muon detection, and such a design would merit consideration by future experiments with similar requirements.

The measurement in the longitudinal direction (denoted by z) is achieved by a 3-stage process. The precision is obtained by signals from sets of tapered cathode pads, however this pattern repeats every 17.1 *cm* down the chamber so the co-ordinate is only defined modulo 17.1 *cm*, and there is a 60-fold ambiguity. A second set of pads of wavelength 171 *cm* provides a 6-fold ambiguous measurement of medium resolution (typically 3 *cm*). The third measurement is provided by a charge-division type method from the signals at the two ends of the wire; this is unambiguous but has a resolution of approximately 30 *cm*. This 3-stage readout: fine, medium, and coarse, leads to a new type of ambiguity problem in hit and track reconstruction. This note describes the best way to combine the 3 measurements.

Figure 1: A hit with the 3 z measurements

The operation of the chambers in determining the z position of a track is sometimes loosely described as using the coarse measurement to decide the correct medium wavelength, the medium measurement to give the fine pad wavelength, and the fine pad measurement then gives the final precision. This is however an oversimplification [4]. This is illustrated in Figure 1. The coarse z measurement indicates some value, taken as zero, with a measurement error of 30 *cm* - this is shown on the plot as a solid line. Only one of the 6 medium z ambiguities is at all relevant to this, and this is shown as a dashed line. 4 of the 60 fine z ambiguities appear in the region, and are shown as dotted lines. The

medium z measurement indicates that the fine z value at $z = 23\text{cm}$ is to be chosen, being more likely than the alternative at $z = 6\text{cm}$. However this preference is very slight, and the information from the coarse z measurement indicates that the $z = 6\text{cm}$ value is more likely than the $z = 23\text{cm}$ one. In such situations the two contradictory indications have to be combined: both coarse and medium measurements must be used to choose the correct fine z ambiguity. The solution adopted in [4] was to consider all 60 possibilities. By contrast, it will be shown that the best result of a z measurement can be found by integer rounding and some simple geometry, without the need for explicit computation of any probabilities.

Suppose that the coarse, medium, and fine z readouts give values z_c, z_m , and z_f . The true z co-ordinate measurements, after resolution of the ambiguities, are given by adding integral numbers of medium and fine wavelengths to the second and third of these. Thus the actual z co-ordinate measurements are

$$Z_c = z_c \quad Z_m = z_m + N_m \lambda_m \quad Z_f = z_f + N_f \lambda_f$$

where $\lambda_m = 171\text{cm}$ and $\lambda_f = 17.1\text{cm}$. The problem is to find the ‘best’ value of the combined measurement Z .

For a given true Z there is some probability density $P(z_c, z_m, z_f; Z)$ of obtaining a particular triplet of measurement values. In accordance with the principle of maximum likelihood, Z can be estimated by finding the value which maximises the probability of the measured triplet. It is assumed that the probability factorises (it is difficult to imagine why this might not be true) and that the 3 individual probability functions are Gaussian. Then

$$P = \frac{1}{\sigma_c \sigma_m \sigma_f (2\pi)^{3/2}} e^{-\frac{1}{2} \left(\frac{Z - Z_c}{\sigma_c} \right)^2} \times e^{-\frac{1}{2} \left(\frac{Z - Z_m}{\sigma_m} \right)^2} \times e^{-\frac{1}{2} \left(\frac{Z - Z_f}{\sigma_f} \right)^2}$$

is the function to be maximised. This amounts to minimising the χ^2 :

$$\chi^2 = \left(\frac{Z - Z_c}{\sigma_c} \right)^2 + \left(\frac{Z - Z_m}{\sigma_m} \right)^2 + \left(\frac{Z - Z_f}{\sigma_f} \right)^2$$

For given N_m, N_f , it follows that the best Z is given by

$$Z \left(\frac{1}{\sigma_c^2} + \frac{1}{\sigma_m^2} + \frac{1}{\sigma_f^2} \right) = \frac{Z_c}{\sigma_c^2} + \frac{Z_m}{\sigma_m^2} + \frac{Z_f}{\sigma_f^2}$$

However, as $\sigma_c \approx 10\sigma_m$ and $\sigma_m \approx 10\sigma_f$ one can safely ignore the first two terms and write

$$Z = Z_f = z_f + N_f \lambda_f$$

i.e. the coarse and medium z information are indeed to be used only in determining the ambiguity. The problem is then to determine N_f (and N_m). With the third term of the χ^2 having been set to zero, it becomes

$$\chi^2 = \left(\frac{z_f + N_f \lambda_f - z_c}{\sigma_c} \right)^2 + \left(\frac{z_f + N_f \lambda_f - z_m - N_m \lambda_m}{\sigma_m} \right)^2$$

Using $\lambda_m = 10\lambda_f$, and introducing n_f :

$$N_f = 10N_m + n_f$$

then

$$\chi^2 = \left(\frac{z_f - z_c + N_f \lambda_f}{\sigma_c} \right)^2 + \left(\frac{z_f - z_m + n_f \lambda_f}{\sigma_m} \right)^2$$

and the problem is to choose integer values of N_m and n_f such that this is minimised.

Writing

$$x = \frac{z_m - z_f}{\lambda_f} \quad X = \frac{z_c - z_f - n_f \lambda_f}{\lambda_m}$$

Then

$$\chi^2 = \chi_c^2 + \chi_m^2 = \left(\frac{(N_m - X) \lambda_m}{\sigma_c} \right)^2 + \left(\frac{(n_f - x) \lambda_f}{\sigma_m} \right)^2$$

Naively one would take for n_f , the integer closest to x , and then N_m as the integer value closest to X . However, as is pointed out in [4] this doesn't always give the best answer. The solution adopted in [4] is to try all 60 possible values of N_f and choose the best. However this approach takes an unnecessarily long time, and for reasons of speed, one would like to be able to go to the correct answer as directly as possible.

The full situation is as shown in figure 2.

Figure 2: χ^2 contours for a hit with the 3 z measurements

With contour ellipses of constant χ^2 as shown, the problem is to find the point on the lattice of allowed (n_f, N_M integer) values for which it is smallest. The reason for using n_f and N_f is because the axes of the ellipses are aligned along the horizontal and vertical axes when plotted like this. The 4 allowed corner points A,B,C,D form a parallelogram enclosing the zero of the χ^2 function at $(n_f = x, N_f = (z_c - z_f)/\lambda_f)$. The point of minimum

χ^2 has to be one of those 4. Any point with the same n_f but different N_f has the same χ_m^2 as two of the corner points but a χ_c^2 which is larger. (E has the same n_f as A and B: F has the same as C and D.) A point like G with the same N_m but different n_f can have a lower χ_c^2 than any corner point, but has a larger χ_m^2 . Comparing such a point with the corresponding corner (here B) the increase in χ_m^2 is at least λ_f^2/σ_m^2 , and the decrease in χ_c^2 is less than $20\lambda_f^2/\sigma_c^2$. Thus G cannot have a lower total χ^2 provided $\sigma_m^2/\sigma_c^2 \leq 1/20$. For the OPAL chambers this figure is generally about 1/100, which is very safe. (This limit can be tightened up by more detailed considerations discussed later, by a factor of about 2).

The problem is then to choose the best of the 4 corner points. This can be expressed by scaling the vertical axis by σ_m/σ_c . The ellipses then become circles, and the problem is to find the corner point closest to a given point inside the parallelogram. After scaling, the situation is as shown. The regions delimited by the dashed lines divide the parallelogram according to the naive ‘nearest integer’ method. For the two horizontal lines this is correct, but the correct vertical division is along the dotted lines, which are the lines of equal distance from the vertices.

Figure 3: Geometry of the parallelogram after scaling

It can be seen that the nearest integer gives the correct answer except in a zig-zag region halfway across. The distance OX is

$$\xi = \frac{1}{2} - \frac{9}{2} \frac{\sigma_m^2}{\sigma_c^2}$$

For typical values this is around 0.45. (The condition that $\xi > 0$ is the more stringent version of the condition that the point of lowest χ^2 is one of the 4 enclosing points, referred

to earlier. In practice there are rare and exceptional cases where the medium z readout has severe problems and σ_m is large enough for ξ to be negative, and the full consideration of all 60 ambiguities is required.)

If n_f and N_m are defined using nearest integers, then if $|x - n_f| < \xi$, this is unambiguously correct. This happens often, as ξ is close to 0.5, and because for good measurements the desired numbers of wavelengths are close to their true values, which are whole numbers. In any case, if not, then the ways in which the ambiguity can work are limited. From the figure it can be seen that

- 1 if x, X is in the lower left quadrant, n_f may have to be increased by 1
- 2 if x, X is in the upper right quadrant, n_f may have to be decreased by 1
- 3 if x, X is in the lower right quadrant, n_f may have to be decreased and N_f increased by 1
- 4 if x, X is in the upper left quadrant, n_f may have to be increased and N_f decreased by 1

Thus the best Z value is usually given by simple integer rounding of numbers of wavelengths. When it is not, a simple comparison of X, x , and ξ to determine where in figure 3 the point lies determines whether the values need changing to prescribed alternatives.

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