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MPI -Ph 94-17

ne 3425

MPI-Ph/94-17

KANAZAWA-93-07

April 1994

Virasoro Anomaly, Trace Anomaly and the Liouville Action for 2D Gravity

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Abstract

The relation among the 2D general covariance, the Weyl invariance, the Virasoro anomaly and the trace anomaly in a conformal field theory coupled to 2D gravity is investigated in the canonical formalism. Making Polyakov's non-local action for 2D gravity local, we obtain a novel Liouville action which is a local functional of 2D metric fields. Its quantization in the conformal and the light-cone gauges is briefly discussed.

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In this note we shall investigate from somewhat novel point of view the Polyakov string [1] as an example of conformal matter coupled to 2D gravity and give a canonical description of the Virasoro anomaly (VA) [2] and the trace anomaly (TA) by examining the reparametrization invariance and the locality of the action.

We will argue that the conservation of the naive stress tensor defined by an ordering prescription is broken by the VA and the 2D covariance is lost in general. We then construct a counteraction which recovers the covariance without affecting the Weyl symmetry. It will be shown that the counteraction possesses $PSL(2, R) \times PSL(2, R)$ symmetry. This action, however, is nonlocal in the metric variables. We shall see that the Polyakov's LA [1] must be introduced to recover the locality. We thus arrive at a local action of Liouville type, which is neither reparametrization invariant nor symmetric under Weyl rescalings. Instead, it correctly produces the trace anomaly. This allows us to interpret the LA as a gravitational Wess-Zumino-Witten (WZW) term [3] converting VA into TA.* In the recent works [6,7] on the BRST quantization of 2D gravity, we have found the same type of LA. The present analysis will make it clear the origin and the structure of this action.

Since the LA is local in the metric variables, canonical quantization procedure can be applied in a straightforward manner [8]. We will discuss briefly some basic consequences of the quantization to get contact with the results of the light-cone gauge of Knizhnik, Polyakov and Zamolodchikov (KPZ) [9,10] and the conformal gauge of David, Distler and Kawai (DDK) [11,12] (see [13] for a review). The LA obtained below turns out to be somewhat intervening between that of Polyakov's and that of DDK's.

The dynamics of the Polyakov string described by the action

$$S_{\text{str}} = \int d^2x \left(-\frac{1}{2} \sqrt{-g} g^{ab} \partial_a X^\mu \partial_b X_\mu \right) \quad (1)$$

is characterized by the Virasoro constraints

$$\varphi_\pm(x) \equiv \frac{1}{4} (P \pm X')^2 \approx 0, \quad (2)$$

where $P_\mu = -\sqrt{-g} g^{0a} \partial_a X_\mu$ is the canonical momentum conjugate to X^μ .† The

* In refs. [4,5], the relationship between VA and TA has been investigated using differential geometric approach.

† The world-sheet coordinates are denoted by x^a with $x^0 = \tau$ and $x^1 = \sigma$. For any field A , $\dot{A} = \partial_\tau A$ and $A' = \partial_\sigma A$. We will take $-\infty < \sigma < +\infty$ for simplicity.

classical equations of motion for $\varphi_{\pm}$ are given by

$$\dot{\varphi}_{\pm} \mp \lambda^{\pm} \varphi'_{\pm} \mp 2\lambda^{\pm} \varphi_{\pm} = 0 \quad \text{with} \quad \lambda^{\pm} = \frac{\sqrt{-g} \pm g_{01}}{g_{11}}. \quad (3)$$

As a result of (3), the stress tensor

$$T_{ab} = -\frac{2}{\sqrt{-g}} \frac{\delta S_{\text{str}}}{\delta g^{ab}} \quad (4)$$

whose components are given by

$$T_{00} = (\lambda^+)^2 \varphi_+ + (\lambda^-)^2 \varphi_-, \quad T_{01} = T_{10} = \lambda^+ \varphi_+ - \lambda^- \varphi_-, \quad T_{11} = \varphi_+ + \varphi_-, \quad (5)$$

is covariantly conserved. In addition to this T_{ab} is manifestly traceless because of the relation $g_{11}(\lambda^{\pm})^2 \mp 2g_{01}\lambda^{\pm} + g_{00} = 0$.

For the most part of this note we consider the world-sheet metric g_{ab} as classical background fields and only string variables X^{μ} are quantized. When we consider X^{μ} as quantized operators, the Virasoro operators $\varphi_{\pm}$ must suitably be defined by employing some ordering prescription. Here we adopt the free field analog of normal ordering [8]. We first decomposed $P \pm X'$ into “positive” and “negative” frequency parts $(P \pm X')^{(\pm)}$ by

$$(P \pm X')^{(\pm)}(\tau, \sigma) = \int d\sigma' \delta^{(\mp)}(\sigma - \sigma') (P \pm X')(\tau, \sigma') \quad \text{with} \quad \delta^{(\pm)}(\sigma) = \frac{1}{2\pi} \frac{\pm i}{\sigma \pm i\epsilon}.$$

We then put the positive frequency parts to the right of the negative frequency parts in operator products. This defines a normal ordering prescription. This definition is obviously independent of the background metric and coincides to the usual normal ordering in the orthonormal gauge $g_{ab} = \eta_{ab} e^{\phi}$ where X 's reduce to free fields.

The Virasoro operators $\varphi_{\pm}$ thus defined are suffered from the VA. The equal-time commutators between $\varphi_{\pm}$ develop Schwinger terms, i.e.,

$$[\varphi_{\pm}(x), \varphi_{\pm}(y)] = \pm i \{ \varphi_{\pm}(x) + \varphi_{\pm}(y) \} \partial_x \delta(x - y) \mp \frac{iD}{24\pi} \partial_x^3 \delta(x - y), \quad (6)$$

where D is the space-time dimensions. The VA in turn leads to modifications of

the classical equations of motion (3) as follows

$$\dot{\varphi}_{\pm} \mp \lambda^{\pm} \varphi'_{\pm} \mp 2\lambda^{\pm'} \varphi_{\pm} = \pm \kappa_0 \lambda^{\pm'''} \quad \text{with} \quad \kappa_0 = -\frac{D}{24\pi}. \quad (7)$$

Due to the appearance of the nonhomogeneous term in the rhs of (7), the naive stress tensor (4) is not covariantly conserved. Instead, we obtain

$$\nabla_a T^{0a} = \kappa_0 \frac{(\lambda^+ - \lambda^-)'''}{\sqrt{-g}}, \quad \nabla_a T^{1a} = -\kappa_0 \frac{(\lambda^+ \lambda^{+'''} + \lambda^- \lambda^{-'''})}{\sqrt{-g}}, \quad (8)$$

with $T_{ab} = \sqrt{-g} T_{ab}$. This implies the absence of general covariance. The origin of the anomaly (8) is clear. The decomposition of $P \pm X'$ into positive and negative frequency parts introduced in defining “normal ordering prescription” is not generally covariant. As a result, T_{ab} is neither covariant, nor covariantly conserved. In the orthonormal gauge, however, the VA does not imply the breakdown of the reparametrization invariance since in this gauge $\lambda^{\pm} = 1$ and T_{ab} is covariantly conserved.

In general background gauges, the VA inevitably leads to the nonconservation of the stress tensor (4). But this does not necessarily imply the breakdown of the reparametrization invariance. In fact, as we shall see, T_{ab} can be made covariantly conserved by a suitable redefinition

$$T_{ab} \rightarrow \tilde{T}_{ab} = T_{ab} + t_{ab} \quad \text{with} \quad t_{ab} = -\frac{2}{\sqrt{-g}} \frac{\delta S_L}{\delta g^{ab}}, \quad (9)$$

where S_L is a functional of g_{ab} . If we include S_L as a counteraction to (1) the induced stress tensor is given by $\tilde{T}_{ab}$.

To find S_L we first look for solution for t^{ab} in (9) which cancels the VA in (8). This can be achieved by solving the following equations

$$\dot{\phi}_{\pm} \mp \lambda^{\pm} \phi'_{\pm} \mp 2\lambda^{\pm'} \phi_{\pm} = \pm \lambda^{\pm'''}, \quad (10)$$

where $\phi_{\pm}$ are functional of g_{ab} , or more precisely of $\lambda^{\pm}$, satisfying the conditions $\phi_{\pm} = 0$ for $\lambda^{\pm} = 1$. We can solve (10) by utilizing the transformation properties

$\lambda^\pm \rightarrow \xi' \lambda^\pm \mp \xi$ and

$$\phi_\pm \rightarrow (\xi')^{-2}(\phi_\pm - \mathcal{D}\xi) \quad (11)$$

under the time- and orientation-preserving reparametrizations

$$\tau \rightarrow \tau, \quad \sigma \rightarrow \xi(\tau, \sigma) \quad \text{with} \quad \xi' > 0, \quad (12)$$

where $\mathcal{D}$ is the Schwarzian derivative defined by

$$\mathcal{D}\xi = \frac{\xi'''}{\xi'} - \frac{3}{2} \left(\frac{\xi''}{\xi'} \right)^2.$$

It is easy to show that

$$\phi_\pm = \mathcal{D}f_\pm \quad (13)$$

solve (10), where $f_\pm$ are given by

$$f_\pm \mp \lambda^\pm f'_\pm = 0. \quad (14)$$

In terms of $\phi_\pm$ the VA in (7) can be cancelled. In fact if we define new Virasoro operators $\Phi_\pm$ by

$$\Phi_\pm = \varphi_\pm - \kappa_0 \phi_\pm,$$

then they satisfy the homogeneous equations as (3). As a result, the new stress tensor $\tilde{T}_{ab}$ defined by (4) with $\varphi_\pm$ replaced with $\Phi_\pm$ is covariantly conserved. We thus obtain a solution for t_{ab} in (9).

We denote, instead of S_L , the counteraction leading to $\tilde{T}_{ab}$ by S_V . To obtain S_V , we return to the defining relation of t_{ab} ,

$$\delta S_V = -\frac{1}{2} \int d^2x \sqrt{-g} t_{ab} \delta g^{ab} = \kappa_0 \int d^2x [\phi_+ \delta \lambda^+ + \phi_- \delta \lambda^-], \quad (15)$$

where use has been made of the relations

$$\delta \lambda^\pm = -\frac{1}{2\sqrt{-g}} (\delta g_{00} \mp 2\lambda^\pm \delta g_{01} + (\lambda^\pm)^2 \delta g_{11}).$$

Eq.(15) can be integrated in a straightforward manner. After a bit of calculation

we obtain

$$\begin{aligned}
S_V &= \frac{\kappa_0}{2} \int d^2x \left[\frac{(f_+''')^2 \dot{f}_+}{(f_+')^3} - \frac{f_+'' \dot{f}_+}{(f_+')^2} - \frac{(f_-''')^2 \dot{f}_-}{(f_-')^3} + \frac{f_-'' \dot{f}_-}{(f_-')^2} \right] \\
&= \frac{\kappa_0}{2} \int d^2x (\lambda^{+''} \ln f_+' + \lambda^{-''} \ln f_-').
\end{aligned} \tag{16}$$

The total effective action $S_{\text{str}} + S_V$ is invariant under the reparametrizations. Furthermore, the stress tensor (9) is traceless, i.e., the trace anomaly is absent. This is because the counteraction S_V depend only on $\lambda^\pm$ which are invariant under the Weyl rescalings $g_{ab} \rightarrow e^\varphi g_{ab}$. It is noteworthy that we can construct an effective action which is not only generally covariant but also invariant under local Weyl rescalings. This conclusion, at first sight, seems to be inconsistent with the generally accepted fact that the classical Weyl symmetry is suffered from trace anomaly in quantum theory. But this is not the case. In general, counterterms introduced in defining quantum effective action should be local functionals of the fields. On the other hand, S_V is a nonlocal functional in g_{ab} since $f_\pm$ defined by (14) is nonlocal. Hence we have subtracted too much by adding S_V as a counteraction.

Before turning to the problem of recovering the locality, we remark here the $PSL(2, R) \times PSL(2, R)$ symmetry of (16). Since the operator ordering is defined on constant time lines, it is worth studying symmetries which leave (16) invariant. Let us again consider the time- and orientation-preserving coordinate transformation (12). Since $f_\pm$ are transformed as scalars under (12), we can easily obtain

$$\int d^2x \left(\frac{(f_\pm''')^2 \dot{f}_\pm}{(f_\pm')^3} - \frac{f_\pm'' \dot{f}_\pm}{(f_\pm')^2} \right) \rightarrow \int d^2x \left(\frac{(f_\pm''')^2 \dot{f}_\pm}{(f_\pm')^3} - \frac{f_\pm'' \dot{f}_\pm}{(f_\pm')^2} - 2 \frac{\dot{f}_\pm}{f_\pm'} \mathcal{D}\xi - \frac{\xi' \xi''}{(\xi')^2} \right).$$

It can be easily seen that (16) is invariant if and only if $\mathcal{D}\xi = 0$, i.e.,

$$\xi(\tau, \sigma) = \frac{a(\tau)\sigma + b(\tau)}{c(\tau)\sigma + d(\tau)} \quad \text{with} \quad ad - bc = 1. \tag{17}$$

For fixed τ such transformations form a group of $PSL(2, R) = SL(2, R)/Z_2$. This also implies that the normal ordered string action (1) is invariant under the $PSL(2, R)$ transformation (17). Since we can consider the coordinate transformation (17) for f_+ and f_- separately, (16) is in fact invariant under a larger symmetry $PSL(2, R) \times PSL(2, R)$.

We now turn to the problem of recovering the locality of the counteraction. To remedy the nonlocality of S_V we utilize the freedom to introduce a reparametrization invariant counteraction S_T , which is chosen to cancel the nonlocal terms in S_V . The reparametrization invariance of S_T ensures the covariant conservation of the induced stress tensor.

The nonlocal terms contained in S_V can be extracted systematically by using weak-field expansion of the metric $g_{ab} = \eta_{ab} + h_{ab}$. To first order in h_{ab} , $\lambda^+ = 1 - 2h_{--}$ and $\lambda^- = 1 - 2h_{++}$.^{*} Putting these in (10), we obtain

$$\begin{aligned}\phi_+ &= -\frac{1}{4}\frac{\partial_+^3}{\partial_-}h_{--} - \frac{1}{4}(-\partial_-^2 + 3\partial_+\partial_- - 3\partial_+^2)h_{--}, \\ \phi_- &= -\frac{1}{4}\frac{\partial_-^3}{\partial_+}h_{++} - \frac{1}{4}(-\partial_+^2 + 3\partial_+\partial_- - 3\partial_-^2)h_{++}.\end{aligned}\tag{18}$$

These together with (15) give, to second order in h_{ab} , the counteraction S_V as

$$S_V = \frac{\kappa_0}{4} \int d^2x \left[h_{++} \frac{\partial_-^3}{\partial_+} h_{++} + h_{--} \frac{\partial_+^3}{\partial_-} h_{--} + \dots \right],$$

where $\dots$ denotes local terms. On the other hand the nonlocal terms in the rhs of this expression can be cancelled by choosing [1,14]

$$S_T = \frac{\kappa_0}{4} \int d^2x \int d^2y \sqrt{-g} R(x) K(x, y) \sqrt{-g} R(y),\tag{19}$$

where $R(x)$ is the scalar curvature and $K(x, y)$ is the scalar Green's function satisfying

$$\partial_a(\sqrt{-g}g^{ab}\partial_b)K(x, y) = \delta^2(x, y).\tag{20}$$

To lowest order in h_{ab} , they are given by

$$\sqrt{-g}R = \partial_-^2 h_{++} + \partial_+^2 h_{--} - 2\partial_+\partial_- h_{+-}\tag{21}$$

and $K = \frac{1}{4\partial_+\partial_-}$. Putting these in the rhs of (19), we see that the nonlocal terms of S_T cancel exactly those of S_V . Similar cancellation also occurs for terms third order in h_{ab} .

^{*} The lighth-cone coordinates are $x^\pm = x^0 \pm x^1$, and the flat-metric are given by $\eta_{++} = \eta_{--} = 0$, $\eta_{+-} = \eta_{-+} = -\frac{1}{2}$. We also use $\partial_\pm = \partial_0 \pm \partial_1$, hence $\partial_\pm x^\pm = 2$.

In addition to this, the stress tensor $\tilde{T}_{ab}$ modified by the counteraction

$$S_L = S_V + S_T \quad (22)$$

satisfies the trace anomaly relation

$$T_a^a = \frac{D}{24\pi} R. \quad (23)$$

We thus arrive at a local action (to third order in h_{ab}) by sacrificing the Weyl symmetry.

At this point the following questions may naturally arise. Does such cancellations persist to all orders? Can one writedown S_L in terms of g_{ab} as a local functional? In what follows we shall show that the nonlocal terms in S_V and S_T exactly cancel and the explict local form of S_L can be obtained.

This can most easily be achieved by working at harmonic coordinates defined by

$$\tilde{x}^\pm = f_\mp(x). \quad (24)$$

In this coordinates the metric tensor reduces to a conformally minkowskian form

$$\tilde{g}_{ab}(\tilde{x}) = \frac{g_{11}(x)}{f'_+(x)f'_-(x)} \eta_{ab}, \quad (25)$$

where $f'_\pm$ denotes the differentiation of $f_\pm$ with respect to $x^1 = \sigma$. Since the metric is conformally minkowskian, the curvature is simply given by

$$\sqrt{-\tilde{g}}\tilde{R} = \eta^{ab}\tilde{\partial}_a\tilde{\partial}_b \ln \frac{g_{11}}{f'_+f'_-}.$$

Eq.(20) takes the simplest form

$$\eta^{ab}\tilde{\partial}_a\tilde{\partial}_b\tilde{K}(\tilde{x},\tilde{y}) = \delta^2(\tilde{x},\tilde{y}).$$

Using these properties, we can simplify (19) as

$$\frac{1}{2} \int d^2x \int d^2y \sqrt{-g} R(x) K(x,y) \sqrt{-g} R(y) = \frac{1}{2} \int d^2x \sqrt{-g} R \ln \frac{g_{11}}{f'_+f'_-}. \quad (26)$$

Integrating by parts and using the relations like

$$\lambda^{\pm'} = -\frac{1}{2\sqrt{-g}}(g'_{00} + 2\lambda^{\pm}g'_{01} \pm 2g_{01}\lambda^{\pm}\xi' - g_{00}\xi'),$$

we can further reduce the rhs of (26) to

$$\frac{1}{2} \int d^2x \left(\frac{1}{2} \sqrt{-g} g^{ab} \partial_a \xi \partial_b \xi - \sqrt{-g} R \xi + \frac{2g_{11}}{\sqrt{-g}} \left\{ \left(\frac{g_{01}}{g_{11}} \right)' \right\}^2 - \lambda^{+'} \frac{f''_+}{f'_+} - \lambda^{-'} \frac{f''_-}{f'_-} \right), \quad (27)$$

where ξ is defined by

$$\xi(x) = \ln(g_{11}(x)). \quad (28)$$

This is essentially the LM of the metric tensor g_{ab} .

Eq.(27) together with (16), (22) and (26) implies that the counteraction (22) is explicitly given by

$$\begin{aligned} S_L &= S_V + S_T \\ &= \frac{\kappa_0}{2} \int d^2x \sqrt{-g} \left(\frac{1}{2} g^{ab} \partial_a \xi \partial_b \xi - R \xi \right) - \kappa_0 \int d^2x \sqrt{-g} g^{00} \left\{ \left(\frac{g_{01}}{g_{11}} \right)' \right\}^2. \end{aligned} \quad (29)$$

This is a main result of this note. The cosmological term may also be included here. The first term is nothing but the LA with $\xi = \ln(g_{11})$ identified with the conformal mode. Under the Weyl transformation $\delta g_{ab}(x) = \sigma(x)g_{ab}(x)$ it reproduces the trace anomaly (23). On the other hand the second term is manifestly invariant. Note that S_L is a local functional of the metric variables g_{ab} in arbitrary gauges. This is in sharp contrast with the Polyakov's LA (19) which is local only in the conformal gauge. The action S_L is, however, noncovariant. This is because our quantum string action (1) is not covariant due to the noncovariant ordering prescription in defining operator products. The noncovariance in (29) exactly cancels the anomaly in (8) originating from the VA. This implies that (29) plays the role of WZW term converting VA into TA. In ref. [6] the counteraction (29) has been obtained in a slightly different context by considering the relation between the BRST anomaly of Kato and Ogawa type [15] and the Weyl anomaly found by Fujikawa [16].

We are now in a position to relate our description with the results of the light-cone gauge [9,10] and of the conformal gauge [11,12]. So far we have been considering g_{ab} as classical background fields. We can elevate g_{ab} to dynamical variables

and straightforwardly investigate the canonical quantization of the action $S_{\text{str}} + S_L$ [8]. This involves gauge fixing procedure and introducing ghost variables. Furthermore, we must employ operator ordering for the 2D gravity sector consistently with the underlying general covariance, i.e., the BRST invariance. We do not, however, intend to show the analysis in full detail because the quantization of an essentially equivalent model has been investigated from a different view point in ref. [7]. Instead, we briefly examine these gauges, seeing that the conformal matter action (1) supplemented by the Liouville term (29) correctly reproduces the basic results of KPZ in the light-cone gauge and those of DDK in the conformal gauge.

(i) Conformal gauge: We separate the conformal mode ϕ from g_{ab} by writing

$$g_{11} = e^\phi \hat{g}_{11}. \quad (30)$$

A fixed background field $\hat{g}_{11}$ together with $\hat{\lambda}^\pm = \lambda^\pm$ determine a conformal class $g_{ab} = e^\phi \hat{g}_{ab}$. Putting (30) in the rhs of (29), we obtain

$$\begin{aligned} S_L = & -\frac{\kappa}{2} \int d^2x \sqrt{-\hat{g}} \left(\frac{1}{2} \hat{g}^{ab} \partial_a \phi \partial_b \phi + \hat{R} \phi \right) \\ & + \frac{\kappa}{2} \int d^2x \sqrt{-\hat{g}} \left(\frac{1}{2} \hat{g}^{ab} \partial_a \hat{\xi} \partial_b \hat{\xi} - \hat{R} \hat{\xi} \right) - \kappa \int d^2x \sqrt{-\hat{g}} \hat{g}^{00} \left\{ \left(\frac{\hat{g}_{01}}{\hat{g}_{11}} \right)' \right\}^2, \end{aligned} \quad (31)$$

where $\hat{\xi} = \ln(\hat{g}_{11})$ and $\hat{R}$ is the scalar curvature of $\hat{g}_{ab}$. The LM as well as the reparametrization ghost variables contributes to the central charge, modifying the coupling κ . By requiring the nilpotency of the BRST charge, we obtain

$$\kappa = \frac{25 - D}{24\pi}. \quad (32)$$

After a rescaling $\sqrt{\frac{25 - D}{12}} \phi \rightarrow \phi$, we see that the ϕ -dependent term in (31) coincides to the LA of refs. [11,12,17] for the vanishing cosmological term.

(ii) Light-cone gauge: The metric components are defined by $ds^2 = -dx^+ dx^- + g_{++}(dx^+)^2$. In terms of the light-cone variables, $\phi_\pm$ given in (14) can easily be

shown to satisfy

$$\partial_- f_+ = 0, \quad \partial_+ f_- + g_{++} \partial_- f_- = 0. \quad (33)$$

We see that f_- is nothing but the Polyakov's f introduced in ref. [9], i.e.,

$$f(x^+, x^-) = f_- \left(\frac{x^+ + x^-}{2}, \frac{x^+ - x^-}{2} \right), \quad (34)$$

while f_+ is independent of g_{++} and does not contribute to S_V . The Liouville action (22) can be expressed in terms of f with

$$S_V = -\frac{\kappa}{2} \int d^2 x \left[\frac{(f'')^2 \dot{f}}{(f')^3} - \frac{f'' \dot{f}'}{(f')^2} \right], \quad S_T = \frac{\kappa}{8} \int d^2 x \left[\frac{(\partial_-^2 f)^2 \partial_+ f}{(\partial_- f)^3} - \frac{\partial_-^2 f \partial_+ \partial_- f}{(\partial_- f)^2} \right]. \quad (35)$$

As we have seen above for the general case, $S_L = S_V + S_T$ is local in g_{++} , in sharp contrast with the Polyakov's nonlocal action S_T [1], and can easily be obtained from (29), i.e.,

$$S_L = \frac{\kappa}{4} \int d^2 x \frac{1}{1 + g_{++}} [(\partial_- g_{++})^2 - 2 \partial_- g_{++} (\ln(1 + g_{++}))' + 4 (\ln(1 + g_{++}))''], \quad (36)$$

In ref. [7], it is shown that the Polyakov's curvature equation

$$\frac{\kappa}{4} \partial_-^3 g_{++} = 0 \quad (37)$$

can be obtained as a result of the consistency between the BRST invariance and the field equations. This in turn naturally leads to $SL(2, R)$ Kac-Moody symmetry [9,10]. By requiring the nilpotency of the BRST charge, we obtain KPZ condition [10] for the central charge $k = 4\pi\kappa$ of the $SL(2, R)$ current algebra, i.e.,

$$\frac{3k}{k-2} + 6k + D - 28 = 0. \quad (38)$$

In this note we have investigated Polyakov string as conformal field theory coupled with 2D gravity and shed some light on the relations between VA and TA by examining the covariance and the locality of the action from canonical theoretical point of view. These apparently different anomalies have shown to be converted into each other by the local Liouville action S_L . Other conformal matters such as massless Majorana fields [10] can also be investigated. The only change appears in the central charge. A nontrivial generalization of our analysis is an application to superstrings. We will discuss this in forthcoming publication.

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