Experimental check of precise predictions of QCD using \( \pi^+ K^- \), \( K^+ \pi^- \), and \( \pi^+ \pi^- \) atoms

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SPSC, October 2018
1. Experimental check of QCD using $\pi^+ K^-$, $\pi^- K^+$ and $\pi^+ \pi^-$ atoms.

2. Long-lived $\pi^+ \pi^-$ atom lifetime first measurement and possibility to check of QCD predictions.


4. Proton-antiproton pair analysis as the new physical method to investigate the particle production in the coordinate space.

5. The short-lived $\pi^+ \pi^-$ atom lifetime measurement.

6. High precision measurement of the multiple scattering in Be, Ti, Ni and Pt.
The QCD Lagrangians use the $SU(3)_L \times SU(3)_R$ and $SU(2)_L \times SU(2)_R$ chiral symmetry breaking.

$\mathcal{L}(u,d,s) = \mathcal{L}(3) = \mathcal{L}_{\text{sym}}(3) + \mathcal{L}_{\text{sym.br.}}(3)$

$\mathcal{L}(u,d) = \mathcal{L}(2) = \mathcal{L}_{\text{sym}}(2) + \mathcal{L}_{\text{sym.br.}}(2)$

$\mathcal{L}_{\text{sym.br.}}$ is proportional to $m_q$

$e^+e^- \rightarrow \text{hadrons}$

QCD provides cross sections with 1% precision

1. Perturbation theory is working at high momentum transfer $Q$.
2. Unitarity condition.

At large $Q$, contribution of $\mathcal{L}_{\text{sym.br.}}$ to the cross section is proportional to $1/Q^4$. Therefore these experiments checked only the $\mathcal{L}_{\text{sym}}$ prediction precision.

To check the total $\mathcal{L}(3)$ Lagrangian predictions, we must study the low momentum transfer $Q$ processes.

**Tools:** Lattice calculations and Chiral Perturbation Theory (ChPT)
Lattice----- $\mathcal{L}(3)$, $\mathcal{L}(2)$
ChPT------Effective Lagrangians.
### Experiment

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Detected atomic pairs ($n_A$)</th>
<th>$\tau$ ($10^{-15}$ sec)</th>
<th>$a^- = \frac{1}{3} (a_{1/2} - a_{3/2})$</th>
<th>Average error</th>
</tr>
</thead>
<tbody>
<tr>
<td>DIRAC</td>
<td>$349 \pm 61$(stat)$\pm 9$(syst) = $349 \pm 62$(tot) (5.6$\sigma$)</td>
<td>$5.5 \pm 5.0$</td>
<td>$0.072 \pm 0.031 - 0.020$</td>
<td>34%</td>
</tr>
</tbody>
</table>

### Theory

<table>
<thead>
<tr>
<th>Method</th>
<th>$\alpha^-$</th>
<th>$\alpha^-$</th>
<th>$\alpha^-$</th>
<th>$\alpha^-$</th>
<th>$\alpha^-$</th>
</tr>
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<tbody>
<tr>
<td>Roy-Steiner equations</td>
<td>0.090$\pm$0.005</td>
<td>0.081</td>
<td>0.077</td>
<td>0.077</td>
<td>0.10</td>
</tr>
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<td>Lattice calculations</td>
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<td>0.077</td>
<td>0.077</td>
<td>0.10</td>
</tr>
</tbody>
</table>

### Method

- **Roy-Steiner equations**
- **Lattice calculations**
- **ChPT, two loops**
The DIRAC collaboration Phys.Lett.(2015) observed $436\pm 61$ pion pairs from the long-lived ($\tau \geq 1 \times 10^{-11}$ sec) $\pi^+\pi^-$ atom breakup in Pt foil (Phys.Lett.(2015)).

The short-lived atoms lifetime measurement allowed to evaluate $\pi\pi$ scattering length combination $a_0 - a_2$. The study of the long-lived atoms will allow to measure the Lamb shift depending on another $\pi\pi$ scattering length combination: $2a_0 + a_2$ and to evaluate the $a_0, a_2$ separately.

At present time:

- $a_0$ precision is 6% (experiment), 4-10% (Lattice), 2.3% (ChPT)
- $a_2$ precision is 22% (experiment), 1% (Lattice), 2.3% (ChPT)
- $a_0 - a_2$ precision is $\approx 4$% (experiment), 1.5% (ChPT)
**π⁺π⁻ atom lifetime**

**π⁺π⁻** atom (pionium) is a hydrogen-like atom consisting of **π⁺** and **π⁻** mesons:

\[ E_B = -1.86 \text{ keV}, \quad r_B = 387 \text{ fm}, \quad p_B \approx 0.5 \text{ MeV/c} \]

The **π⁺π⁻** atom lifetime is dominated by the decay into **π⁰π⁰** mesons:

\[
\Gamma = \frac{1}{\tau} = \Gamma_{2\pi^0} + \Gamma_{2\gamma}, \quad \frac{\Gamma_{2\gamma}}{\Gamma_{2\pi^0}} \approx 4 \times 10^{-3}
\]

\[
\tau_{1s} = (2.9 \pm 0.1) \times 10^{-15} \text{ s}
\]

\[
\Gamma_{ns \rightarrow 2\pi^0} = R |\psi_{ns}(0)|^2 |a_0 - a_2|^2
\]

\[ a_0 \text{ and } a_2 \text{ are the } \pi\pi \text{ S-wave scattering lengths for isospin } I=0 \text{ and } I=2. \]

\[ \psi_{nl}(0) \begin{cases} 
\neq 0 \text{ for } l = 0 & \text{A}_{2\pi}(1s, 2s, \ldots, ns) \rightarrow \pi^0\pi^0 \\
= 0 \text{ for } l \neq 0 & \text{A}_{2\pi}(np) \gamma \rightarrow \text{A}_{2\pi}(1s, 2s, \ldots (n-1)s) \rightarrow \pi^0\pi^0 
\end{cases} \]

The **np** state lifetime depends on the transition **np → 1s, 2s, \ldots, (n-1)s** probability. This probability is about 3 orders of magnitude less than for **ns → π⁰π⁰**.
The $\pi^+\pi^-$ atoms production in Be target

Fig. 1 Method to observe long-lived $A_{2\pi}^L$ by means of a breakup foil ($Pt$). Most (70%) of the produced $\pi^+\pi^-$ atoms decay and 6% are ionized in the $Be$ target. 6% are long-lived and 18% are short-lived atoms.

$N_A^L \approx 0.07N_A, N_A^L \approx 0.05N_A$
Experimental $|Q_L|$ distributions of $\pi^+\pi^-$ pairs

a) The experimental distribution (points with statistical error) and the simulated background (solid line).

b) The experimental distribution after background subtraction (points with statistical error) and the simulated distribution of atomic pairs (dot-dashed line).

The fit procedure has been applied to the 2-dimensional ($|Q_L|$, $Q_T$) distribution.
### Atomic states population for \( n=8 \)

<table>
<thead>
<tr>
<th>( n )</th>
<th>( l )</th>
<th>( m )</th>
<th>( P )</th>
<th>( \Sigma_m )</th>
<th>( \Sigma^L_{l,m} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.008</td>
<td>0.008</td>
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</tr>
<tr>
<td>1</td>
<td>-1 , 1</td>
<td>2 × 0.0068</td>
<td>0.014</td>
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<td></td>
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<tr>
<td>2</td>
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<td>2 × 0.0063</td>
<td>0.016</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>0.0038</td>
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<tr>
<td>3</td>
<td>-3 , 3</td>
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<td>0.019</td>
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<td></td>
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<tr>
<td></td>
<td>-1 , 1</td>
<td>2 × 0.0032</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>4</td>
<td>-4 , 4</td>
<td>2 × 0.0058</td>
<td>0.020</td>
<td></td>
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<tr>
<td></td>
<td>-2 , 2</td>
<td>2 × 0.0028</td>
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<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>0.0023</td>
<td></td>
<td></td>
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<tr>
<td>8</td>
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<td>2 × 0.0056</td>
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<tr>
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<td>-3 , 3</td>
<td>2 × 0.0025</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>-1 , 1</td>
<td>2 × 0.0019</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>6</td>
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<td></td>
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<tr>
<td></td>
<td>-4 , 4</td>
<td>2 × 0.0023</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-2 , 2</td>
<td>2 × 0.0016</td>
<td></td>
<td></td>
<td></td>
</tr>
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<td></td>
<td>0</td>
<td>0.0015</td>
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</tr>
<tr>
<td>7</td>
<td>-7 , 7</td>
<td>2 × 0.0051</td>
<td>0.020</td>
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<tr>
<td></td>
<td>-5 , 5</td>
<td>2 × 0.0021</td>
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</tr>
<tr>
<td></td>
<td>-3 , 3</td>
<td>2 × 0.0014</td>
<td></td>
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<td></td>
</tr>
<tr>
<td></td>
<td>-1 , 1</td>
<td>2 × 0.0012</td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>
Populations $P_n^L$ of long-lived states $A_{2\pi}^L$ versus $n$, summed over $l$ and $m$, at the exit of the Be target (●) and at the Pt foil entry (■).

Summed populations $\sum P_n^L$ of all long-lived atomic states at the exit of the Be target as a function of $n$ used for “tail” estimation. For each $n$, the two upper curves show the sum of state populations for the given $n$ plus different “tail” estimations calculated from populations for $n$ and $n-1$ (exponential “tail” – ●, hyperbolic “tail” – ▲). The lower curve (■) presents the sum of the population for the given $n$ plus the population for $n+1$ instead of the “tail”.

10/15/2018
Probability $P_{br}^{tot}(Pt)$ calculated as a function of $\alpha$. The horizontal lines correspond to the measured value $P_{br}^{tot}(Pt) = 0.0257^{+0.0097}_{-0.0036}\times 10^{-11}$, together with the total errors. The value $\alpha = 1$, which corresponds to pure QED calculations, is within the error band of the measurement.
Lifetime of long-lived $\pi^+\pi^-$ atoms

Number of atoms: generated on Be target $N_A = 16960 \pm 290|_{\text{tot}}$

Number of atomic pairs after Pt foil: $n_A = 436^{+157}_{-61}|_{\text{tot}}$

The lifetime of the long-lived atom in 2p state is:

$$\tau_{2p} = 0.45^{+1.08}_{-0.30}|_{\text{tot}} 10^{-11}\text{s (1)}, \quad \tau_{2p} = 0.22^{+1.42}_{-0.18}|_{\text{tot}} 10^{-11}\text{s (2)}$$

QED: $\tau_{2p} = 1.17 \times 10^{-11}\text{s}$

The measured ground state lifetime is: $\tau_{1s} = 3.15^{+0.28}_{-0.26}|_{\text{tot}} \times 10^{-15}\text{s}$

The 90% of the long-lived atoms have decay length in l.s. from 40 cm. up to 140 cm. It opens the possibility to measure the Lamb shift and $\pi\pi$ scattering lengths. The experimental results were presented as section report on the Rochester 2018, submitted as CERN preprint and in
For charged pairs from short-lived sources and with small relative momenta $Q$, Coulomb final state interaction has to be taken into account. This interaction increases the production yield of the free pairs with $Q$ decreasing and creates atoms.

There is a precise ratio between the number of produced Coulomb pairs ($N_C$) with small $Q$ and the number of atoms ($N_A$) produced simultaneously with Coulomb pairs:

$$N_A = K(Q_0)N_C(Q \leq Q_0), \frac{\delta K(Q_0)}{K(Q_0)} \leq 10^{-2}$$

$n_A$ - atomic pairs number, $P_{br} = \frac{n_A}{N_A}$
The $A_{2K}$ lifetime is strongly reduced by strong interaction (OBE, scalar meson $f_0$ and $a_0$) as compared to the annihilation of a purely Coulomb-bound system ($K^+K^-$).

<table>
<thead>
<tr>
<th>$\tau (A_{2K} \rightarrow \pi\pi,\pi\eta)$</th>
<th>K$^+$K$^-$ interaction</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1.2 \times 10^{-18}$ s [1]</td>
<td>Coulomb-bound</td>
</tr>
<tr>
<td>$8.5 \times 10^{-18}$ s [3]</td>
<td>momentum dependent potential</td>
</tr>
<tr>
<td>$3.2 \times 10^{-18}$ s [2]</td>
<td>+ one-boson exchange (OBE)</td>
</tr>
<tr>
<td>$1.1 \times 10^{-18}$ s [2]</td>
<td>+ $f'_0$ (I=0) + $\pi\eta$-channel (I=1)</td>
</tr>
<tr>
<td>$2.2 \times 10^{-18}$ s [4]</td>
<td>ChPT</td>
</tr>
</tbody>
</table>

References:  
$K^+K^-$ Coulomb pairs.

30% $K^+K^-$ pairs

50% $K^+K^-$ pairs

70% $K^+K^-$ pairs
Predicted number of $K^+K^-$ pairs with $Q_t < 4$ MeV/c and $Q_t < 6$ MeV/c according to fits of $Q_t$ distributions of given samples

<table>
<thead>
<tr>
<th></th>
<th>$Q_t &lt; 4$ MeV/c</th>
<th>$Q_t &lt; 6$ MeV/c</th>
</tr>
</thead>
<tbody>
<tr>
<td>k70:</td>
<td>13906</td>
<td>31457</td>
</tr>
<tr>
<td>k50:</td>
<td>12666</td>
<td>28653</td>
</tr>
<tr>
<td>k30:</td>
<td>14834</td>
<td>33556</td>
</tr>
<tr>
<td>average:</td>
<td>13802</td>
<td>31222</td>
</tr>
</tbody>
</table>

k70, k50, k30 – samples with ratio of at least 70%, 50%, 30% of $K^+K^-$ pairs in individual momentum and time intervals
Coulomb correlations

<table>
<thead>
<tr>
<th>Atom</th>
<th>Borh radius $a_B$ [fm]</th>
<th>Resonance $cτ$ [fm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^+\pi^-$</td>
<td>387</td>
<td>$\omega(782)$ 23</td>
</tr>
<tr>
<td>$\pi K$</td>
<td>248</td>
<td>$\omega(782) + \phi(1020)$</td>
</tr>
<tr>
<td>$K^+K^-$</td>
<td>109</td>
<td>$\phi(1020)$ 46</td>
</tr>
<tr>
<td>$p\bar{p}$</td>
<td>58</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Z</th>
<th>A</th>
<th>Nublear radius [fm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Be</td>
<td>04</td>
<td>9.012 2.56</td>
</tr>
<tr>
<td>Ni</td>
<td>28</td>
<td>58.69 4.78</td>
</tr>
<tr>
<td>Pt</td>
<td>78</td>
<td>195.08 7.13</td>
</tr>
</tbody>
</table>

Coulomb correlation with account of size of pair production region $r^*$

$$A_c(r^*, a_B) = A_c(0) \left[1 - \frac{2r^*}{a_B} + \cdots \right], \quad A_c(0) \sim \frac{1}{q}$$

Point-like Coulomb correlation
Number of $p$-antiproton pairs

$p\bar{p}$ pairs
(total 21000 events)
1. In April 2019 the theoretical investigation of KK pairs and KK atoms production will be finished.
2. In June 2019 the preliminary measurement of the KK atoms number generated simultaneously with detected KK pairs will be evaluated.
3. In October 2019 the dedicated paper will be submitted.
4. The experimental conditions needed for the KK atoms lifetime measurement on SPS and LHC will be formulated.
5. The proton-antiproton Coulomb pairs investigation will be done in June 2019.
The $S$-wave $\pi K$ scattering lengths $a_{1/2}$ and $a_{3/2}$ in the chiral symmetry world are zero. Therefore the scattering length values $a_{1/2}$ and $a_{3/2}$ are very sensitive to the $\mathcal{L}_{\text{sym.br.}}(3)$.

For Lattice QCD the $\pi K$ interaction at threshold is a relatively simple process. It gives $\pi K$ scattering length values with an average precision of 5%. This precision will be improved in the near future.

There is only one experimental data: DIRAC collaboration observed $349 \pm 62$ $\pi K$ atomic pairs (Phys.Rev.Lett. 2016) and measured $|a_{1/2}-a_{3/2}|$ with an average precision of 34% (Phys.Rev.D 2017).
Table 1: Population $P$ of atomic states with the quantum numbers $n$, $l$ and $m$ at the exit of the 103 $\mu$m thick $Be$ target. The calculations are performed for the average atom momentum $4.44$ GeV/c and the ground state lifetime $\tau = 3.15 \cdot 10^{-15}$ s. $\Sigma_m$ is the population summed over the quantum number $m$, and $\Sigma_{l,m}$ the long-lived state population summed over $l$ and $m$. All numbers are given in % of the total number $N_A$ of produced atoms.
Table 2: Summed populations of long-lived atomic states versus n given in % of the total number of produced $A_{2\pi}$ in the Be target. The values are calculated in approach 1 (A1) and minimum/maximum values in approach 2 (A2$_{\text{min}}$/A2$_{\text{max}}$).

<table>
<thead>
<tr>
<th>n</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>$\sum n \leq 8$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>2.38</td>
<td>1.56</td>
<td>0.85</td>
<td>0.49</td>
<td>0.30</td>
<td>0.19</td>
<td>0.12</td>
<td>5.91</td>
</tr>
<tr>
<td>A2$_{\text{min}}$</td>
<td>2.46</td>
<td>1.51</td>
<td>0.81</td>
<td>0.46</td>
<td>0.27</td>
<td>0.16</td>
<td>0.08</td>
<td>5.75</td>
</tr>
<tr>
<td>A2$_{\text{max}}$</td>
<td>2.46</td>
<td>1.54</td>
<td>0.91</td>
<td>0.67</td>
<td>0.64</td>
<td>0.73</td>
<td>0.92</td>
<td>7.87</td>
</tr>
</tbody>
</table>

Table 3: $P_n$ (in %) is the population of long-lived atomic states versus n (summed over $l$ and $m$) at the entry in the Pt foil. $P_{\text{br}}(np)$ is the breakup probability of the $A_{2\pi} np$ states in the 2.1μm thick Pt foil. The values are calculated in approach 1 for the average atom momentum 4.44 GeV/c and the ground state lifetime $\tau = 3.15 \cdot 10^{-15}$ s.

<table>
<thead>
<tr>
<th>n</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_n$</td>
<td>0.48</td>
<td>1.10</td>
<td>0.76</td>
<td>0.47</td>
<td>0.30</td>
<td>0.19</td>
</tr>
<tr>
<td>$P_{\text{br}}(np)$</td>
<td>0.763</td>
<td>0.933</td>
<td>0.978</td>
<td>0.991</td>
<td>0.996</td>
<td>0.998</td>
</tr>
</tbody>
</table>
K\(^+\)K\(^-\) atoms ionization probability

K\(^+\)K\(^-\) atoms Lorentz factor is \(\gamma = 18\)
DIRAC future Experimental setup

- Resonators
- Forward Detectors
- Pt foil
- Collimator
- Proton beam
- Secondaries
Energy splitting measurement

For Coulomb potential, \( E \) depends only on \( n \):

\[
\begin{align*}
\Delta^{\text{vac}}_{2s-2p} & = -0.111 \text{eV} \\
\Delta^{\text{str}}_{2s-2p} & = -0.47 \pm 0.01 \text{eV} \\
\Delta^{\text{em}}_{2s-2p} & = -0.012 \text{eV}
\end{align*}
\]

\( \Delta^{\text{vac}}_{2s-2p} \) can be calculated with relative precision \( \approx 10^{-5} \) (S. Karshtenbom)

Notation:

\[
E_{2s} - E_{2p} = \Delta_{2s-2p}
\]

\[
\Rightarrow \Delta^{\text{vac+str+em}}_{2s-2p} = -0.59 \pm 0.01 \text{eV}
\]

\[
\Delta^{\text{str}}_{2s-2p} = -\frac{\alpha^3 m_\pi}{8} \frac{1}{6} (2a_0 + a_2) + \cdots
\]


\[
\Delta^{\text{str}}_{ns-np} = -\frac{\Delta^{\text{str}}_{2s-2p}}{n^3} \cdot 8
\]

CONCLUSION: one parameter \( 2a_0 + a_2 \) allows to calculate all \( \Delta^{\text{str}}_{ns-np} \) values

J. Schweizer [PL B (2004)]
Number of atomic pairs

\[ \pi^+ \pi^- \text{ pairs} \]

\[ K^+ K^- \text{ pairs} \]

\[ p^+ \pi^- \text{ pairs} \]
$Q_l$ for $Q_T$ between 0-1 MeV (accidentals subtracted) MeV/c
Distribution of $K^+K^-$ pairs in the RUN 2009 + 2010 over the full pair momentum in laboratory system.

$K^+K^-$ Coulomb pairs signal

$N$

2009 + 2010

Sum 184350
III. The short-lived $\pi^+\pi^-$ atom lifetime measurement

Preliminary results on the short-lived atom lifetime measurement based on all available 2008-2010 data are presented in Fig. 1 and 2.

Fig.1. Distribution over $|Q_L|$ for events, selected with criterion $Q_T < 4$ MeV/c. Fractions of atomic, Coulomb and non-Coulomb pairs were obtained by fitting the distribution over $(|Q_L|,Q_T)$ with criteria: $|Q_L|<15$ MeV/c, $Q_T < 4$ MeV/c.

$N_A=51091. \pm 214.$

$n_A=24226. \pm 444.$

$P_{Br}=0.474 \pm 0.010$
III. The short-lived $\pi^+\pi^-$ atom lifetime measurement

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Fig. 1. Distribution over $|Q_L|$ for events, selected with criterion $Q_T < 4$ MeV/c. Fractions of atomic, Coulomb and non-Coulomb pairs were obtained by fitting the distribution over $(|Q_L|, Q_T)$ with criteria: $|Q_L| < 15$ MeV/c, $Q_T < 4$ MeV/c. 

$N_A = 51091. \pm 214.$

$n_A = 24226. \pm 444.$

$P_{Br} = 0.474 \pm 0.010$

$Q_L$ [MeV/c]