Triviality, perturbation theory and $Z \rightarrow b\bar{b}$

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Abstract

We compute the large- m_t contributions to the $Zb\bar{b}$ vertex in the Standard Model non-perturbatively using both the $1/N_c$ and the $1/N_F$ expansions. Triviality and its consequences for the perturbative result are systematically analyzed. In particular, we point out that the effect of triviality is already important at the two-loop level for a heavy Higgs ($600 \text{ GeV} < M_H < 1 \text{ TeV}$) even when the top mass is below 200 GeV .

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1. Introduction

A peculiar feature of the Standard Model (SM) is the appearance of non-decoupling effects in radiative corrections. This means that certain contributions from heavy particles do not decrease (and sometimes even grow) as their mass increases. The most striking examples of this are the contribution of the top quark and the Higgs boson to the ρ parameter [1] and the $Zb\bar{b}$ vertex [2]. Moreover, we are presently witnessing the test of SM radiative corrections against high-precision experiments, where impressive accuracies are being attained [3]. On the theoretical side, this experimental accomplishment has spurred the quest for more precise calculations with a better control over higher-order contributions. To this end, two-loop electroweak effects are now being obtained and analyzed in a rather systematic way [4]. Given this state of affairs, we think it is important to understand the behavior of the perturbative series itself and assess its validity and limitations. With this idea in mind, we previously studied the dependence of the ρ parameter on the top and the Higgs mass non-perturbatively using a $1/N$ expansion [5]. In the present letter, we shall analyze the leading contribution of the top quark to the $Zb\bar{b}$ vertex. This is of interest since the $Zb\bar{b}$ vertex, besides the ρ parameter, is the only other independent example where an m_t^2 growth is seen in perturbation theory.

The $1/N$ approach is a systematic and controlled expansion of the Schwinger–Dyson equations and is non-perturbative in that it resums an infinite set of Feynman diagrams [6]. Consequently, we can analyze certain non-perturbative properties, such as triviality, that cannot be treated in a perturbative approach. There are a variety of analyses (most noticeably the lattice) leading to the result that the SM is a trivial theory [7]. Triviality, succinctly, means that the SM has a physical scale Λ_{triv} beyond which the theory ceases to make sense so that it becomes an effective theory valid *only* for momentum scales smaller than Λ_{triv} [8]. Using the vertex $Zb\bar{b}$ as an example, our objective here will be to systematically analyze how the finite energy scale Λ_{triv} appears in physical quantities in the SM and to assess its consequences for perturbation theory.

Commonly, radiative effects are studied using perturbation theory. While this is indeed a reasonable approximation for small couplings, non-decoupling effects become more appreciable when these coupling constants are larger and, in an extreme case, even leave the perturbative regime. Perturbation theory leads one to believe that, as these couplings grow within the perturbative domain, more and more loops are to be computed to attain the desired degree of precision in the calculation. The result we find is that for large masses of the Higgs (or the top) this is not true. Because of non-perturbative effects, namely triviality and the corresponding appearance of a physical cutoff, when the Higgs mass becomes heavy (but still lighter than a TeV, say) cutoff effects introduce an intrinsic uncertainty in the perturbative expression that grows very fast with the Higgs mass. Similar conclusions were reached in ref. [9]. This ambiguity can be a sizeable fraction of the two-loop top contribution to the $Zb\bar{b}$ vertex if $M_H \gtrsim 600$ GeV, even when the top mass is smaller than 200 GeV, for instance. At this point the calculation of higher orders does not improve the precision of the results. Clearly, the situation would change if the underlying theory for $\mu \gtrsim \Lambda_{triv}$ were known since, in principle, these cutoff effects would then be precisely calculable.

To consistently study how the triviality of the Higgs system afflicts physical quantities,

one clearly needs to go beyond perturbation theory since otherwise there is no triviality at all. Here, we shall use a large- N approach because it is a very convenient way to implement triviality in the dynamics and still keep the treatment analytical. Using the fact that the Yukawa sector is by itself also trivial [10][11] [12][13], we shall consider non-perturbative large- N calculations (for N_c and N_F) of the $Z\bar{b}b$ vertex in order to gain some understanding of the interplay between triviality and perturbation theory. This will allow us to estimate the effects of triviality for the more interesting case of a heavy Higgs. On the more theoretical side, we are also interested in finding out whether the growth with the top mass seen in perturbation theory in the $Z\bar{b}b$ vertex is bounded due to non-perturbative effects, as happens with the ρ parameter [5]. As we shall see, this is indeed the case.

The first observation is that leading m_t contributions are entirely due to the dynamics of the Yukawa sector of the SM [14]; they, in essence, have nothing to do with the gauge field dynamics even though we always measure them in the theory once it is “gauged”. This is also true for the $Z\bar{b}b$ vertex, as we will now briefly review. The starting SM Lagrangian (with $g = g' = 0$) reads

$$-\mathcal{L} = \bar{\psi}_L \not{\partial} \psi_L + \bar{t}_R \not{\partial} t_R + y_t (\bar{\psi}_L \phi t_R + \text{h.c.}) - \mathcal{L}(\phi) \quad , \quad (1)$$

with $\phi^T = ((v + H + i\chi^0)/\sqrt{2}, -\chi^-)$; $\chi^{\pm,0}$ being the Nambu–Goldstone bosons and $\mathcal{L}(\phi)$ the usual kinetic term and the potential that yields a vacuum expectation value $v/\sqrt{2}$ for the scalar field ϕ ; ψ_L is the top-bottom left-handed doublet, and t_R the right-handed top singlet. We consider the bottom quark as massless whereas the top has a mass $m_t = y_t v/\sqrt{2}$ at tree level. Quantum corrections induce an effective derivative coupling between the Nambu–Goldstone boson and the b quark that is given by

$$-\mathcal{L} = \tau(y_t) \bar{b}_L \gamma^\mu b_L \partial_\mu \frac{\chi^0}{v} + \bar{b} \not{\partial} b + \dots \quad (2)$$

When one gauges the theory by the usual replacement $\partial_\mu \rightarrow D_\mu$ it leads to the following interactions with the Z^0 ,

$$-\mathcal{L} = \tau(y_t) \bar{b}_L \gamma^\mu b_L \left(\partial_\mu \frac{\chi^0}{v} - \frac{g}{2c} Z_\mu \right) + \frac{g}{c} \bar{b} \gamma^\mu \left(-\frac{1}{2} P_L + \frac{1}{3} s^2 \right) b Z_\mu + \dots \quad (3)$$

so that, in the unitary gauge, one obtains

$$-\mathcal{L} = \frac{g}{c} \bar{b} \gamma^\mu [g_V + g_A \gamma_5] b Z_\mu + \dots \quad (4)$$

with

$$g_V = -\frac{(1 + \tau(y_t))}{4} + \frac{s^2}{3} \quad ; \quad g_A = \frac{(1 + \tau(y_t))}{4} \quad . \quad (5)$$

Here, the notation $c \equiv \cos \theta_W$, $s \equiv \sin \theta_W$ is used. A non-vanishing value of $\tau(y_t)$ modifies the ratio of vector to axial couplings of the bottom to the Z^0 with respect to the same ratio for the lighter fermions. This can be measured, for instance, by comparing forward–backward asymmetries for the bottom quark and for the other fermions [15]. In perturbation theory, the leading-order contribution in the expansion with respect to $y_t^2 \equiv 2\sqrt{2}G_F m_t^2$ is

$$\tau(y_t) = -\frac{\sqrt{2}G_F m_t^2}{8\pi^2} \quad . \quad (6)$$

2. Triviality and the $Zb\bar{b}$ vertex in the $1/N_c$ expansion

We shall now compute $\tau(y_t)$ non-perturbatively in y_t^2 . The $1/N_c$ expansion is performed by letting $N_c \rightarrow \infty$ while taking $y_t^2 N_c$ and v^2/N_c to be quantities of order one. To leading order in $1/N_c$, the top mass is not corrected and the diagram of fig. 1 corrects the charged Nambu–Goldstone boson propagator.

fig. 1 *Leading-order correction to the charged Nambu–Goldstone boson propagator in the $1/N_c$ expansion.*

When this quantum effect is taken into account, the effective Lagrangian for the charged Nambu–Goldstone boson reads

$$-\mathcal{L} = Z_+(\partial_\mu \chi^+ - \frac{gv}{2}W_\mu^+)^2 + \dots \quad , \quad \text{where} \quad Z_+ = 1 - \frac{y_t^2 N_c}{(4\pi)^2} \log \frac{m_t^2}{L^2} \quad ; \quad (7)$$

L may be thought of as a momentum cutoff¹ ; W^+ enters eq. (7) in this precise combination because of gauge invariance. The presence of Z_+ implies that the W^+ mass term in the Lagrangian is now $Z_+ g^2 v^2/4$ and consequently, the Fermi constant appearing for instance in μ decay is

$$\left(\sqrt{2}G_F\right)^{-1} = Z_+ v^2 \equiv V^2 \quad (8)$$

This equation expresses how the bare parameter v is related to the physical observable G_F . This is all we need, for now the Yukawa coupling y_t renormalizes (*i.e.* it can be expressed in terms of physical parameters) as

$$y_t^2 = 2\frac{m_t^2}{v^2} = 2\frac{(m_t^2)^{phys}}{v^2} = 2\sqrt{2}G_F(m_t^2)^{phys}Z_+ \equiv (y_t^R)^2 Z_+ \quad . \quad (9)$$

This last equation may be reinterpreted as a running coupling constant

$$y_t^2(\mu') = \frac{y_t^2(\mu)}{1 - y_t^2(\mu)N_c/(4\pi)^2 \log(\mu'^2/\mu^2)} \quad , \quad (10)$$

with $y_t \equiv y_t(\mu' = L)$ and $y_t^R \equiv y_t(\mu = m_t)$. It should be stressed that this large- N_c equation is non-perturbative in the Yukawa coupling.² Here, triviality manifests itself

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¹ $\log(L^2/4\pi) = 2/(4-n) - \gamma_E + 1/2$ in dimensional regularization.

² Usual one-loop renormalization group arguments also lead to an equation that looks like (10), but with $N_c \rightarrow N_c + 3/2$.

through the appearance of a scale Λ_{triv} at which y_t diverges. This scale is to be interpreted as the maximum energy scale up to which the theory, which now becomes an effective theory, is valid. Equation (10) tells us that

$$\Lambda_{triv}^2 = m_t^2 \exp\left(\frac{(4\pi)^2}{2\sqrt{2}G_F m_t^2 N_c}\right) \quad , \quad (11)$$

which is a fantastically large scale for small values of m_t . Imposing the condition that the top mass needs to be substantially below the cutoff of the theory, a triviality mass bound for the top may be obtained in the $1/N_c$ limit. For instance, imposing $m_t/\Lambda_{triv} < 0.4$, as is sometimes done, we obtain $m_t < 3.7 V$, which is consistent with the mass bound we obtain from the $1/N_F$ expansion, $m_t < 3.5 V$ [11],[12], from more sophisticated $1/N$ approaches [13], and also from the lattice [10].

Let us now turn to the actual calculation of $\tau(y_t^R)$. The leading $1/N_c$ contribution is given by fig. 2A,B.

fig. 2A,B *Leading-order contributions to $\tau(y_t^R)$ in $1/N_c$ expansion. Lines with blobs denote the full propagators.*

A straightforward calculation, taking care of the due renormalization, secures

$$\begin{aligned} \tau(y_t) &= \tau(\text{fig. 2A}) + \tau(\text{fig. 2B}) \\ \tau(\text{fig. 2A}) &= -\left(\frac{y_t^R}{4\pi}\right)^2 \int_0^\infty \frac{d\xi}{(1+\xi)^2 f(\xi)} \\ \tau(\text{fig. 2B}) &= -\frac{N_c}{2} \left(\frac{y_t^R}{4\pi}\right)^4 \int_0^\infty d\xi \frac{1 - (1 + \frac{1}{\xi}) \log(1 + \xi)}{\xi (1 + \xi) [f(\xi)]^2} \quad , \end{aligned} \quad (12)$$

where

$$f(\xi) \equiv 1 - 2N_c \left(\frac{y_t^R}{4\pi}\right)^2 \int_0^1 dt \, t \log[1 + \xi (1 - t)] \quad (13)$$

and $\xi = l_E^2/m_t^2$ for a Euclidean momentum l_E routing inside the loop³. Equations. (12) are well behaved in the ultraviolet ($\xi \rightarrow \infty$) and in the infrared ($\xi \rightarrow 0$). However there

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³ In fig. 2B the fermion loop connecting the three Nambu–Goldstone bosons is finite.

is a pole in the integrand at a finite momentum ξ_0 . This fact may be traced back to the appearance of denominators such as the one of eq. (10) and is therefore a *direct* consequence of the triviality of the Yukawa theory. This is how triviality manifests itself in Green functions⁴. If one expands eqs. (12) in a power series in y_t^R , there is no pole and these expressions are well behaved. But, then, there is no triviality in eq. (10) either!

Notice that, in fact, the pole is located at a momentum scale that is beyond the triviality cutoff defined by eqs. (10), (11); i.e. beyond the physical cutoff up to which the effective field theory is valid. In other words, we already “knew” because of eqs. (10), (11) that the integrals in eqs. (12) could not be taken up to infinity but up to $\Lambda \leq \Lambda_{triv}$. This introduces a dependence on Λ since the result will depend on the cutoff procedure. This cutoff dependence on Λ may be understood as follows. When one has accepted the impossibility of removing the cutoff Λ , in general there will be higher-dimensional operators (weighted with the appropriate inverse powers of the cutoff) that are to be added to the initial Lagrangian. Since the SM is an effective theory, presumably there will have to be a more fundamental Lagrangian that sets in at scales $\mu \gtrsim \Lambda$. Integrating out these fundamental degrees of freedom above Λ is what produces the tower of higher-dimensional operators alluded to above. In practice, since we do not know what the underlying theory is for $\mu \gtrsim \Lambda$, this introduces an uncertainty in the result in the form of a cutoff dependence.

Now, what is the magnitude of this cutoff dependence? In the limiting case $g = g' = \lambda = 0$ ⁵, the *maximum* cutoff Λ is given by Λ_{triv} of eq. (11). However, not having any information on the underlying theory for $\mu \gtrsim \Lambda$, we cannot know the right way to implement this physical cutoff Λ . We estimate this uncertainty by choosing $\Lambda/\Lambda_{triv} = 0.6$ and 0.4. The integrals in eqs. (12) are then carried out between 0 and Λ (instead of ∞) and one obtains for τ the curves of fig. 3, in which we also plot the perturbative result for comparison (see below). We only plot in the region $m_t < \Lambda$.

fig. 3 τ versus the top mass to leading order in the $1/N_c$ expansion for the cutoff values $\Lambda/\Lambda_{triv} = 0.4, 0.6$. The two-loop result is extracted from Barbieri et al. and Fleischer et al. in ref. [4].

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⁴ We may also use the location of this pole as the definition of the triviality scale. This would differ from Λ_{triv} obtained in (10) approximately by a factor of $e^{3/2}$. Which scale we use will make no difference in our subsequent discussion.

⁵ g and g' are the SM gauge couplings, and λ is the quartic scalar coupling.

As one can see, the $1/N_c$ curves tend to saturate, and they deviate from the perturbative result in the region of large masses, where their behavior is dominated by cutoff effects.

On the other hand, things go very differently in perturbation theory. One obtains the perturbative series by expanding eqs. (12) in y_t^R . This yields

$$\begin{aligned} \tau(y_t^R) = & -\frac{\alpha_t}{N_c} - \frac{1}{N_c} \sum_{n=0}^{\infty} \alpha_t^{2+n} 2^n \int_0^{\infty} \frac{d\xi}{(1+\xi)^2} \left[\frac{n-2}{2} + \frac{n-1}{2\xi} + \frac{1-n}{2} \left(1 + \frac{1}{\xi}\right)^2 \log(1+\xi) \right] \\ & \times \left[\int_0^1 dt \, t \log(1+\xi(1-t)) \right]^n \end{aligned} \quad (14)$$

where $\alpha_t \equiv N_c(y_t^R/4\pi)^2$. The first few terms are

$$\tau(y_t^R) = -\frac{\alpha_t}{N_c} + 0 + \frac{\alpha_t^3}{4N_c} + \dots \quad (15)$$

The α_t term in eq. (15) of course agrees with the one-loop result. The α_t^2 term fortuitously vanishes, which means that this term is of higher order in the $1/N_c$ expansion. This is in agreement with the calculation of Barbieri et al. in ref. [2], who find no N_c dependence in the two-loop result, and explains why the two-loop curve of fig. 3 tends to deviate from the rest. The α_t^3 term in eq. (15) can be considered as an estimate of the three loop contribution.

Higher-order terms can also be computed easily using eq. (14). For large n one finds the following asymptotic behavior

$$\tau(y_t) \sim \frac{1}{N_c} \sum_n \alpha_t^n C_n, \quad C_n \sim (n+1)! \quad (16)$$

Due to the factorial growth of C_n with n , the perturbative expansion not only has a zero radius of convergence but is also Borel non-summable. This is the consequence of triviality that may be gleaned from the large-order behavior of perturbation theory [16].

We see that no trace of the need for a physical cutoff exists within perturbation theory, since by expanding the denominators in eqs. (12) one misses the pole that exists in the loop momentum. For a small Yukawa coupling, when the perturbative expansion is supposed to work, the physical cutoff given by eq. (11) is fantastically large. Cutoff effects in τ are of $\mathcal{O}(V^2/\Lambda_{triv}^2)$, whose precise numerical value depends on the details of the cutoff procedure, i.e. on the dynamics of the underlying theory that is supposed to take over the SM for $\mu > \Lambda_{triv}$. Notice that the cutoff effects depend on the symmetry breaking scale. This is why non-decoupling effects are interesting for studying triviality-related issues. In an ordinary physical process without non-decoupling effects, it would have appeared, instead of V , the energy scale of the process, which is typically (much) smaller.

Of course, the $\mathcal{O}(V^2/\Lambda_{triv}^2)$ correction is minute and, for all practical purposes, completely irrelevant even for a top mass as large as 250 GeV; this is why one forgets about triviality altogether when computing radiative corrections. Moreover, for a top this heavy

or lighter, one may compute higher orders in perturbation theory and add to the perturbative series in y_t^R the radiative corrections coming from the Higgs boson, for instance. This was first done, at the two-loop level, by Barbieri et al. in ref. [4]. The output of this calculation is that the Higgs contributions eventually grow with the Higgs mass, clearly manifesting non-decoupling. In perturbation theory, one is led to believe that as the Higgs becomes heavier, namely in the range $400 \text{ GeV} < M_H < 1 \text{ TeV}$, one has to keep adding loop corrections in order to obtain a more and more precise answer. However, as the Higgs becomes heavier, the λ coupling becomes larger and its associated triviality scale goes down exponentially as⁶

$$\Lambda_{triv}^2 \approx M_H^2 \exp \left(\frac{4\pi^2 \sqrt{2}}{3M_H^2 G_F} \right) . \quad (17)$$

Clearly, for a sufficiently heavy Higgs (with $M_H < 1 \text{ TeV}$, nonetheless), but still $m_t < 200 \text{ GeV}$, the physical cutoff of the theory will no longer be given in terms of the Yukawa, eq. (11), but in terms of the Higgs, eq. (17). Regrettably our simple $1/N_c$ approach does not allow us to calculate non-perturbatively the contribution of the Higgs to the $Z\bar{b}b$ vertex τ , because it is a subleading effect in $1/N_c$. However our previous calculation with the top quark shows that we should expect new denominators to appear in eq. (12), of a form similar to that showed by the function $f(\xi)$ of eq. (13), i.e. producing a pole in eq. (12). We may anticipate that this will be the effect because, as we have seen in the Yukawa system, this is the way triviality modifies perturbation theory, and the Higgs system is also a trivial one.

These new denominators will force us to cutoff the integrals, but now at the scale (17). We may estimate that

$$\tau(m_t, M_H) \sim -\frac{\alpha_t}{N_c} \left(1 - \gamma \frac{m_t^2}{\Lambda_{triv}^2} \right) - \frac{\alpha_t^2}{2N_c^2} \tau^{(2)}(M_H/m_t) + \dots \quad (18)$$

where the ellipses stand for terms with higher powers of α_t that we can safely neglect since, for $m_t < 200 \text{ GeV}$, α_t is small. Here, γ is of order 1 and parametrizes the dependence of τ on the precise cutoff procedure, i.e. on the physics beyond the SM. The $\tau^{(2)}$ term is the two-loop calculation of Barbieri et al. and of Fleischer et al. in ref. [4]. Rearranging a little, one obtains from eq. (18)

$$\tau(m_t, M_H) \sim -\frac{\alpha_t}{N_c} \left\{ 1 + \frac{\alpha_t}{2N_c} \left[\tau^{(2)} - \gamma \left(\frac{4\pi V}{M_H} \right)^2 \exp \left(-\frac{8\pi^2 V^2}{3M_H^2} \right) \right] \right\} + \dots \quad (19)$$

The crucial point here is that in a non-perturbative expression including both the top and the Higgs contributions, there is no way to separate these contributions, so that we need to impose an overall cutoff for the whole expression. In table 1 we present the values of the

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⁶ This equation is the result of the combined information extracted from $1/N$, renormalization group and lattice techniques [17]. It is a trick of Nature that, for $M_H < 1 \text{ TeV}$, it essentially coincides with the result of naively integrating the one-loop perturbative β function for λ .

cutoff contribution to τ , which we call $(\Delta\tau)_{triv}$ and which correspond to the γ -dependent term in eq. (19). We also show the corresponding two-loop perturbative contribution, $\tau^{(2)}$, as extracted from the work of Barbieri et al. and Fleischer et al in ref. [4]. We chose $m_t = 200$ GeV, or $\alpha_t/N_c = 0.0084$. As one can see, even for $\gamma \sim 1$, $(\Delta\tau)_{triv}$ is about 16% of the two-loop perturbative contribution $\tau^{(2)}$ for $M_H = 600$ GeV. It can be even higher if γ turns out to be two or three, or if the Higgs is heavier.

M_H	400 GeV	500 GeV	600 GeV	700 GeV	800 GeV	900 GeV
$\tau^{(2)}$	1.33	1.60	1.95	2.36	2.79	3.23
$(\Delta\tau)_{triv}$	0.003	0.07	0.32	0.75	1.2	1.7

Table 1 *Comparison of the two-loop contribution to τ and the ambiguity due to triviality for various Higgs masses.*

3. $Zb\bar{b}$ vertex in the $1/N_F$ expansion

The large- N_c expansion selects diagrams with closed fermion loops over other types of topologies because these always have the factor N_c out front. However it is possible that other types of diagrams turn out to be numerically more important and that consequently the large- N_c expansion is misleading. In order to get an idea of how much our conclusions depend on the precise $1/N$ expansion used, we shall next present the results of a $1/N_F$ expansion where N_F is now the number of flavors (in the SM, $N_F = 2$). The large- N_F expansion selects loop diagrams with scalars and is somewhat complementary to the large- N_c expansion. It should be straightforward, although much more complicated, to generalize our result to the so-called Veneziano limit, where we let both N_c and N_F go to infinity with the ratio N_c/N_F held fixed [13].

In order to consider the generalization of the SM to an arbitrary number of flavors N_F , one takes the scalar field to be $\phi^T = ((v + H + i\chi^0)/\sqrt{2}, -\chi^1, -\chi^2, \dots, -\chi^{N_F-1})$ and the fermion field to be $\psi_L = (t_L, b_L^1, \dots, b_L^{N_F-1})$. The Lagrangian with only the scalar fields is the $O(2N_F)$ model [18]. The charged Nambu–Goldstone boson may be chosen to be any of the $N_F - 1$ χ 's due to the residual symmetry. We refer the reader to [12] for details. We will again compute to lowest order in the weak gauge couplings but non-perturbatively in the Yukawa coupling. In this case, unlike the large- N_c case, the top propagator is corrected at leading order and reads

$$\left[i\not{p} (A_{R,bare}(p^2) P_R + P_L) + \frac{y_t v}{\sqrt{2}} \right]^{-1}, \quad \text{where} \quad A_{R,bare}(p^2) \equiv 1 - \frac{y_t^2 N_F}{32\pi^2} \log \frac{p^2}{L^2}, \quad (20)$$

where L^2 is a momentum cutoff. Then the Yukawa coupling y_t is renormalized in the following way

$$y_t^2(\mu) = \frac{y_t^2(L)}{1 - y_t^2(L) N_F / (32\pi^2) \log(\mu^2/L^2)}, \quad (21)$$

whereas v remains unrenormalized to this order, i.e. $v = V$ in this case. One can solve eq. (20) for the physical mass and width by looking for a pole in the complex plane in the

form $p^2 = (m_t - i\Gamma_t/2)^2$, as a function of $y_t^2(\mu_0)$. The renormalization scale μ_0 is arbitrary and will not affect any physical results.

Now we can go about computing $\tau(y_t)$. To leading order in $1/N_F$, the only contribution is the one represented in fig. 4.

fig. 4 *Leading-order contributions to $\tau(y_t)$ in the $1/N_F$ expansion. Lines with blobs denote the full propagators.*

A straightforward calculation yields

$$\tau(y_t) = -\frac{y_t^4(\mu)v^2}{2(4\pi)^2} \int_0^{\Lambda^2} \frac{dl^2}{[l^2 A_R(l^2) + y_t^2(\mu)v^2/2]^2} \quad \text{where} \quad A_R(l^2) \equiv 1 - \frac{N_F}{2} \left(\frac{y_t(\mu)}{4\pi} \right)^2 \log \frac{l^2}{\mu^2} \quad (22)$$

Again, this expression for τ is ill-defined unless we introduce a cutoff $\Lambda^2 \lesssim \Lambda_{triv}^2 \equiv \mu^2 \exp(32\pi^2/y_t(\mu)^2 N_F)$, which produces a cutoff dependence of order v^2/Λ^2 analogously to the result of the $1/N_c$ expansion. For a certain arbitrarily chosen μ , this equation gives $\tau(y_t(\mu))$ and since the physical mass is determined by $y_t(\mu)$, one can also obtain $\tau(m_t)$. Nevertheless, since the relationship between $y_t(\mu)$ and m_t is highly non-linear it is not possible to write $\tau(m_t)$ in a simple analytical form and we shall give the result in the plot of fig. 5, again for the cutoff values $\Lambda/\Lambda_{triv} = 0.4, 0.6$. Clearly, only the region $m_t < \Lambda$ is of physical significance; however, in this case with a non-vanishing width, we find it more convenient to use $m_t^2 + \Gamma_t^2/4 < \Lambda^2$ [5], and the $1/N_F$ curves are plotted within this region. One then sees that, although the details of fig. 3 and fig. 5 are clearly different, their gross features are similar. Firstly, one observes clear deviations from the perturbative contribution only in the region where cutoff effects are also large. And secondly, although the perturbative curves would grow on forever with the particle's mass, the $1/N$ curves are bounded in both cases. We notice that these were also the main characteristics of the non-perturbative calculation of the ρ parameter done in ref. [5].

Comparison of $\tau(y_t(\mu))$ in eq. (22) and the result for the ρ parameter obtained in ref [5] leads us to establish the following amusing non-perturbative relation to leading order in $1/N_F$:

$$\tau(m_t) = -\frac{2}{3} (\rho(m_t) - 1) \quad . \quad (23)$$

fig. 5 τ plotted against the physical top mass for the cutoff values $\Lambda/\Lambda_{triv} = 0.4, 0.6$.

In order to compare eq. (22) to the two-loop perturbative result of ref. [4], we can also expand eq. (22) in powers of the top “mass”, defined through the pole of the real part of the propagator, which is $y_t(\mu = m_t)v/\sqrt{2}$, and that we shall call $\hat{m}_t$. Clearly this is not the same as the physical mass (which is defined through the pole of the propagator in the complex plane) but it suffices for our limited purpose of a two-loop comparison. The expansion of eq. (22) in powers of $G_F \hat{m}_t^2$ yields

$$\tau(\hat{m}_t) = -\frac{\sqrt{2}G_F \hat{m}_t^2}{8\pi^2} \left[1 + \frac{N_F \sqrt{2}G_F \hat{m}_t^2}{16\pi^2} + \dots \right]_{N_F=2}. \quad (24)$$

In this case, the $1/N_F$ result does not have the fortuitous cancellation of the $(G_F \hat{m}_t^2)^2$ term, unlike the $1/N_c$ expansion. The perturbative calculations of Barbieri et al. and Fleischer et al. in ref. [4] produce the coefficient $(27 - \pi^2)/3 \simeq 5.7$ instead of N_F in eq. (24), so that the numerical coefficient turns out to be different, but of the same order and with the same sign. The large-order behavior of the perturbative expansion for τ in (22), similar to that of (16), is again consistent with triviality.

Let us consider the case of a heavy Higgs ($400 \text{ GeV} < M_H < 1 \text{ TeV}$) when the top is lighter than 200 GeV . Again, the role of the Higgs will be to force the cutoff of the theory to be below the triviality scale due to the top, so that the effects of new physics may again be estimated to be of $\mathcal{O}(\alpha_t^2 v^2 / \Lambda_{triv}^2)$ as in (18), where Λ_{triv}^2 is given by the Higgs coupling (17). The size of this dependence is the same as the one given in table 1.

4. Discussion

In this work we have used $1/N$ techniques to study how a known non-perturbative property of the SM, namely triviality, modifies results obtained in perturbation theory. We took the $Zb\bar{b}$ vertex as an example.

From figs. 3 and 5 one sees that the most apparent change, with respect to perturbation theory, is the appearance of cutoff effects. We also see that the perturbative growth of τ with the particle’s mass, characteristic of a non-decoupling effect, tends to saturate non-perturbatively and therefore deviates from the perturbative result, but only in the region

where the cutoff effects are large. These cutoff effects are akin to the scaling violations found in lattice studies.

There of course remains the open question of how much the SM ($N_F = 2, N_c = 3$) resembles the two limits we have taken ($N_F = \infty, N_c = 3$ and $N_F = 2, N_c = \infty$). While we cannot claim, and do not claim, that this resemblance be quantitatively good, we think that it is very reasonable to expect that the large- N limits we take will not change the gross features qualitatively. After all, all the large- N limits do is implement triviality in a consistent way. The cutoff ambiguity and other related effects are just natural consequences of triviality.

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