Beam optics calculations for CLIC
Master thesis
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Abstract

This Master’s thesis was done in the framework of the Master 2 Grands Instruments program of Université Paris Sud and was conducted at the European Organization for Nuclear Research (CERN). It is dedicated to the Compact LInear Collider (CLIC) beam optics studies for initial energy stage of 380 GeV. The thesis focuses on the Final Focus System (FFS) optics, the very last section of the CLIC Beam Delivery System (BDS). The FFS squeezes the beam strongly at the IP, benefiting from the local chromaticity correction scheme, to reach the target luminosity of $10^{34}\text{cm}^{-2}\text{s}^{-1}$ order.

Initially, the FFS nominal optics with $\beta_y^* = 100 \, \mu\text{m}$ was analyzed to identify the possible ways to optimize the lattice. Then the FFS optics was optimized with MadX and MAPCLASS to achieve a smaller vertical IP beta function of $\beta_y^* = 70 \, \mu\text{m}$ while preserving small nonlinear contributions to the IP beam size. The analysis of such optics in terms of luminosity and energy bandwidth was conducted with PLACET and GUINEA-PIG, with synchrotron radiation and beam-beam effects taken into account. Following the obtained results, possibilities for further optimization of the FFS were identified.
Acknowledgements

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Chapter 1

Introduction

1.1 Future particle colliders

In 2012 the Higgs boson was discovered at CERN by ATLAS [1] and CMS [2] experiments. LHC provided enough precision to discover a new particle, but a more precise measurement of the Higgs field is needed, meaning a so called “Higgs factory” is required. Following the HL-LHC upgrade [3], the next collider could be lepton collider. Such colliders, and more particularly $e^-e^+$ machines, are more suitable for precise measurements as they provide much cleaner experimental conditions, compared to the hadron colliders. More details of physics potential of $e^-e^+$ collider can be found in [4].

In recent years, the scientific community developed the requirements and the concepts for the future lepton collider. Studies for the Future Lepton Collider were held considering both circular and linear machines. Linear colliders are represented by CLIC [5] and ILC [6, 7]. Both projects are consolidated together in the frame of the Linear Collider Collaboration (LCC). Other projects are under study such as FCC-ee [8] and CEPC [9], which are circular electron-positron colliders.

In Fig. 1.1 target luminosities are presented for projects under consideration, including the planned staging upgrades. The advantage of circular lepton collider over linear lepton collider is larger luminosities for collisions with low center-of-mass energy of $\sqrt{s} < 400$ GeV, due to the high collision rate. On the other hand, linear lepton colliders can provide high center-of-mass energy up to few TeV, which is prohibitive to achieve with a circular machine, as the rate of the beam energy loss becomes too high: power radiated as a synchrotron light scales as $P_\gamma \propto \frac{E^4}{\rho^2m^3}$ (see Sec. 2.5), where $E$ is energy of the particle, $m$ is mass of the particle, and $\rho$ is the accelerator curvature radius.

1.2 Linear lepton colliders

In linear colliders, the beam is accelerated in a single pass through the main accelerating structures, so it has to be very efficient in terms of cost and effectiveness. Another
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Figure 1.1: Design luminosities as a function of center-of-mass energy for lepton colliders, currently under consideration. Figure taken from [10].

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<td>1.5</td>
<td>5.9</td>
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<tr>
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<td>0.9</td>
<td>2</td>
<td>1.05</td>
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<tr>
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<td>5.2</td>
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<td>1312</td>
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<td>144/2.9</td>
<td>40/0.9</td>
<td>474/5.9</td>
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Table 1.1: CLIC and ILC design parameters.

A key point that is common for every collider is the luminosity at the IP. Reaching high luminosity is especially important for the linear collider as the repetition rate in such an accelerator is lower than in the circular machine. That means that the beam size at the Interaction Point (IP) has to be reduced to the order of nanometers to reach the design luminosity value of order $\mathcal{L} = 10^{34}$ cm$^{-2}$s$^{-1}$. The general parameters of two projects of linear colliders are shown in Tab. 1.1.

Both proposals are studied in large international collaborations and have many in common. Although CLIC and ILC use different technologies, they are composed of the similar components [5] such as:

- **Electron and positron source.**
  Electrons are extracted from a cathode with a Q-switched laser, accelerated and bunched for the further usage. Positrons are generated in an $e^-e^+$ pair production process.
• **Damping ring.**
  Electron and positron beams are transported to the damping rings, where beams transverse emittance is reduced by several orders.

• **Main Accelerating structure.**
  Beams are accelerated to the design energy while preserving the small normalized transverse emittance.

• **Beam Delivery System (BDS).**
  BDS guides the beam towards the IP. It is composed of Diagnostic, Collimation sections, which increase the beam quality for the collision, and Final Focus System (FFS), in which the beam is squeezed to the design, sub-nanometer in the case of CLIC, size.

### 1.3 CLIC

The Compact Linear Collider or CLIC is a project of the Future Lepton Collider led by CERN. The final goal of CLIC is achieving $5.9 \times 10^{34} \text{cm}^{-2}\text{s}^{-1}$ total luminosity and $2.0 \times 10^{34} \text{cm}^{-2}\text{s}^{-1}$ peak luminosity for $e^-e^+$ collisions with 3 TeV center-of-mass energy. The optimization of the staging strategy was done \[1\], three main stages were chosen: 380 GeV, 1.5 TeV and 3 TeV center-of-mass energy (see Fig. 1.2), where the initial stage c.o.m. energy was decreased from 500 GeV \[5\]. Physics potential of the 380 GeV stage is discussed in \[12\]. Current staging plan foresees the possible second usage of the sections from previous energy stage, see Fig. 1.3.

CLIC benefits from the novel two-beam acceleration (TBA) (see Fig. 1.4), where the drive beam, decelerated in Power Extraction and Transfer structure (PETS), generates RF power, which is delivered to the Main Beam. For the initial stage of 380 GeV, one drive beam is used with an upgrade to two drive beams in the next energy stages.

#### 1.3.1 Final Focus System

The Final Focus System (FFS) is the part of the Beam Delivery System responsible for decreasing the horizontal and vertical beam sizes by several orders at the IP location. Optimization of the FFS is crucial for the performance of the whole collider and it is the main subject of study in this thesis.

The key part of the FFS is the Final Doublet (FD), which is composed of one focusing and one defocusing quadrupoles. The FD has to focus the beam at the IP by applying large kicks to the beam. Since the beam coming from the linac is not monochromatic (0.35 % RMS momentum spread), particles with different momentums will be focused differently, resulting in large beam size dilution at the IP. Since the beam size is expected to be of the nanometer order, any chromatic and other contributions to the beam size, especially ones that are generated by FD, have to be minimized. Traditional non-local chromaticity correction scheme allows to correct
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Figure 1.2: CLIC layout featuring 3 energy stages.

Figure 1.3: Concept of the main linac foresees the usage of the sections from previous energy stage, such that the previous linac is the beginning of new linac. Figure taken from [5].
horizontal and vertical chromaticities in two dedicated sections upstream of the FD, but it requires a lot of space in the case of multi-TeV colliders. In order to mitigate the chromaticity generated by the FD at the IP - CLIC FFS adopted the local chromaticity correction scheme \[13\]. In such a scheme, sextupoles are interleaved with the FD quadrupoles to correct the horizontal and vertical chromaticity locally and simultaneously at the IP; dispersion is generated upstream the FD with a bending magnet; two more sextupoles are placed in phase to cancel geometrical aberrations (see Fig. 1.5). More details on chromaticity correction are described in Sec. 2.4.

Local chromaticity correction scheme demonstrated its effectiveness at Accelerator Test Facility 2 (ATF2) \[14\]–\[16\], based at KEK, allowed to reach 41 nm vertical beam size at the virtual IP \[17\]. Although the Local correction is planned to be used at CLIC, the performance analysis of the Traditional correction scheme was also
done [18], which has slightly lower luminosity but it is much easier to tune.

1.3.2 FFS for CLIC 380 GeV

It is planned to have $\sqrt{s} = 380$ GeV center-of-mass energy as the initial energy stage of CLIC. Analysis of the possible FFS optics for this energy stage was done in [19]. Two different values of $L^*$ were considered: 4.5 m and 6 m. The main advantage of the longer $L^*$ optics is that the final quadrupole $QD0$ is outside the detector acceptance, compared to the shorter $L^*$, where $QD0$ needs additional shielding: an anti-solenoid to cancel the solenoid field inside the quadrupole. Such shielding is required since $QD0$ and solenoid fields will overlap, breaking the symmetry and produce aberrations at the IP. As a result, it was agreed to have $L^* = 6$ m as a base design for CLIC FFS 380 GeV and 3 TeV to benefit from having the whole FD outside the detector.
Chapter 2

Beam Dynamics in the FFS

2.1 General conventions

In the presence of an electromagnetic field, a charged particle is affected by the Lorentz force:

\[ \vec{F} = q(\vec{E} + \vec{v} \times \vec{B}). \]  

(2.1)

where \( q \) is the charge of the particle, \( \vec{v} \) is velocity of the particle, \( \vec{B} \) is magnetic field vector and \( \vec{E} \) is electric field vector.

Thus, its trajectory, generally, is described with Newton’s second law \( \dot{\vec{p}} = q(\vec{E} + \vec{v} \times \vec{B}) \). An accelerator beam line, like FFS, is the combination of elements that produce a specific EM field to accelerate/decelerate, guide and focus the beam. To describe the movement of the particle in such a beam line, the natural coordinate system was adopted, with coordinate \( s \), the position along the reference orbit, as an independent coordinate instead of time \( t \), as shown in Fig. 2.1. In such a system, each particle at the arbitrary position \( s \) is described by 6 coordinates: \( (x, p_x, y, p_y, z, p_z) \). The reference orbit is the trajectory of the particle with design momentum \( p_0 \) through ideal magnets. Such a particle is called the reference particle.

Furthermore, coordinates were modified to a more convenient form for accelerator physics, such that a particle coordinate is read as \( \vec{u}(s) = (x, x', y, y', z, \delta p) \). Here \( x \) and \( y \) are the particle offsets, \( x' = \frac{dx}{ds} \) and \( y' = \frac{dy}{ds} \) are particle slopes in horizontal and vertical planes respectively. \( z \) is the longitudinal position of the particle with respect to the bunch center and \( \delta p = \frac{\Delta p}{p_0} \) is the relative momentum deviation. In such a definition, the reference particle has a zero coordinate in 6D phase space.

In the FFS, there is no longitudinal acceleration, which means that no static electric field exists there \( (\vec{E}(s) = 0) \), and each particle in the beam already has a velocity close to the speed of light \( c \). This simplifies the equation of motion to the following:

\[ m\gamma \ddot{v} = -e\vec{v} \times \vec{B}. \]  

(2.2)

Here, we assume, the particle is an electron and \( |\vec{v}| = c \).
2.2 Linear transverse beam optics

In the linear approximation, each of the particle coordinates transforms linearly through the beam line, as follows:

\[ u_{i}^{\text{final}} = \sum_{j=1}^{5} R_{ij} u_{j}^{\text{initial}}. \]  

(2.3)

The longitudinal position of the particle is ignored as it does not affect other coordinates in the FFS. Each section of the beam line can be associated with a matrix \( R_{ij} \), which is called a transfer matrix. In this case, each element in the beamline, such as a quadrupole, a drift, etc. is described with its transfer matrix. The transfer matrix \( R \) of the beam line, which is composed of several simple elements, is the result of multiplying the matrices of these elements: \( R = \prod_{i=1}^{N} R_{i} \).

2.2.1 On-momentum particle motion

In most cases, the motion in the vertical plane is independent of the motion in the horizontal plane. Such motion is called uncoupled. If we write the Eq. (2.2) in the linear approximation for the case of the uncoupled the on-momentum (\( \Delta p = 0 \)) particle we get Hill’s equations:

\[ x'' + \left( \frac{1}{\rho^2} - k(s) \right) x = 0, \]  

(2.4)

\[ y'' + k(s)y = 0, \]  

(2.5)
where \( k = \frac{eg}{p_0} \) is the focusing strength and \( g \) is the magnetic field gradient. The linear approximation we used previously automatically means that the magnetic field does not have any modes higher than the quadrupole mode.

We have the term \( \frac{1}{\rho} \) in Eq. (2.4) because the motion equation is written in curvilinear coordinates. It appears only when we have the bending of the reference orbit; thus in magnets with a dipole moment. If we consider the pure quadrupole, the orbit curvature is zero and the solution can be written as a function of the initial coordinates \( x_0 = x(0) \), \( y_0 = y(0) \), \( x'_0 = x'(0) \) and \( y'_0 = y'(0) \):

When \( k > 0 \):

\[
\begin{align*}
x(s) &= \cos \left( \sqrt{k}s \right) x_0 + \frac{1}{\sqrt{k}} \sin \left( \sqrt{k}s \right) x'_0, \\
y(s) &= \cosh \left( \sqrt{k}s \right) y_0 + \frac{1}{\sqrt{k}} \sinh \left( \sqrt{k}s \right) y'_0,
\end{align*}
\]  

(2.6)  

When \( k < 0 \):

\[
\begin{align*}
x(s) &= \cosh \left( \sqrt{|k|}s \right) x_0 + \frac{1}{\sqrt{|k|}} \sinh \left( \sqrt{|k|}s \right) x'_0, \\
y(s) &= \cos \left( \sqrt{|k|}s \right) y_0 + \frac{1}{\sqrt{|k|}} \sin \left( \sqrt{|k|}s \right) y'_0.
\end{align*}
\]  

(2.8)  

The quadrupole focuses the beam in one transverse plane and defocuses it in the other plane, so the quadrupole with \( k > 0 \) is called a focusing quadrupole and the one with \( k < 0 \) the defocusing quadrupole.

One can easy construct a \( 2 \times 2 \) the transfer matrix for the quadrupole of length \( L \) that focuses the beam in horizontal plane (\( k > 0 \)):

\[
R_{\text{focusing}} = \begin{bmatrix} \cos \sqrt{kL} & \frac{1}{\sqrt{k}} \sin \sqrt{kL} \\ -\sqrt{k} \sin \sqrt{kL} & \cos \sqrt{kL} \end{bmatrix}, \tag{2.10}
\]

\[
R_{\text{defocusing}} = \begin{bmatrix} \cosh \sqrt{kL} & \frac{1}{\sqrt{k}} \sinh \sqrt{kL} \\ \sqrt{k} \sinh \sqrt{kL} & \cosh \sqrt{kL} \end{bmatrix}. \tag{2.11}
\]

If a quadrupole, which focuses the beam in the horizontal plane, is considered, one has to take the focusing transfer matrix for a horizontal plane and defocusing transfer matrix for a vertical plane and vice versa.

The drift space can be considered as a quadrupole with the field strength of \( k = 0 \), so the transfer matrix would be the same for both planes:

\[
R_{\text{drift}} = \begin{bmatrix} 1 & L \\ 0 & 1 \end{bmatrix}. \tag{2.12}
\]
From Eqs. (2.4) and (2.5) one can construct the transfer matrix for the pure sector dipole \( k = 0 \), that steers the beam in the horizontal plane:

\[
R_{\text{dipole}} = \begin{bmatrix}
\cos \theta & \rho \sin \theta \\
-\frac{1}{\rho} \sin \theta & \cos \theta
\end{bmatrix},
\]  

(2.13)

where \( \theta = \frac{L}{\rho} \) is the bending angle of the dipole magnet. In the vertical plane, the dipole behaves like a drift.

### 2.2.2 Off-momentum particle motion

When a particle has a different momentum than the reference particle, the dipole magnet will deflect it differently. An off-momentum particle that enters the sector dipole with no horizontal divergence will have gained some at the exit of the magnet, as the rotation angle would be different from the design dipole bending angle for the reference particle. Considering small momentum deviation \( \Delta p \ll 1 \), one can develop Eq. (2.2) and get:

\[
x'' + \left( \frac{1}{\rho^2} - k(s) \right) x = \frac{1}{\rho} \frac{\Delta p}{p_0},
\]

(2.14)

\[
y'' + k(s) y = 0.
\]

(2.15)

The general solution of this equation when the right hand side is zero is already known. One has to find the particular solution of the inhomogeneous Eq. (2.14) in a form \( x_p = D(s) \frac{\Delta p}{p_0} \). \( D(s) \) satisfies the equation:

\[
D'' + \left( \frac{1}{\rho^2} - k(s) \right) D = \frac{1}{\rho},
\]

(2.16)

The function \( D(s) \) is called a dispersion function. When the general solution of the homogeneous equation is in the form \( x(s) = C(s)x_0 + S(s)x'_0 \), where \( C(s) \) and \( S(s) \) are two linearly independent solutions of the homogeneous equation, the solution of the Eq. (2.16) reads

\[
D(s) = S(s) \int_{s_0}^{s} \frac{1}{\rho(t)} C(t) dt - C(s) \int_{s_0}^{s} \frac{1}{\rho(t)} S(t) dt.
\]

(2.17)

The final solution of Eq. (2.14) is:

\[
x(s) = C(s)x_0 + S(s)x'_0 + D(s) \frac{\Delta p}{p_0}.
\]

(2.18)

A more general transfer matrix can be defined, including the dispersion term:

\[
R = \begin{bmatrix}
C(s) & S(s) & D(s) \\
C'(s) & S'(s) & D'(s) \\
0 & 0 & 1
\end{bmatrix},
\]

(2.19)
to be used as:

\[
\begin{bmatrix}
  x \\
  x' \\
  \Delta p \\
  p_0
\end{bmatrix}_s = R
\begin{bmatrix}
  x \\
  x' \\
  \Delta p \\
  p_0
\end{bmatrix}_{s_0}.
\]

### 2.2.3 Twiss formalism

The beams that are used in accelerators consist of billions of particles such that it is not possible nor reasonable to track the coordinates of each individual particle through the accelerator. The overall behavior of the beam is tracked instead, such as beam size and divergence. A so-called Twiss formalism was developed and is very convenient to describe the beam envelope and divergence evolution along the accelerator. In this section, we assume a monochromatic beam with uncoupled motion in horizontal and vertical planes.

We look for the solution of the general Hill’s equation

\[ u'' + K(s)u(s) = 0 \]

in a form:

\[ u = \sqrt{\epsilon \beta(s)} \cos(\phi(s) - \phi_0) \],

where \( K(s) \) has a specific value, corresponding to the magnet considered and plane in which it is solved. For example, \( K(s) = \frac{1}{\beta^2} \) for a dipole magnet plane in the horizontal plane.

The function \( \beta(s) \) is called beta function, it describes the evolution of the beam size along the beam line and is exclusively defined by magnetic lattice:

\[ \frac{1}{2} \beta'' - \frac{1}{4} \beta'^2 + \beta^2 K = 1. \]

The function \( \phi(s) \) is called phase advance and is related to the beta function:

\[ \phi(s) = \int_0^s \frac{1}{\beta(s)} ds. \]

Other useful functions are introduced: \( \alpha = -\frac{1}{2} \beta' \), which is called the alpha function, describes the beam divergence and \( \gamma = \frac{1 + \alpha^2}{\beta} \), the gamma function. Altogether with \( \beta \)-function are the Twiss parameters.

Another important parameter in Eq. (2.21) is \( \epsilon \), which is called the single particle emittance. It fulfills the equation:

\[ \epsilon = \gamma u^2 + 2 \alpha uu' + \beta u'^2. \]

Equation (2.24) describes an ellipse with area \( \pi \epsilon \) in the phase space \( x - x' \) as shown in Fig. 2.2. At a given location \( s \) each particle has a corresponding ellipse. Single particle emittance is also called Courant-Snyder invariant. An important property of such an invariant is that it remains constant when we transform \( x \) and \( x' \) coordinates through the accelerator, thus the area of the ellipse is preserved as well. A very
important result comes when we consider a beam composed of multiple particles. Each of the particles has its own ellipse in the phase space.

Following the definition of single particle emittance, one can construct the beam emittance $\epsilon_{\text{beam}}$. The beam ellipse in the phase space and the corresponding emittance are defined statistically, as an ellipse covering 46% of the particles following the 2D one-sigma rule.

Due to the area preservation, the phase-space particles density is invariant (Liouville’s theorem). This is true only for a closed system when there is no acceleration. To have the similar emittance preservation rule when the beam changes its energy, one can define the normalized emittance:

$$\epsilon_n = \gamma \beta \epsilon_{\text{beam}}. \tag{2.25}$$

which is invariant even under the presence of an accelerating structure in the beam line.

![Figure 2.2: The phase ellipse of a particle.](image)

The single particle is described with the vector in 5D phase space, the ensemble of the particles or beam is characterized with the Sigma or $\sigma$ matrix instead, defined as:

$$\Sigma = \begin{bmatrix}
\sigma_{xx} & \sigma'_{xx} & \sigma_{xy} & \sigma_{xy'} & \sigma_{x\delta_p} \\
\sigma'_{xx} & \sigma'_{xx} & \sigma'_{xy} & \sigma'_{xy'} & \sigma'_{x\delta_p} \\
\sigma_{yx} & \sigma_{yx'} & \sigma_{yy} & \sigma_{yy'} & \sigma_{y\delta_p} \\
\sigma'_{yx} & \sigma'_{yx'} & \sigma'_{yy} & \sigma'_{yy'} & \sigma'_{y\delta_p} \\
\sigma_{\delta_p x} & \sigma_{\delta_p x'} & \sigma_{\delta_p y} & \sigma_{\delta_p y'} & \sigma_{\delta_p \delta_p}
\end{bmatrix}. \tag{2.26}$$
Here $\delta p = \frac{\Delta p}{p_0}$ is the particle momentum deviation.

It is a symmetric matrix, constructed from the second central moments of all possible coordinates among all the particles in the beam. In the absence of coupling, the matrix will be block diagonal. In general, statistical moments are defined as follows:

\[
\langle x \rangle = \int \int x \rho(x, x') dx dx', \tag{2.27}
\]
\[
\sigma_{xx} \equiv \sigma_x^2 = \int \int (x - \langle x \rangle)^2 \rho(x, x') dx dx', \tag{2.28}
\]
\[
\sigma_{xx'} = \int \int (x - \langle x \rangle)(x' - \langle x' \rangle) \rho(x, x') dx dx'. \tag{2.29}
\]

where $\rho(x, x')$ is the normalized phase density, $\int \rho(x, x') dx dx' = 1$. The equations were simplified for the case of the phase space $x - x'$ with the $2 \times 2$ Sigma matrix, but one can construct the same equations for a higher number of coordinates, with the general phase space density.

The Sigma matrix transforms along the beam line as follows [21]:

\[
\Sigma(s) = R \Sigma_0 R^T. \tag{2.30}
\]

The emittance of the ensemble of particles, as it was mentioned before, in the absence of dispersion, is defined as:

\[
\epsilon_{beam} \equiv \epsilon_{rms} = \sqrt{\sigma_x^2 \sigma_{xx'}^2 - \sigma_{xx'}^2}. \tag{2.31}
\]

Henceforward, only the beam emittance is used, $\epsilon_x$ and $\epsilon_y$ for horizontal and vertical beam emittance respectively. Normalized emittance writes as $\epsilon_{nx}$ and $\epsilon_{ny}$.

Following the beam emittance definition, one can write the beam matrix for the transverse plane in connection with the Twiss parameters:

\[
\Sigma_x = \begin{bmatrix}
\sigma_x^2 & \sigma_{xx'} \\
\sigma_{xx'} & \sigma_{x'}^2
\end{bmatrix} = \epsilon_x \begin{bmatrix}
\beta_x & -\alpha_x \\
-\alpha_x & \gamma_x
\end{bmatrix}. \tag{2.32}
\]

The smallest beam size is achieved in the so-called “beam waist”, when the beta function reaches its local minimum, as shown in Fig. 2.3. The phase ellipse of the beam is oriented vertically and the alpha function is zero in this position. The beam size at the position $s$ is obtained from Eq. (2.32):

\[
\sigma(x) = \sqrt{\sigma_x^2} = \sqrt{\epsilon_x \beta_x}. \tag{2.33}
\]

When the beam is not monochromatic, each particle has a different momentum from a design one and deviates from its initial orbit by an additional $D(s) \frac{\Delta p}{p_0}$ as given by Eq. (2.18). The momentum spread of the beam, defined as $\delta p_{beam} = \sqrt{\sigma_{\delta p}^2}$ will contribute to the total beam size in quadrature:
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Figure 2.3: Beam envelope along the beam line. Figure comes from [22].

\[ \sigma(x) = \sqrt{\epsilon\beta(s) + (D(s)\delta p_{beam})^2}. \]  
(2.34)

Henceforward, the beam momentum spread, by analogy to the emittance, writes as \( \delta \).

## 2.3 Nonlinear beam dynamics

By analogy to Eq. (2.3) it is possible to extend such formalism and take into account second, third or even higher order contributions to the particle coordinate:

\[ u_i^{\text{final}} = \sum_{j=1}^{5} R_{ij} u_j^{\text{initial}} + \sum_{j,k=1}^{5} T_{ijk} u_j^{\text{initial}} u_k^{\text{initial}} + \sum_{j,k,l=1}^{5} U_{ijkl} u_j^{\text{initial}} u_k^{\text{initial}} u_l^{\text{initial}} + O(4). \]  
(2.35)

Higher order transfer maps are no longer matrices, but tensors. Generally, this is called map formalism, as the higher-order tensors give the information on how to calculate the particle coordinates at the final state based on its initial state. It can be summed up in one equation in a slightly different form with no limit on order [23]:

\[ u_i^{\text{final}} = \sum_{j,k,l,m,n} X_{ijklmn} (u_1^{\text{initial}})^j (u_2^{\text{initial}})^k (u_3^{\text{initial}})^l (u_4^{\text{initial}})^m (u_6^{\text{initial}})^n. \]  
(2.36)

These nonlinear contributions come from the magnets with magnetic moment higher than quadrupole, such as sextupoles, octupole, decapoles and others, which
will be introduced by design or represent imperfections in the magnetic lattice. Higher order magnets are dedicated to correcting the nonlinear aberrations introduced by lower order magnets such as chromaticity and are important for achieving the small beam sizes.

2.3.1 Multipole field expansion

In Cartesian coordinates, multipole field expansion reads [24]:

\[ B_y + iB_x = B_{\text{ref}} \sum_{n=1}^{+\infty} (b_n + ia_n) \left( \frac{x + iy}{R_{\text{ref}}} \right)^{n-1}. \]  

(2.37)

Since no longitudinal magnetic field is present in the accelerator, the vector potential has a longitudinal component \( A_s \) which is related to the magnetic field as

\[ B_x = \frac{\partial A_s}{\partial y}, \]

(2.38)

\[ B_y = -\frac{\partial A_s}{\partial x}. \]

(2.39)

Vector potential giving Eq. (2.37) reads:

\[ A_s = -B_{\text{ref}} \Re \left( \sum_{n=1}^{+\infty} (b_n + ia_n) \left( \frac{x + iy}{R_{\text{ref}}} \right)^n \right). \]

(2.40)

The component \( b_n \) is called the normal component and \( a_n \), the skew component.

In most accelerators, pure magnets are used, with the dipole, quadrupole, etc. magnetic moment only. Such magnet has a magnetic field like:

\[ B_{ny} + iB_{nx} = B_{\text{ref}} (b_n + ia_n) \left( \frac{x + iy}{R_{\text{ref}}} \right)^{n-1}. \]

(2.41)

Two major types of multipoles are distinguished - skew \((b_n = 0)\) and normal \((a_n = 0)\) multipoles. To characterize each of them, multipole strength is defined as follows:

\[ k_{n-1}^N = \frac{1}{B_0 \rho} \frac{\partial^{n-1} B_y}{\partial \rho^{n-1}}, \]

(2.42)

\[ k_{n-1}^S = \frac{1}{B_0 \rho} \frac{\partial^{n-1} B_x}{\partial \rho^{n-1}}, \]

(2.43)

where \( B_0 \rho = \frac{e}{q} \) is the magnetic rigidity.

As a result of such a convention, one can notice that \( k_1 \) corresponds to the quadratic magnetic field of the quadrupole, \( k_2 \) is the sextupole strength etc. Following these definitions, one can calculate the vector potential as:
\[ A_{s,n}^N = -B_0 \rho \frac{k_n^N}{n!} \Re\{(x + iy)^n\}, \quad (2.44) \]

\[ A_{s,n}^S = -B_0 \rho \frac{k_{n-1}^S}{n!} \Im\{(x + iy)^n\}. \quad (2.45) \]

### 2.3.2 Hamiltonian formalism

The Hamiltonian of a relativistic particle in the Frenet-Serret coordinate system (with a coordinate \( s \) as the independent variable) has the following form \[25\]:

\[ H = e p A_s (1 - \delta_p) - \left( 1 + \frac{x}{\rho} \right) \sqrt{1 - (p_x^2 + p_y^2)} \quad (2.46) \]

where \( \delta_p = \frac{\Delta p}{p_0} \) is a particle momentum deviation. With kinematic term simplified:

\[ H = -\frac{e}{p} A_s (1 - \delta_p) + \left( 1 + \frac{x}{\rho} \right) \frac{p_x^2 + p_y^2}{2}. \quad (2.47) \]

Motion equations can be derived from Hamiltonian equations:

\[ \frac{dp_x}{ds} = -\frac{\partial H}{\partial x}, \quad (2.48) \]
\[ \frac{dx}{ds} = \frac{\partial H}{\partial p_x}, \quad (2.49) \]

where \( \{p_x, x\} \) is the pair of canonical coordinates. The canonical momentum for the most of the fields in beam dynamics, with no longitudinal magnetic field, is \( p_x = x' \) \[26\]. This is obtained from the simplified Hamiltonian, as the kinematic term is always the same and equal to \( \frac{x'^2 + y'^2}{2} \) when \( p_x \) and \( p_y \) are small. From Eq. \(2.48\) one can get the motion equation:

\[ x'' = -\frac{\partial H}{\partial x}. \quad (2.50) \]

One can calculate the Hamiltonian for the most widely used magnets in accelerator physics:

- **Normal \( n^{th} \) multipole**
  
  **Quadrupole (\( n = 2 \))**:
  
  \[ H = \frac{k_1^N}{2} (x^2 - y^2)(1 - \delta_p) + \frac{x'^2 + y'^2}{2}, \quad (2.51) \]
  
  with \( \frac{x'^2 + y'^2}{2} \) as a kinematic term.

**Sextupole (\( n = 3 \))**:

\[ H = \frac{k_2^N}{6} (x^3 - 3xy^2)(1 - \delta_p) + \frac{x'^2 + y'^2}{2}. \quad (2.52) \]
2.4 Chromaticity

Octupole \( (n = 4) \):
\[
H = \frac{k_3^N}{24} (x^4 - 6x^2y^2 + y^4)(1 - \delta_p) + \frac{x'^2 + y'^2}{2}.
\] (2.53)

- Skew \( n^{th} \) multipole

Quadrupole \( (n = 2) \):
\[
H = k_1^S xy(1 - \delta_p) + \frac{x'^2 + y'^2}{2}.
\] (2.54)

Sextupole \( (n = 3) \):
\[
H = \frac{k_2^S}{6} (y^3 - 3yx^2)(1 - \delta_p) + \frac{x'^2 + y'^2}{2}.
\] (2.55)

Octupole \( (n = 4) \):
\[
H = \frac{k_3^S}{6} xy(x^2 - y^2)(1 - \delta_p) + \frac{x'^2 + y'^2}{2}.
\] (2.56)

The Hamiltonian of the particle in a dipole magnetic field has the form:
\[
H = (\frac{x^2}{2\rho^2} - \frac{x\delta_p}{\rho})(1 - \delta_p) + \frac{x'^2 + y'^2}{2}.
\] (2.57)

2.4 Chromaticity

The equation of motion for a particle inside a quadrupole magnet, with the Hamiltonian given by Eq. (2.51) is:
\[
x'' = -k_1(1 - \delta_p)x \Rightarrow x'' + k_1x = k_1x\delta_p.
\] (2.58)

This result shows that the particle, which has a relative momentum deviation \( \delta_p \), is “kicked” differently by the quadrupole, compared to the reference particle, thus producing chromatic aberrations, by analogy to the aberrations in white light. This effect is undesirable, as the particles in a beam will be focused differently, as shown in Fig. 2.4, leading to larger beam size at the IP. The consideration of this effect is rather important for CLIC FFS, as the Final Doublet has to squeeze the beam strongly to reach the sub-nanometer beam size.

Quadrupoles are not the only type of magnet, which produces chromatic aberrations; sextupoles can produce chromaticity as well. The equations of motion of a particle in the magnetic field with quadrupole and sextupole magnetic moments reads:
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Figure 2.4: Chromatic aberrations in the Final Doublet, which lead to an increased beam size at the IP. Figure taken from [27].

\[ x'' + k_1 x = k_1 \delta_p x - \frac{k_2}{2} (x^2 - y^2), \]  
\[ x'' - k_1 x = k_1 \delta_p y + k_2 xy. \]  

Following the same procedure as in [26] and putting \( x = x_\beta + D_x \delta \) and \( y = y_\beta \), we get the equations on \( x_\beta \) and \( y_\beta \) in order to get the phase advance shift caused by the chromatic aberrations:

\[ x''_\beta + k_1 x_\beta = (k_1 - k_2 D_x) \delta_p x_\beta - \frac{k_2}{2} (x_\beta^2 - y_\beta^2) + O(3), \]  
\[ y''_\beta - k_1 y_\beta = (-k_1 + k_2 D_x) \delta_p y_\beta + k_2 x_\beta y_\beta + O(3). \]  

Here, no vertical dispersion is considered.

One can notice, that apart from chromatic aberration terms, which are proportional to momentum deviation \( \delta_p \), there are also pure geometrical terms. These kicks are produced by sextupoles and higher order magnets, and are called geometrical aberrations. They are the subject for correction as well as chromatic aberrations.

Chromaticity is related to the chromatic aberrations, so considering chromatic terms only one can connect the phase advance shift with change of the optics parameters:

\[ \Delta \Psi_x = -\frac{1}{2} \delta_p \int \beta_x (k_1 - D_x k_2) ds, \]  
\[ \Delta \Psi_y = \frac{1}{2} \delta_p \int \beta_y (k_1 - D_x k_2) ds. \]
Almost all the chromatic aberrations in the FFS are produced in the Final Doublet. To characterize it, chromaticity is introduced as follows:

\[ \xi_y \equiv \frac{L_y^C}{\beta_y^*} \approx - \int k_1 \beta_y d s \approx - \frac{1}{\beta_y^*} \int k_1 R_{34}^2 d s, \]  

(2.65)

where \( L_y^C \) is the distance from the magnet to the IP and \( R_{34} \) is the transport matrix element from position \( s \) to IP. A simple approximation for the distance \( L_y^C \) is the distance from the corresponding magnet center to the IP. For the defocusing magnet in the FD, the length \( L_y^C \) can also be approximated to the drift length \( L^* \).

If the chromaticity is left uncontrolled, it will lead to the beam size dilution:

\[ \frac{\Delta \sigma_{x,y}^*}{\sigma_{x,y}^*} \approx \xi_{x,y} \delta. \]  

(2.66)

It adds to the total beam size in quadrature:

\[ \sigma_y^* \sqrt{1 + \xi_y^2 \delta^2}. \]  

(2.67)

Since the beam for CLIC is flat when \( \sigma_x \gg \sigma_y \), the vertical beam size has to be small and it is crucial to eliminate beam size contributions coming from aberrations.

Following, the definition of the map formalism in Eq. (2.36) one can express the natural chromaticity in terms of map coefficients:

\[ \xi_y^* = \frac{1}{\beta_y^*} \left( X_{y,00101}^2 \beta_y^0 + X_{y,00011}^2 \frac{1}{\beta_y^0} \right), \]  

(2.68)

where \( \beta_y^* \) is the vertical beta function at the IP and \( \beta_y^0 \) is the beta function at the beginning of the line (usually in calculations, the entrance of BDS is meant).

Eqs. (2.61) and (2.62) are written for the magnet with quadrupole and sextupole moments at the same time. One can rewrite them in the thin lens approximation, considering small kicks applied to the beam by each magnet with the integrated strengths \( K_1 \) and \( K_2 \).

Quadrupole, excluding the focusing, kick gives an additional kick:

\[ \Delta x' = K_1 x \delta_p + K_1 D_x \delta_p^2, \]  

\[ \Delta y' = -K_1 y \delta_p. \]  

(2.69)

(2.70)

The kick introduced by sextupole:

\[ \Delta x' = K_2 x D_x \delta_p + \frac{K_2}{2} D_x^2 \delta_p^2 + \frac{K_2}{2} (x^2 - y^2), \]  

\[ \Delta y' = K_2 xy + K_2 D_x y \delta_p. \]  

(2.71)

(2.72)

Quadrupoles in the Final Doublet of the FFS have a large magnetic strength. Thus, according to Eq. (2.68), chromaticity produced upstream of the IP is high,
resulting in the beam size and luminosity dilution. One can notice from Eqs. (2.69) and (2.71) that the sextupole, which is put in the dispersive region with strength of $k_2 = \frac{k_1}{D_x}$, can cancel the first order chromatic term. Unfortunately, in this case the second order chromatic contribution is left uncontrolled, as shown:

\[
\Delta x'_{\text{total}} = K_1 D_x \frac{\delta^2}{2} - \frac{K_2}{2} (x^2 - y^2),
\]
\[
\Delta y'_{\text{total}} = K_2 xy.
\]

The second term purely depends on coordinates, and it is called a geometrical aberration. It is possible to cancel it by using a second sextupole of the same strength, which is separated by $-\pi$ transformation from the first one, which corresponds to $\pi$ phase advance between them.

It is possible to cancel the second order chromatic term by increasing the magnetic strength of the sextupoles by a factor of 2, but then an overcompensation of the first order chromatic term will happen:

\[
\Delta x'_{\text{total}} = -K_1 x \delta_p - \frac{K_2}{2} (x^2 - y^2),
\]
\[
\Delta y'_{\text{total}} = K_1 y \delta_p + K_2 xy.
\]

In this case one has to generate the natural chromaticity in the nondispersive region, prior to the chromaticity correction sextupoles, thus the overcompensated term and natural chromaticity term will cancel each other.

Following such a chromaticity cancellation scheme, one can also get rid of the second order geometrical terms terms, introduced by sextupoles, and cancel the natural chromaticity.

### 2.5 Synchrotron radiation effect

When the charged particle experiences the deceleration, it emits electromagnetic radiation. The same effect is observed when the velocity vector of the particle changes the orientation, and the radiation emitted in this case is called synchrotron radiation. Instantaneous radiated power by a particle in the dipole magnet reads:

\[
P_\gamma = \frac{2}{3} r_e mc^3 (\beta \gamma)^{\frac{1}{2}} \propto \frac{E^4}{\rho^2 m^3}.
\]

Here $\rho$ is the curvature radius of the reference orbit in the dipole, $\beta = \frac{v}{c} \approx 1$ is the reduced velocity, $m$ is the mass of the particle, $\gamma = \frac{E}{mc^2}$ is the Lorentz factor, and $r_e$ is classical electron radius.

The particle of the smaller mass will lose more energy than a heavier particle of the same energy. For the ultra-relativistic electron, such energy losses can be large,
and if they occur in the dispersive region, the change of the orbit of the particles leads to the increase of the transverse emittance. At the IP it is observed as a dilution of the beam size. According to \[21\], the vertical beam size dilution reads:

\[
\Delta \sigma_y^* \sim (\gamma \epsilon_y)^{3/2} L^{*3} \left( \frac{D_x^*}{\epsilon_x} \right)^{3/2} \left( \frac{\epsilon_x}{\epsilon_y} \right)^{3/2} \frac{\gamma^2}{L_{F,F,S}^5}.
\] (2.78)

Where \( D_x^* \equiv \frac{dD_x}{ds} \big|_{IP} \) is the angular horizontal dispersion at the IP.

### 2.6 Oide effect

The synchrotron radiation emitted by the high energy electron leads to an energy loss, and thus to a chromatic kick by a quadrupole. At the end, it leads to the dilution of the beam size at the IP. Such contribution to the beam size is called an Oide effect \[30\]. The beam size dilution caused by this effect adds in quadrature:

\[
(\sigma_y^*)^2 = \epsilon_y \beta_y^* + \frac{110}{3 \sqrt{6\pi}} r_e \lambda_e \gamma^5 F(\sqrt{K}L, \sqrt{K}l^*) \left( \frac{\epsilon_y}{\beta_y^*} \right)^{5/2},
\] (2.79)

where \( \lambda_e = \frac{\hbar}{m_e c} \) is the electron Compton wavelength, \( r_e \) is the classic electron radius, \( \gamma \) is the Lorentz factor, \( \epsilon_y \) is the vertical emittance, \( \beta_y^* \) is the IP vertical beta function and \( F \) is a dimensionless function defined as:

\[
F(\sqrt{K}L, \sqrt{K}l^*) \equiv \int_0^{\sqrt{KL}} |\sin \phi + \sqrt{KL} \cos \phi| \left[ \int_0^{\phi} (\sin \phi' + \sqrt{KL} \cos \phi')^2 d\phi' \right]^2 d\phi.
\] (2.80)

### 2.7 Luminosity

The center of mass energy and luminosity are the two main parameters that describe the collider potential. The particle collider has to provide enough center of mass energy to the particles prior to collisions, to produce the physical effects. The quantity that relates the cross section and number of collision events is introduced and is called luminosity \[31\]:

\[
\frac{dR}{dt} = \mathcal{L} \sigma_p.
\] (2.81)

The number of interactions per unit time (event rate) is proportional to the luminosity and the cross section \( \sigma_p \). The common unit of luminosity is cm\(^{-2}\)s\(^{-1}\). As the cross section of the events under study is extremely small, one has to provide large enough luminosity to keep a large event rate.

For two Gaussian beams colliding head-on with no offset at the IP, luminosity is given by:
\[
L_0 = \frac{N^2 f N_b}{4\pi \sigma_x \sigma_y}.
\] (2.82)

Here, \(N\) is the number of particles per colliding bunch, \(f\) is repetition frequency (for a linear collider) and \(N_b\) the number of bunches in a train. This equation is valid for beams colliding head-on at the speed of light.

In a real machine, the actual luminosity differs due to the different effects, such as crossing angle, collision offset, hourglass effect and so on. The final luminosity is expressed as:

\[
L = L_0 H_D,
\] (2.83)

where \(H_D\) is the factor that includes the effects of the beam-beam interactions and other effects mentioned earlier.

Each beam in the accelerator produces a strong electromagnetic field and in the electron-positron collider, they will focus each other at the IP, reducing the effective IP beam size and increasing the luminosity. This is called a pinch effect \[32\]. This effect also leads to radiation emission called beamstrahlung and leads to the loss of beam energy.

The focusing of the colliding beams can be described with the disruption parameter:

\[
D_{x,y} = \frac{2N r_e \sigma_z}{\gamma \sigma_{x,y} (\sigma_x^2 + \sigma_y^2)},
\] (2.84)

where \(r_e \approx 2.8\) fm is the classical electron radius and \(\gamma\) is the relativistic factor of the particles. The disruption parameter is connected with the focusing distance \(f_{x,y}\) as \(D_{x,y} = \frac{\sigma_z}{f_{x,y}}\). The larger disruption parameter is, the stronger focusing of the beams is.

Since the particles will lose energy at the IP due to the beamstrahlung, it is convenient to distinguish the total luminosity \(L_{total}\), where all the collisions are taken into account regardless of the energy, and peak luminosity \(L_{1\%}\), where the collisions with the energy higher than 99\% of the nominal energy are taken into account.

The beamstrahlung is characterized by its critical energy \(h\omega_c\), which is given by

\[
h\omega_c = \frac{3}{2} \frac{h\gamma^3 c}{\rho},
\] (2.85)

where \(\rho\) is the bending radius of the particle trajectory due to the pinch effect and \(\gamma\) is the Lorentz factor. Furthermore a beamstrahlung parameter \(\Upsilon\) is introduced:

\[
\Upsilon = \frac{2 h\omega_c}{3 E},
\] (2.86)

where \(E\) is the beam energy.

The average Beamstrahlung parameter is estimated as:
\[ \langle \gamma \rangle = \frac{5}{6} \frac{N r_e}{\alpha \sigma_z (\sigma_x + \sigma_y)}. \]  

(2.87)

where \( \alpha = \frac{1}{137} \) is the fine structure constant. The number of photons emitted per particle \( n_\gamma \) is defined as:

\[ n_\gamma \propto \gamma \frac{\sigma_z}{\gamma} \propto \frac{N}{\sigma_x^* + \sigma_y^*}. \]  

(2.88)

The average energy of each photon is

\[ E_\gamma \propto \frac{1}{\gamma} \propto \frac{N}{\sigma_z (\sigma_x^* + \sigma_y^*)}. \]  

(2.89)

To decrease the effect of the pinch effect on the beam, while keeping the high enough luminosity, one should simultaneously increase the sum of the IP transverse beam sizes \( \sigma_x^* + \sigma_y^* \) and reduce the product \( \sigma_x^* \sigma_y^* \). It is possible to achieve that for a flat beam \( (\sigma_x^* \gg \sigma_y^*) \). A small vertical beam size \( \sigma_y^* \) at the IP means a small IP beta function \( \beta_y^* \). The beta function will change around the waist as follows:

\[ \beta_y(s) = \beta_y^* \left( 1 + \left( \frac{s}{\beta_y^*} \right)^2 \right). \]  

(2.90)

Reducing the beta function \( \beta_y^* \) at the waist leads to the high divergence around the waist. The beam size \( \sigma_y(s) = \sqrt{\beta_y(s) \epsilon_y} \) depends on the longitudinal position, and increases with the distance from the waist. This effect is called the hourglass effect because of the shape of the beta function from Eq. (2.90), see Fig. 2.5. When \( \beta_y^* \) is close to the bunch length \( \sigma_z \), the beam size will strongly depend on the distance to the waist within the bunch, thus not all the particles collide at the smallest beam size. This leads to the luminosity reduction, which is expressed in a more complex and general form for zero crossing angle \( \frac{28}{A} \):

\[ \mathcal{L} = \mathcal{L}_0 H_D = \mathcal{L}_0 \sqrt{\frac{2}{\pi} a e^2 K_0(a^2)}, \]  

(2.91)

with \( a = \frac{\beta_y^*}{\sqrt{2} \epsilon_y} \); \( K_0 \) is modified Bessel function of the second kind. Using Eq. (2.82) the luminosity can be rewritten as follows:

\[ \mathcal{L} = \frac{N^2 f N_b}{4 \pi \sigma_x^* \epsilon_y \beta_y^*} \sqrt{\frac{2}{\pi} a e^2 K_0(a^2)} = A \sqrt{a e^2} K_0(a^2), \]  

(2.92)

where \( A \) is a proportionality parameter. The luminosity dependence on IP vertical beta function is shown in Fig. 2.6. The maximum luminosity value is obtained for \( a \approx 0.18 \), which gives \( \beta_y^* \approx 0.26 \sigma_z \).

In most cases, the IP vertical beta function is chosen to have roughly the same value as the bunch length, \( \beta_y^* \approx \sigma_z \). The final luminosity will depend on the strength of beam-beam effects and the best luminosity might be obtained for larger \( \beta_y^* \). Reduction
Figure 2.5: The vertical beta function as a function of the distance from the waist for different values of the beta function at the IP. Bunch length is $\sigma_z = 70 \, \mu m$.

Figure 2.6: Luminosity dependence on IP vertical beta function.
of the beta function less than the bunch length might lead to the reduction of the luminosity, as the beam size at the head and tail of the bunch would differ significantly from the value at the waist.
Chapter 3

CLIC 380 GeV optics optimization

Currently, CLIC [5] is planned to have several energy stages: 380 GeV, 1.5 TeV and the final stage of 3.0 TeV center of mass energies [11]. The FFS was modified in order to move the last quadrupole QD0 outside the detector acceptance for the 3 TeV case [33]. For the case of $\sqrt{s} = 380$ GeV CLIC two options were analyzed [34]: $L^* = 4.3$ m, where the last quadrupole is partially is located inside the detector, and $L^* = 6$ m, where the quadrupole is entirely outside the detector. In this thesis, the aim is the optimization of the FFS optics for $L^* = 6$ m.

3.1 Overview of the current BDS optics

The initial CLIC energy stage was changed from 500 GeV to 380 GeV [11]. This lead to the re-matching of the BDS optics, to provide the aimed beam properties at the IP which fulfill the luminosity requirements. The design parameters for 380 GeV CLIC are presented on Tab. 3.1.

Normalized emittance, following the definition Eq. (2.25) is $\epsilon_n = \gamma \beta \epsilon$. In CLIC, particles travel with a speed very close to the speed of light and approximately $\beta \approx 1$. In Tab. 3.1 two normalized emittances are presented: at the end of the linac and at the IP. For the optics calculations of the FFS and IP luminosity calculations, the one at the IP is used with $\gamma \epsilon_x = 950$ nm and $\gamma \epsilon_x = 30$ nm. The Twiss parameters $\beta_x$ and $\beta_y$ along the BDS were calculated in MADX [35] and are shown in Fig. 3.1.

MADX with integrated PTC (Polymorphic Transfer Code) [36] is used, which allows to produce the map of coefficients $\{X_{ijklmp}\}$ as defined in Eq. (2.36) up to a desired order. These are the input for another code written in Python, MAPCLASS [37–39] that estimates the beam parameters at the IP. The vertical and horizontal beam sizes at the IP, calculated with MAPCLASS, are shown in Figs. 3.2 and 3.3. One can notice a large contribution to the beam size coming from the chromatic terms, almost 5 nm from 2$^{nd}$ order terms in the horizontal plane. MAPCLASS calculates the beam size order by order, up to the 8$^{th}$ order, giving the overview of the different contributions to the beam size. To reduce the effect of different nonlinearities one has to aim to get the beam size as close as possible to the linear beam size. For that
Distance between FD and IP, $L^* = 6 \text{ m}$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>BDS length [m]</td>
<td>1949</td>
</tr>
<tr>
<td>FFS length [m]</td>
<td>770</td>
</tr>
<tr>
<td>Normalized emittance (end of linacs) $\epsilon_{n,x}/\epsilon_{n,y}$ [nm]</td>
<td>920/20</td>
</tr>
<tr>
<td>Normalized emittance (IP) $\epsilon_{n,x}/\epsilon_{n,y}$ [nm]</td>
<td>950/30</td>
</tr>
<tr>
<td>Beta function (IP) $\beta^<em>_x/\beta^</em>_y$ [mm]/[mm]</td>
<td>8/0.1</td>
</tr>
<tr>
<td>IP beam size $\sigma^<em>_x/\sigma^</em>_y$ [mm]</td>
<td>144/2.9</td>
</tr>
<tr>
<td>Bunch length $\sigma_z$ [$\mu$m]</td>
<td>70</td>
</tr>
<tr>
<td>RMS energy spread $\delta_p$ [%]</td>
<td>0.35</td>
</tr>
<tr>
<td>Number of particles in one bunch $N$ [$\times 10^9$]</td>
<td>5.2</td>
</tr>
<tr>
<td>Number of bunches in one train $N_b$</td>
<td>352</td>
</tr>
<tr>
<td>Repetition rate $f$ [Hz]</td>
<td>50</td>
</tr>
<tr>
<td>Total luminosity $\mathcal{L}$ [$10^{34} \text{cm}^2\text{s}^{-1}$]</td>
<td>1.5</td>
</tr>
<tr>
<td>Peak luminosity $\mathcal{L}_{1%}$ [$10^{34} \text{cm}^2\text{s}^{-1}$]</td>
<td>0.9</td>
</tr>
</tbody>
</table>

Table 3.1: CLIC 380 GeV design parameters

Figure 3.1: $\beta_x$, $\beta_y$ and $D_x$ calculated for the CLIC 380 GeV BDS nominal optics.
purpose, the local chromaticity correction scheme with sextupoles and octupoles is
implemented.

3.2 Aperture calculations for the nominal optics

The aperture is the transverse size of the inner part of the magnet (invisible) for the
beam. Depending on the exact definition, it can be radius or diameter. In this thesis,
the radius is used as an aperture.

The bunch particles should not interact with the beam pipe either directly or
indirectly, and the aperture has to take into account the beam size, possible offsets
of the beam and magnets, and wakefields. The aperture has to be large enough to
mitigate the single bunch wakefield effects imposed in the beam pipe. Studies of the
wakefields for the CLIC 500 GeV stage were conducted in \[40\] and the results obtained
there can be scaled for the case of CLIC 380 GeV. Following this, the minimum
aperture, which allows neglecting the wakefield effects, is 15 mm.

The beam size is to define the aperture of the beamline and is:

\[
\sigma_x = \sqrt{\epsilon_x \beta_x + (D_x \delta_p)^2},
\]

\[
\sigma_y = \sqrt{\epsilon_y \beta_y}. \tag{3.2}
\]

Here, no vertical dispersion is assumed.

The linear approximation for the beam size gives a satisfactory precision for the
aperture calculations as the nonlinear components are small compared to the value
of the minimum required aperture. For given transverse beam sizes \(\sigma_x\) and \(\sigma_y\) the
aperture required to store the beam is calculated as \(\max(14\sigma_x, 55\sigma_y)\) \[3\]. Also, an
extra length of 1.1 mm has to be added to the aperture value, to include the beam

![Figure 3.2: Horizontal beam size versus order.](image1)

![Figure 3.3: Vertical beam size versus order.](image2)
pipe thickness of 1.0 mm and possible misalignment of the magnets of maximum 0.1 mm. In Fig. 3.4 the final results are presented.

![Graph of aperture from beam size and final aperture over distance]

Figure 3.4: The final aperture is the maximum value of the two values given by the wakefields and by the beam size. The minimal value allowed is 15 mm.

### 3.2.1 BDS magnets maximum field

CLIC is composed of normal conducting magnets of different properties, such as dipoles, quadrupoles, sextupoles and others. The maximum magnetic field in these magnets is limited and for CLIC the top value is set to $B_{\text{max}} = 1.5$ T. This value cannot be exceeded and the calculated optics should respect it. The magnetic field created by dipole magnets is homogeneous to some extent and is related to the beam rigidity.

$$B_{\text{dipole}} = \frac{1}{\rho} (B \rho) = \frac{1}{\rho} \frac{p}{e}$$  \hspace{1cm} (3.3)

It can be rewritten in a simpler form that is more useful for the calculations:

$$B_{\text{dipole}}[T] = 3.33 \frac{1}{\rho[m]} [p[GeV/c]]$$  \hspace{1cm} (3.4)

For other magnets, the field is not homogeneous and depends on the position inside the magnet. Following the definitions introduced in Section 2.3.1, the pole tip field for a general normal multipole is:
3.2. APERTURE CALCULATIONS FOR THE NOMINAL OPTICS

\[ B_{ny}^N = B \rho k_n^N \frac{1}{(n-1)!} A^n, \]  

(3.5)

where \( A \) is the aperture.

Tip field for the other magnets in BDS with the simplification as in Eq. (3.4) is:

\[ B_{\text{quadrupole}} = 3.333p[GeV/c]k_1pA, \]  

(3.6)

\[ B_{\text{sextupole}} = 3.333p[GeV/c]k_2 \frac{A^2}{2}, \]  

(3.7)

\[ B_{\text{octupole}} = 3.333p[GeV/c]k_3 \frac{A^3}{6}, \]  

(3.8)

where \( k_1, k_2 \) and \( k_3 \) are the quadrupole, sextupole and octupole strengths respectively.

Apertures calculated in this section are used to estimate the pole tip field for each element of the BDS, see Fig. 3.5.

Figure 3.5: The pole tip field calculated for each element in the BDS. The maximum field is \( B_{\text{max}} = 1.5 \) T.

The dipoles, which are primarily used in the Collimation section and in the FFS, as expected, generate a small field, as they are used as a source of dispersion for collimation and chromaticity correction respectively. Quadrupoles, on the contrary, in most cases have a large pole tip field and the quads in the diagnostic section exceed the \( B_{\text{max}} = 1.5 \) T limit. The other magnets have a field which is below the limit.
To decrease the peak field in the problem quads, one wants to decrease the magnetic strength of the corresponding magnet but keep the same optical properties. The impact of the quadrupole magnet can be described as a transverse velocity kick and simplified as following:

\[ \Delta x' = - \int_{s_0}^{s_0 + L} k_1 x dt \approx -k_1 L x, \]  

(3.9)

for a magnet of a length \( L \) in a hard edge model. The magnetic strength of the magnet can be decreased, if the length of the magnet is increased in a way that \( k_1 L \) is kept constant.

Another interesting result that comes from these calculations is that the quadrupoles that form the Final Doublet have a small pole tip field, below 0.7 T. They are expected to give a large kick to the particles and focus them at the IP in both dimensions. Initially, their length was chosen large: \( QF1 \) is around 5.59 m long and \( QD0 \) is 4.69 m long in the current design. As it was shown, their field is far from the maximum and it allows to optimize the FFS optics by decreasing the length of both magnets and increasing their gradient. Modification of the length of the FD quads would have a great impact on the local chromaticity correction scheme, which would lead to the need of rematching of the whole set of the magnets in the FFS, but can save space before the IP.

### 3.3 Reducing the IP vertical beta function \( \beta_y^* \)

In this section, the calculations for the 380 GeV optics are done. In the current design 380 GeV beamline is aligned with linac, and has increased bending angles of some dipoles in the BDS.

#### 3.3.1 Matching the Twiss functions

In the nominal design, as it is shown in Tab. 3.1, the IP vertical beta function is chosen to be \( \beta_y^* = 100 \) \( \mu m \). As it was discussed in Section 2.7 one can expect to achieve the highest luminosity when \( \beta_y^* \approx \sigma_z \), so the aim is to achieve the vertical beta of a value 70 \( \mu m \).

Initially, MADX is used to match the beta function to the desired value. Apart from \( \beta_y^* \) other optics parameters were strictly constrained at the IP as well to preserve beam properties. The results of the matching are shown in Tab. 3.2.

The quadrupoles that are located upstream from the local chromaticity correction part of the FFS were used for the matching: \( QD7, QD6C, QD6B, QD6B, QF5B, QF5B, QF5A, QF5A \), such that they do not disrupt the phase advance needed to provide the \(-J\) transformation between pairs of sextupoles and octupoles to mitigate the geometrical aberrations. The Twiss functions calculated for the new optics are shown in Fig. 3.6.
3.3. REDUCING THE IP VERTICAL BETA FUNCTION $\beta_y^*$

<table>
<thead>
<tr>
<th>Nominal</th>
<th>$\beta_x^*$ [mm]</th>
<th>$\beta_y^*$ [μm]</th>
<th>$D_x^*$ [m]</th>
<th>$\alpha_x^*$</th>
<th>$\alpha_y^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reduced $\beta_y^*$, target</td>
<td>8</td>
<td>100</td>
<td>$5 \times 10^{-7}$</td>
<td>-0.005</td>
<td>-0.021</td>
</tr>
<tr>
<td>Reduced $\beta_y^*$, calculated</td>
<td>8</td>
<td>70</td>
<td>$4 \times 10^{-9}$</td>
<td>$-4 \times 10^{-5}$</td>
<td>$-3 \times 10^{-5}$</td>
</tr>
</tbody>
</table>

Table 3.2: Comparison of the linear optics parameters at the IP

Figure 3.6: Horizontal beta function $\beta_x$, vertical beta function $\beta_y$ and horizontal dispersion $D_x$ along the FFS, calculated for optics with $\beta_y^* = 70$ μm.
CHAPTER 3. CLIC 380 GEV OPTICS OPTIMIZATION

Figure 3.7: Horizontal beta function for the beam with relative momentum offset. The final value of $\beta_x^* = 8.6$ mm is for momentum offset of 0.5 % and $\beta_x^* = 5.7$ mm for momentum offset -0.5 %. No offset value is $\beta_x^* = 8$ mm.

Analysis of this optics, when momentum error is present, is shown in Figs. 3.7, 3.8 and 3.9.

3.3.2 Nonlinear matching

The next step is to match the nonlinear magnets, sextupoles, and octupoles, to reduce the nonlinear terms in the beam size, otherwise the vertical beam size might be bigger than the design value by several orders. As it was mentioned before, a tandem of MADX-PTC and Python with MAPCLASS is used to reduce the beam size to the linear value as much as possible. The matching module from MADX uses a Simplex method for the beam size optimization and the beam size, calculated with the help of PTC and MAPCLASS, as the figure of merit.

The beam sizes matched and analyzed up to the 8th order are presented in Figs. 3.10 and 3.11.

The transverse beam size reaches the final value of $\sigma_x^* = 143.66$ nm and $\sigma_y^* = 2.67$ nm. The Oide effect contribution was evaluated to be 0.211 nm using MAPCLASS. Comparing these results with the target value, the horizontal beam size is 0.5% larger, while the vertical has 12% increase compared to the 2.38 nm target value.

It is important to have a realistic beam distribution at the IP, to account for different processes that might contribute to the luminosity, such as the pinch effect. PLACET [41, 42] is used to track the beam, composed of macroparticles, through the BDS to the IP and to provide the beam distribution at the IP. To give PLACET the
3.3. REDUCING THE IP VERTICAL BETA FUNCTION $\beta_y^*$

Figure 3.8: Vertical beta function for the beam with relative momentum offset. The final value of $\beta_y^* = 105 \text{ $\mu$m}$ is for momentum offset of 0.5 % and $\beta_y^* = 94 \text{ $\mu$m}$ for momentum offset -0.5 %. No offset value is $\beta_y^* = 70 \text{ $\mu$m}$.

Figure 3.9: Horizontal dispersion for the beam with relative momentum offset. The final value of $D_x^* = -3.7 \times 10^{-5} \text{ m}$ is for momentum offset of 0.5 % and $D_x^* = -1.1 \times 10^{-5} \text{ m}$ for momentum offset -0.5 %. No offset value is $D_x^* = 4.1 \times 10^{-9} \text{ m}$. 
CHAPTER 3. CLIC 380 GEV OPTICS OPTIMIZATION

Figure 3.10: Horizontal beam size versus order.

Figure 3.11: Vertical beam size versus order.

<table>
<thead>
<tr>
<th></th>
<th>$\sigma_x^*$ [nm]</th>
<th>$\sigma_y^*$ [nm]</th>
<th>Oide effect [nm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>MAPCLASS</td>
<td>143.66</td>
<td>2.67</td>
<td>0.21</td>
</tr>
<tr>
<td>Placet without SR</td>
<td>143.27</td>
<td>2.74</td>
<td></td>
</tr>
<tr>
<td>Placet with SR</td>
<td>145.76</td>
<td>2.75</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.3: IP beam sizes calculated with different software for CLIC 380 GeV with reduced $\beta_y^*$.

initial distributions at the entrance of the BDS, transverse Gaussian beams are used with a 1% full-width flat distribution for energy distribution, as shown in Fig. 3.12.

Calculations in PLACET were done with Synchrotron Radiation (SR) switched on and off. IP beam size obtained with MAPCLASS and PLACET without SR are almost the same. Horizontal beam size is around 0.27% higher for MAPCLASS, compared to PLACET, but vertical beam size is 2.5% smaller, see Tab. 3.3.

When the emission of the synchrotron light is taken into account, the dilution of the horizontal beam size is expected, compared to the MAPCLASS result, which is actually observed.

To track how the beam size changes around the waist, the location scan with PLACET was done by modifying the length of the final drift, see Fig. 3.13. Since horizontal IP beta function is much larger than bunch length, the beam horizontal beam size can be considered constant for the whole bunch. The opposite is observed for the vertical plane as the vertical beam size at the head and tail are different from the beam size at the waist by around 35%.
3.3. REDUCING THE IP VERTICAL BETA FUNCTION $\beta_y$

Figure 3.12: Longitudinal phase space at the entrance of the BDS, 1% full width energy spread is assumed.

Figure 3.13: The Hourglass effect - the bunch particles interact with each other with different transverse beam size which changes significantly along the bunch. The bunch length is 70 $\mu$m.
3.4 Luminosity calculations

The performance of a collider is evaluated by the luminosity provided at the IP. To calculate the luminosity, the Guinea-Pig [43, 44] is used, which simulates the interaction of 2 beams, taking into account the beamstrahlung emission. The beam is tracked through the whole BDS with PLACET until the IP, producing the beam distribution, which serves as an input for Guinea-Pig. Two interacting beams are assumed identical, with no transverse offset. Obtained results are shown in Table 3.4. Obtained optics gave larger total and peak luminosity and also larger ratio \( \frac{L_{1%}}{L_{\text{total}}} \), compared to the nominal optics with \( \beta_y = 100 \) μm.

The effect of the waist shift was calculated for 380 GeV case, where one would expect a luminosity growth when the IP is located off the waist. A waist scan with PLACET and Guinea-Pig, presented in Fig. 3.14 for peak luminosity and in Fig. 3.15 for total luminosity, show a luminosity increase of around 10 % when the waist is shifted by around 50 μm from its original position. Compared to the nominal optics at the optimal waist, the total luminosity was increased by around 4.5 % and peak luminosity by around 4 %.

This effect is totally related to the beam-beam effects, as at zero shift the beam is at its waist. In general, the luminosity calculated at the IP for a smaller \( \beta_y \) does not have to necessarily be larger than for the larger beta, and one should always check the location of the waist for higher luminosity.

3.5 Energy Bandwidth

The magnetic rigidity of the beam line is defined as following \( B\rho = \frac{q}{p} \), where the left part is the machine property and the right part is the particle beam property. When these values are matched to one another, the reference particle, with the nominal momentum, follows the reference orbit along the beam line and, ideally, the beam should produce the highest luminosity at the IP. But if one changes the nominal beam energy, the beam changes rigidity. This leads to the different beam behavior and to a different luminosity at the IP. The energy bandwidth of the machine describes how the luminosity changes with the relative change of the beam energy.

Since the possible energy mismatch coming from the linac is expected, the analysis of the energy bandwidth for a given optics is an important characteristic of such a
3.5. ENERGY BANDWIDTH

Figure 3.14: The peak luminosity calculated with Guinea-Pig as a function of the waist shift.

Figure 3.15: The total luminosity calculated with Guinea-Pig as a function of the waist shift.
The energy of the beam was varied in the range $\frac{\Delta p}{p} \in [-1\%, 1\%]$. Peak and total luminosities were calculated with Guinea-Pig for both nominal and new optics, see Figs. 3.16 and 3.17 as a function of $\frac{\Delta p}{p}$. Generally, not symmetric behavior is observed, where maximum total luminosity is achieved for beams with $\frac{\Delta p}{p} < 0$ for both new and nominal optics. The maximum value of the peak luminosity is obtained for zero energy shift in the case of new optics and for $\frac{\Delta p}{p} > 0$ in the case of nominal optics.

Comparing 2 optics, it is obvious that the energy bandwidth is smaller for the new optics with $\beta_y^* = 70$ $\mu$m, compared to the nominal one. This reduction is arising, mainly, from the vertical beam size strong dependence on the momentum offset, see Fig. 3.18, but the horizontal beam size dependence is also observed. Although the chromaticity and the second order geometrical aberrations are corrected, higher-order dispersion adds a large contribution to the beam size for a large momentum deviation. Another reason is a large chromatic variation of the beta function (see Fig. 3.7 and 3.8) along FFS, which affects greatly the preservation of the $-J$ transformation between septupole pairs: geometrical aberrations have to be reduced as much as possible for the beam with momentum offset.

One can compare the obtained beam size bandwidth with the one for a nominal optics (Fig. 3.19), where the vertical beam size has larger bandwidth. The horizontal beam size also depends stronger on the momentum offset for the reduced $\beta_y^*$ optics, the main factor that reduces the energy bandwidth is small vertical beam size bandwidth.

Although the calculated IP beam size is smaller for the optics with reduced $\beta_y^*$, it increases for the energy deviated beams, resulting in smaller energy bandwidth compared with the nominal optics. A way of optimization of the energy bandwidth
3.5. ENERGY BANDWIDTH

Figure 3.17: CLIC 380 GeV energy bandwidth for total luminosity.

Figure 3.18: Relative variation of the IP beam size as a function of the beam momentum offset for the new optics.
Figure 3.19: Relative variation of the IP beam size as a function of the beam momentum offset for the nominal optics.

is described in [45]: one has to find the distribution of the additional sextupoles in FFS that optimizes the beam cross section for a given range of beam momentum offset. Further improvements in the optics for CLIC 380 GeV is the optimization of the sextupoles to get the highest possible energy bandwidth.
Chapter 4

Summary and conclusions

In this thesis, the analysis and optimization of the magnetic lattice for Final Focus System of CLIC 380 GeV were performed. Initially, nominal optics was examined for possible improvements. Pole tip field calculations were performed and obtained results showed that magnetic field in FD is far from 1.5 T limit and in general is even smaller than the pole tip field in most of the quadrupoles in Diagnostic and Collimation sections. It indicates that in current design the quadrupoles in FD are too long and can be shortened in order to optimize the chromaticity and the allocated space.

In the nominal optics design, vertical beta function at the IP is 100 $\mu$m, in this thesis, it was reduced to 70 $\mu$m to have the same value as the bunch length. For such a small value of the vertical beta function, Hourglass effect is observed, when the beam size at the head and tail differs significantly from the value at the waist. Since modification of the beta function along the FFS disrupts the conditions established to mitigate aberrations, the optimization of the beam size at the IP was made. The target was to get the beam size as close as possible to the linear value, given by the beta function, using available sextupoles and octupoles to minimize the high order beam size contributions. After the optimization with MADX and MAPCLASS, the horizontal beam size of 143.66 nm was obtained, which is 0.5% larger than the linear value and 2.67 nm vertical beam size, which is 12% larger than the corresponding linear value. Beam tracked with PLACET through the BDS gave more or less the same IP beam size without synchrotron radiation. With SR taken into account in PLACET, horizontal beam size increased to 145.76 nm, vertical beam size did not change much and is 2.75 nm.

Luminosity calculations were performed with GUINEA-PIG for the new optics. The increase of both total and peak luminosities at the waist is around 2% and 3% respectively, reaching $\mathcal{L}_{\text{total}} = 1.66 \times 10^{34} \text{cm}^{-2}\text{s}^{-1}$ and $\mathcal{L}_{1\%} = 0.96 \times 10^{34} \text{cm}^{-2}\text{s}^{-1}$. Pinch effect is strong for such small beams and greatly affects the collider performance: each of the beams is focused upstream the IP, meaning that expected waist position does not correspond to the actual waist and to the highest luminosity. The waist scan proved that it is true and that maximum luminosity is obtained at 50 $\mu$m distance from the initial waist location. Luminosities obtained for such a so-called “optimal
waist” are $L_{\text{total}} = 1.83 \times 10^{34} \text{cm}^2\text{s}^{-1}$ and $L_{1\%} = 1.03 \times 10^{34} \text{cm}^2\text{s}^{-1}$ which is around 10% and 7% gain for total and peak luminosities respectively. Comparing to the values at optimal waist for nominal optics, new optics give 4.5 % and 4 % gain for total and peak luminosities respectively.

Furthermore, the energy bandwidth was analyzed for such an optics and it showed to be smaller than for the nominal optics. The main reason for this is decreased vertical beam size bandwidth in the new FFS design. Horizontal beam size bandwidth decreased as well, but it is not such a significant change compared to the vertical plane. To increase the energy bandwidth, the additional sextupoles presented in FFS have to redistributed and optimized to decrease the aberrations for the beams with momentum offsets.

In the frame of this thesis, CLIC 380 GeV FFS was optimized to produce larger luminosity. Further options to optimize the lattice in the future has been identified in this thesis, such as decreasing the length of the magnets in FD and optimization of the energy bandwidth.
Bibliography


