Statistical Constraints on the Inflation Effective Potential from the COBE DMR Results

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Abstract

We explore constraints on various forms for the effective potential during inflation based upon a statistical comparison between inflation-generated fluctuations in the cosmic microwave background temperature and the COBE DMR results. We show that the confidence limits of fits to the angular correlation are more sensitive to the inflation generating potential than implied by the uncertainty in the spectral index alone. This is due to the effects of cosmic variance on the statistical weights of the fit. The fact that the cosmic variance diminishes as the tensor and scalar contributions become comparable, and the fact that the cosmic variance diminishes for higher multipoles reduces the statistical significance of the fit for inflation potentials which allow comparable scalar and tensor terms and/or contributions from higher multipoles. As an illustration, we construct fits to the first year $53A + B \times 90A + B$ cross correlation function. Using an effective potential of the form $V(\phi) = \lambda \phi^x / x!$, we infer upper limits of $x \leq 68$ and $x \leq 303$ at the $1\sigma$ and $2\sigma$ confidence levels. The same analysis produces a power law of index of $n = 0.95^{+0.47}_{-0.63}$ which would imply an over estimate of $x \leq 80$ from a naive application of the simple analytic relation between $x$ and $n$. Using this maximum likelihood analysis we also quantify new limits on the parameters for polynomial effective potentials as well as the potential amplitude $\lambda$. This work highlights the importance of a careful statistical treatment when seeking constraints on the inflation-generating effective potential.

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I. INTRODUCTION

Measurements of the large-scale anisotropy in the cosmic microwave background (CMB) by the Cosmic Background Explorer (COBE) Differential Microwave Radiometer (DMR) experiment have provided important support for the hot big bang model with inflation [1]. A favored explanation for the generation of the observed fluctuations in the CMB temperature is by the expansion of quantum fluctuations of a scalar field during the inflationary epoch [2]. The fact that the observed angular correlation function is more or less consistent with a scale-invariant Harrison-Zel’dovich spectrum of power on various angular scales is consistent with the predictions of inflationary models. This is particularly true since the fluctuations resolved by COBE are larger than the horizon at recombination and not yet distorted from the inflation-generated spectrum by gravitational clustering on subhorizon scales.

However, the COBE observations are not exactly scale invariant (i.e. the optimum power law index deviates slightly from unity), and neither are the predictions of inflation. This is because inflation probably occurs as the universe is rolling down an effective potential of the inflaton field. Thus, fluctuations at different angular scales are related to slightly different parts of the potential. The change in angular anisotropy of the cosmic microwave background from scales of a few degrees to the full sky is, therefore, a measurement of the rate of change in the effective potential during that short interval of inflation for which those angular scales were stretched beyond the apparent horizon. The fact that the amplitude of the fluctuations depends upon the overall amplitude of the effective potential is well known [3]. In this paper we discuss how the functional form of the potential is further constrained from the statistics of weighted fits to observed angular correlation function.

Several papers [6]-[16] have explored the basic relations between the shape of the effective potential during inflation and the observed power spectral index $n$ (i.e. $\langle |\delta_k|^2 \rangle \propto k^n$) for fluctuations on various scales. Although attempts have also been made to reconstruct the inflation-generating potential from the COBE DMR results [12]-[16], it is generally concluded that such a reconstruction is subject to large uncertainties and that it is probably not possible [12] in the foreseeable future to obtain an accurate reconstruction of the potential. The reason being that the spectral index is rather insensitive to the potential. For example, for an effective potential of the form $V(\phi) = \lambda \phi^x/\phi!$, it can be shown analytically [5] that $x = 120(1 - n) - 2$. Thus, the limits on the spectral index obtained [1] from fits to the COBE angular correlation, $n = 1.1 \pm 0.5$ imply the not very restrictive limits on $x$ of $-50 \lesssim x \lesssim 70$ at the one sigma level.

Most previous analyses have concentrated on the spectral index and the effects of the scalar and tensor contributions to the quadrupole moment (e.g. [6], [9], [11], [15]). In this paper, however, we explicitly consider fits of various numerical inflation models to the COBE DMR cross correlation function. In this way confidence limits on various parameters for specific forms for the inflation generating potential can be better quantified. There are some important differences between this approach and methods which only consider the spectral index and quadrupole moment. In particular, we include the specific parametric dependence of the contribution of the cosmic variance to the determination of levels of confidence for fits to the data. We show here that the optimum correlation function is more sensitive to the shape of the inflation generating potential when the effects of cosmic variance are correctly included. This makes the prospect of recovering the inflation
generating potential more promising as the data become better determined in the future.

Our work also differs from previous studies [6]-[16] in that we are using the COBE data only. That is, we do not consider microwave background measurements on smaller angular scales or the clustering of galaxies which also contains useful information. However, since fluctuations on smaller scales have experienced gravitational clustering, this generalization requires a knowledge of the influence of dark-matter components. By restricting ourselves to the COBE data alone, we considerably simplify the present analysis the focus of which is to highlight the merits of a statistical analysis of inflation generating potentials.

II. INFLATION GENERATING POTENTIALS

The shape of the inflation effective potential is not known. A large number of different models, with different potentials, some with many free parameters, have been proposed, (cf. ref.’s [3]-[5]). While most models lead to a microwave fluctuation spectrum which is nearly scale invariant on the COBE angular scales, this scale-invariance is not exact, and some models lead to significant deviations from scale invariance. Of course, the constraints which can be placed upon effective potentials are limited by the uncertainties in the observed correlation function, and effects of cosmic variance [17], [18]. Nevertheless, these data represent the only direct observation of the shape of the inflation effective potential, and the statistical analysis of these data as described here provides at least some information as to which inflation models can be excluded.

For this study we consider two simple forms for the effective potential,

\[ V(\phi) = \frac{1}{x!} \lambda \phi^x \]  

(1)

and

\[ V(\phi) = \lambda \left( \frac{1}{5} \beta \phi^2 + \frac{1}{3} \alpha \phi^3 + \frac{1}{4} \phi^4 \right) , \]  

(2)

where we use Planck units, \( c = m_{\text{Pl}} = 1 \).

The first potential form is of a kind often employed in "Chaotic" inflation models [19], where \( x \) is usually taken to have a value of 2 or 4. As previously noted [5] this form is convenient as there is a simple analytic relation between \( x \) and the spectral index \( n \).

The polynomial potential of Equation (2) represents the most general renormalizable potential with just one scalar field [20]. Although more complicated forms for the effective potential have been proposed (cf. [3]-[5]), this form of the potential is sufficiently general for our purpose as it can represent the leading terms in an expansion of many different potentials. This analytic form for the effective potential has been previously discussed by Hodges et al. [20] in the context of generating non-Zel’ dovich spectra over scales of galactic clustering. Here we apply it to the scales sampled by the COBE correlation function.

The calculations performed here are in the context of chaotic inflation models [19] in the sense that we evolve the scalar field along fixed potentials starting from initial values which are removed from the global potential minimum. Nevertheless, the results presented here can be applied by a coordinate transformation to most inflation scenarios during the epoch that the COBE angular scales were being generated (i.e. the last 50-60 e-folds).
III. METHOD

Our goal is to find the optimum values and projected confidence limits for the parameters \(x\) (or \(\alpha, \beta\)) and \(\lambda\) from specific inflation models. The limits on \(x\) can then be compared with the limits inferred from the uncertainty in spectral index. To find these confidence limits we use the maximum likelihood \(\chi^2\) goodness of fit to the \textit{COBE} correlation function. We then use the chi-squared distribution function to set confidence limits. The application of maximum likelihood approaches for determining the spectral index and the ensemble-averaged quadrupole, \(Q_{\text{rms}}\), has been previously discussed by other authors ([21], [22]). Some caveats regarding previous statistical analyses are summarized in [22]. In particular, it is important to use a maximum likelihood function in which nondiagonal contributions to the covariance matrix are included. This we do below.

This approach of course contingent upon assuming a normal distribution of experimental errors and the cosmic variance. It has been shown [18], [23], [24] that this is an adequate approximation when determining confidence limits.

The generation of the CMB anisotropy in inflationary models has been discussed extensively in the literature (e.g. [2], [3], [5], [25]). Here we briefly review the various steps in the calculation from the generation of the fluctuations through the statistical weighting of the fit.

The equations governing the evolution of the scalar field and the universal expansion are,

\[
\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0
\]

and

\[
H^2 \equiv \left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi}{3} \left[V(\phi) + \frac{1}{2} \dot{\phi}^2\right].
\]

For simplicity, we assume instantaneous reheating as the scalar field approaches the minimum of the potential. That is, we approximate the end inflation by converting all of the kinetic energy from the motion of the field into thermal energy when the scalar field reaches the global minimum of the potential. We neglect oscillations about the potential minimum. This approximates the case when the coupling of the scalar field to the matter field is strong. This has the effect of maximizing the sensitivity of the fit to the shape of the effective potential. If the field oscillates around the global minimum, the point on the potential corresponding to the number of e-foldings from the end of inflation which are currently entering the horizon occurs closer to the minimum. Although this approximation to the reheating undoubtedly introduces some uncertainty in the analysis, it presents the most optimistic case, and hence the best scenario for determining the underlying potential. Also, we expect that the uncertainty due to the instantaneous reheating approximation makes a negligible additional contribution to the uncertainty in the effective potential parameters. This is because, for strong enough coupling, the universe experiences little inflation during the oscillation epoch.

The perturbations responsible for the CMB anisotropy originate from quantum fluctuations which are stretched beyond the apparent horizon during inflation. Two kinds of fluctuation can contribute to the CMB. Scalar fluctuations (i.e. fluctuations of the inflaton \(\phi\)) become density perturbations. Tensor fluctuations (i.e. fluctuations of spacetime curvature) become gravitational waves. The fluctuations of interest here have wavelengths of
order the horizon scale. The amplitudes for a multipole expansion of the CMB anisotropy can be written,

$$\langle a_i^2 \rangle = \langle a_i^2 \rangle_S + \langle a_i^2 \rangle_T ,$$

(5)

where the scalar contribution is

$$\langle a_i^2 \rangle_S = \frac{2l + 1}{25\pi} \int_0^{\omega_{\text{max}}} d\omega \frac{\omega j_1(\omega)}{j_0(\omega)}^2 \frac{H^4}{\dot{\phi}^2} ,$$

(6)

and the tensor contribution is

$$\langle a_i^2 \rangle_T = 36l(l-1)(l+1)(l+2)(2l+1) \int_0^{\omega_{\text{max}}} d\omega F_l(\omega)^2 H^2 ,$$

(7)

where from ref. [17]

$$F_l(\omega) = \int_0^{s_{\text{dec}}} ds \left\{ j_2 [\omega(1-s)] - \frac{j_1(\omega)}{j_0(\omega)} \right\} \left[ \frac{2}{(2l+1)(2l+3)} j_l(\omega) + \frac{1}{(2l+1)(2l+3)} j_{l-2}(\omega) \right] + \frac{1}{(2l+1)(2l+3)} j_{l+2}(\omega) ,$$

(8)

where $\omega_{\text{max}} = 2(1+z_{\text{eq}})^{1/2}$ and $s_{\text{dec}} = 1-(1+z_{\text{dec}})^{-1/2}$. The integrals in eqs. (5-7) are over scales $\omega \equiv kr_0$, where $k$ is the comoving wavelength of the perturbation, and $r_0 = 2/H_0$ is the radius of the presently observable universe. We use $H_0 = 50$ km s$^{-1}$ Mpc$^{-1}$ and the values $z_{\text{eq}} = 10500$ and $z_{\text{dec}} = 1100$ for the redshifts of matter-radiation equality and of hydrogen recombination, respectively. The quantities $H^4/\dot{\phi}^2$ and $H^2$ are evaluated at the epoch during inflation, when the scale in question exits the horizon ($k_{\text{Phys}} = H$).

It should be noted, that the use of $H^4/\dot{\phi}^2$ and $H^2$ in the integrands for the fluctuation amplitudes is based upon perturbation theory results obtained for exponentially expanding spacetimes, and is thus an approximation. However, the solution of Eqs. (3) and (4), as well as the integrals (6), (7), and (8), were done numerically. Thus, we do not make use of the ‘slow roll’ approximation.

To compare to the COBE DMR results, we convert from multipole expansion to the angular correlation function with the COBE resolution [1],

$$C(\theta) = \frac{1}{4\pi} \sum_{l>2} \langle a_i^2 \rangle W_i^2 P_l(\cos \theta) ,$$

(9)

where we use the window function in [1]

$$W_i^2 = \exp \left[ -\frac{k(l+1)}{17.8^2} \right] .$$

(10)

Since the amplitudes of the multipole moments arise from quantum fluctuations, they themselves are fluctuating quantities with a variance given by [17],

$$<\sigma_i^2 > = <a_i^2 > - <a_i^2>^2 = \frac{2}{2l+1} <a_i^2>^2 .$$

(11)
This defines the cosmic variance of the correlation function
\[ \sigma^2_{S,T} = \frac{1}{(4\pi)^2} \sum_{l \geq 2} <a_l^2>_S<a_l^2>_T \left[ W_l^2 P_l(\cos \theta)^2 \right]. \] (12)

where the subscripts \(S, T\) denote that the scalar or tensor components must be computed separately. In analyzing the COBE DMR data, we have found it sufficient to calculate multipoles up to \(l = 40\) in Eqs. (9-12).

Regarding the cosmic variance, Eq. (11-12), we note that when inflationary models generate significant amplitude for the higher multipoles, the cosmic variance diminishes due to the \((2l + 1)^{-1}\) factor. This tends to happen when the effective potential flattens during the descent. Another point is that the tensor and scalar components contribute independent random fluctuations which must be separately added in quadrature to the observational uncertainties to determine the goodness of fit to the observed correlation function. As these two components become equal the total cosmic variance tends to be smaller than the variance due to the scalar component alone by a factor of \(1/\sqrt{2}\). Both of these features affect the confidence level for any particular fit to the observed angular correlation function.

Indeed, if the cosmic variance could be treated as an uncorrelated error, then it could be used to place stringent constraints on the potential. However, since the cosmic variance affects the multipoles and not the correlation function the errors in the cosmic variance must be treated as a correlated error. To do this we utilize the generalization [27] of the \(\chi^2\) distribution function for correlated errors. That is, we define \(\chi^2\) by,
\[ \chi^2 = \sum_i \sum_j \Delta C_i (C^{-1})_{ij} \Delta C_j, \] (13)

where
\[ \Delta C_i = C(\theta_i)_\text{obs} - C(\theta_i)_\text{calc}, \] (14)

and the covariance matrix, \(C_{ij}\) is given by,
\[ C_{ij} = \sigma(\theta_i)_\text{obs} \delta_{ij} + \left( \frac{T_{2,7}}{4\pi} \right)^2 \sum_l \frac{2}{2l + 1} \left[ <a_l^2>_S + <a_l^2>_T \right] W_l^4 P_l(\cos \theta_i)P_l(\cos \theta_j), \] (15)

where \(\sigma(\theta_i)_\text{obs}\) is the COBE experimental error bar and \(T_{2,7}\) is the monopole temperature. Here we make use of the important fact that the experimental errors in the cross correlation function are uncorrelated to an excellent approximation [1]. Eq. (13) allows one to infer confidence limits in various parameters from the contours of constant \(\Delta \chi^2 = \chi^2 - \chi^2_{\text{min}}\) in the multi-dimensional parameter space [28].

The assignment of confidence limits here is based upon the additivity theorem for statistics, i.e. difference \((\Delta \chi^2)\) between the \(\chi^2\) for a fit with some number \((v)\) degrees of freedom held fixed and a fit which minimizes minimizes \(\chi^2\) is described by a \(\chi^2\) distribution function in \(\Delta \chi^2\) with \(v\) degrees of freedom. For a proof of this the reader is referred to any one of a number of texts in which the chi-squared distribution function is derived (cf. [28]- [29] and references therein). Here we apply the theorem with our generalized \(\chi^2\) distribution function. The confidence limits so determined should be similar to that obtained from the curvature of the likelihood function around the maximum [22].
IV. RESULTS

Fig. 1 shows the $53A + B \times 90A + B$ COBE cross correlation function of Smoot et al. [1]. As noted previously, the cross correlation function is preferred here as the errors are uncorrelated for this data set to a good approximation. These data are compared with a scale invariant spectrum, i.e., $|\delta|^2 \propto k^n$ with $n = 1$. Including cosmic variance in our fit we obtain an optimum spectral index of $n = 0.95^{+0.47}_{-0.63}$ consistent with the result quoted in [1] of $n = 1 \pm 0.6$ for the COBE analysis of the cross correlation function. We find, as noted in [21], that the projected confidence limits in $n$ are large due to the correlation of $n$ with the rms quadrupole moment.

For the optimum spectral index (including both the cosmic variance and the measurement uncertainties in the weighting) we obtain a minimum $\chi^2 = 59.21$. For $\phi^2$ potentials of the type of Eq. (1) we get a minimum $\chi^2 = 59.50$. For polynomial potentials of the type of Eq. (2) we get almost the same minimum value $\chi^2 = 59.49$. Thus, the best fit power law gives a slightly better fit than any inflation model of the above variety. Also, the minimum for the $\phi^2$ potentials occurs for $x \lesssim 0.1$ implying that the best fit potential is actually one which does not vary at all during inflation.

We mention here a caveat regarding the statistical analysis of these data. Since there are 70 measured points, a two-parameter fit of the correct model should have an expectation value of $\chi^2 \lesssim 68$. Thus, the $\chi^2$ per degree of freedom for the fit to the COBE correlation function is exceedingly good and may indicate that there is less scatter than one would expect from a purely statistical distribution of cosmic variance and experimental error. Nevertheless, a small minimum $\chi^2$ for a particular fit can be regarded as an artifact of the particular way in which the data happened to fall, and the maximum likelihood analysis which we apply is valid in this case as long as one believes that the data are distributed normally about the optimum curve.

For each $x$ or $(\alpha, \beta)$, we have therefore done inflation calculations of the fluctuation spectrum as a function of the potential amplitude, $\lambda$. We then identify which value for $\lambda$ minimizes the $\chi^2$. The contours of minimum $\chi^2$ as a function of the fixed parameters then can be used to set the confidence limits.

Fig. 2 shows $\chi^2$ as a function of $x$ and contours of optimum values for the amplitude $\lambda$ for inflation effective potentials of the form $V(\phi) = \frac{1}{2!} \lambda \phi^x$. Note that larger values for $x$ give a worse fit to the COBE results. A smaller $x$ means a smaller change in the potential when the relevant scales are generated, and thus a flatter (closer to scale invariant) spectrum. Indeed, the correlation function for $x = 0.1$ (lowest value on Fig. 2) is nearly indistinguishable the $n = 1$ case (Fig. 1). Since the scale-invariant spectrum fits the COBE results so well, the larger deviation from the scale-invariant spectrum which is obtained for the larger $x$, leads to a worse fit to the COBE results. Also, the weight of a particular fit decreases as the tensor and scalar contributions to the cosmic variance become more equal. This decreases the statistical confidence in a fit even when the calculated correlation function varies little. Both of these effects are illustrated in Figs. 3 - 5.

In Fig. 3 we show the correlation function for $x = 4$, which is an often assumed self coupling [19] in chaotic inflation models. The optimum correlation function is nearly indistinguishable from the power-law fit. The fact that this potential changes little over the angular scales sampled by COBE is illustrated on Figure 4 which shows the values of the scalar field and potential as various angular scales exited the horizon during inflation. The
amplitude of the various multipoles in the correlation function essentially depends upon the first and second derivatives of the effective potential as the scale corresponding to each multipole exits the horizon [5]. It is apparent on Figure 4 that the derivatives change little over the relevant angular scales. Hence, this potential is not particularly distinguishable from the $n = 1$ power spectrum.

However, the upper and lower curves, which give the cosmic variance, are slightly closer to the optimum curve for the $x = 4$ potential than for the $n = 1$ fit. This implies less statistical confidence in the $x = 4$ fit to the data even though the curve itself is not much different from the optimum curve.

These effects are demonstrated even more dramatically on Fig. 5 which shows an example of a fit for an effective potential with $x = 1000$. Here the inflation-generated correlation function deviates significantly from the COBE correlation function, particularly for angular scales less than $50^\circ$. This can be traced to a significant decrease in the amplitude of fluctuations exiting the horizon at later times (smaller angular scales) for a potential this steep. Also note, that the contributions from the cosmic variance to the statistical weights are much smaller near 40, 90, and 140 degrees. These two effects together imply a very small confidence level for this particular fit.

The confidence limits on $x$ and $\lambda$ can be extracted from their effect on $\chi^2$ as shown in Fig. 2. The $1\sigma$ and $2\sigma$ confidence limits on $x$, allowing for any $\lambda$, are shown on this figure. They are simply given [28] by the value of $x$ for which $\chi^2$ increases by 1.0 and 4.0 while $\lambda$ is varied to minimize $\chi^2$. From this we obtain the upper limits $x \leq 68.2$ ($1\sigma$ or 68% C.L.) and $x \leq 303$ ($2\sigma$ or 95% C.L.). Conversely, if $x$ is optimized and the confidence limits on $\lambda$ are searched for, we find on Figure 2 that $\lambda$ is determined to better than 25% ($1\sigma$), or 50% ($2\sigma$) accuracy. The $\pm 1\sigma$ limits for $\lambda$ are shown as contours above and below the line of optimum $\lambda$ on Figure 2.

It is at least gratifying that a $\chi^2$ analysis does imply better fits for the smallest coupling orders of the scalar field. This is consistent with what one expects physically since a self coupled scalar field in 4 dimensions is not renormalizable unless $x \lesssim 4$ [26].

Regarding the polynomial potential (Eq. 2), as in [20], we can assume with no loss of generality that the initial value for the scalar field is always to the right of the global minimum. This is because any negative initial condition can be made equivalent to a positive initial value by an $\alpha \rightarrow -\alpha$ coordinate transformation.

Following Hodges et al., we can exclude values for $\alpha$ and $\beta$ such that,

$$\alpha > 0, \beta < \frac{8}{9} \alpha^2, \quad (16)$$

and

$$\alpha < 0, \frac{8}{9} \alpha^2 < \beta < \alpha^2, \quad (17)$$

for which the universe must pass through a false-vacuum secondary minimum of $V(\phi)$ when descending from positive values of the scalar field. Such cases are excluded as, either the universe becomes trapped in the false vacuum and can not exit inflation, or (for a small false vacuum) they produce unacceptable large-scale structure due to the wall energy associated with nucleated bubbles.

As noted by Hodges et al. [20], significant deviations from a scale-invariant spectrum occur as one approaches the line $\beta = \alpha^2$, $\alpha < 0$. We therefore present our results as a
contour plot on the \((\alpha, \beta')\)-plane in Fig. 6, where

\[
\beta' = (\beta - \alpha^2)/\alpha^6 .
\]  

(18)

This transformation expands the critical region near \(\beta \geq \alpha^2\).

The shaded regions on Fig. 6 identify excluded values of \(\alpha\) and \(\beta'\) at the 1\(\sigma\) (light shade) and 2\(\sigma\) (dark shade) level based upon the COBE data. The thin lines show contours of optimum values of \(\lambda\) in the \(\beta'\)-\(\alpha\) plane as labeled.

The reason for the excluded regions can be seen in Figs. 7 and 8. Fig. 7 shows the point at which various scales exited the horizon during inflation for a potential which is excluded at the 95\% C. L. in Fig. 6 \((\alpha = -0.9\) and \(\beta' = 6.3 \times 10^{-4}\)). Fig. 8 shows the fitted correlation function for this potential. Here, one can see that there is a flattening of \(V(\phi)\) to the right of the global minimum during the epoch that the small scale \((< 10\) deg\)) structure was formed. However, there was a step descent down the potential when the scales between 10 and 180 deg were formed. In this case, fluctuations on smaller scale dominate over the larger scales causing the correlation function at large angles to show almost no structure. Just as important, however, is the fact that the cosmic variance for these fits is much smaller due to the dominance of contributions from higher multipoles (smaller angular scales) with a smaller cosmic variance. This increases the \(\chi^2\) of the fit.

V. CONCLUSIONS

Although there remain large uncertainties from the cosmic variance and from measurement errors of the angular correlation function for fluctuations in the microwave background, we have shown that it is possible to place slightly more stringent constraints on parameters of the effective potential for inflation by applying a maximum likelihood analysis rather than a naive analytic estimate from the uncertainty in the spectral index \(n\). This follows from the fact that the contribution from the cosmic variance to the statistical weight of the fits is slightly more sensitive to the underlying inflation generating potential than that reflected in the spectral index alone.

Also, since we have numerically integrated each inflation model, we have not resorted to the usual ‘slow roll’ approximation. As Kolb and Vadas[30]have recently discussed, corrections to the slow-roll approximation are important in some inflation models, including chaotic inflation.

The fact that inflation effective potentials with the lowest orders of self coupling best fit the observed correlation function is at least consistent with what one expects, since if the data had required a large power of \(\phi^5\), then the implied potential would not have been renormalizable.

We have also shown that the physics of the underlying potential is constrained, in that terms in the effective potential which produce too much flattening near the global minimum are excluded by the data at the 1 and 2\(\sigma\) level.

Although most of this discussion has centered on the shape of the effective potential, it is also worth noting that the strongest constraints are actually upon the overall amplitude of the effective potential. This is consistently constrained to be \(\lesssim 10^{-12}\), although larger values can occur for some polynomial potentials (cf. Fig. 6).
We have demonstrated that the sensitivity of the cosmic variance to the inflation effective potential can be used to better constrain the parameters of inflation potentials. Clearly, even stronger constraints on the inflation-generating effective potential than those remarked here will be possible as the statistics of the observed correlation function improve with more observing time. Hence, we eagerly await the availability of the second year data.

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References

Figure Captions

FIG. 1. The COBE DMR $53A + B \times 90A + B$ cross correlation function (points) compared with the correlation function generated from scale-invariant $n = 1$ spectrum (central solid line). The upper and lower solid lines illustrate the effects of cosmic variance.

FIG. 2. Computed $\chi^2$ (thick single line) and optimum $\lambda$ (dashed line) as a function of $x$ for an inflation potential with $V(\phi) = \lambda \phi^2/x!$. The scales for $\chi^2$ and $\lambda$ are on the left and right respectively. The arrows indicate the 68% and 95% confidence limits for $x$. The upper and lower thin lines for $\lambda$ show the 68% confidence limit region on the $(x, \lambda)$ plane.

FIG. 3. The COBE DMR $53A + B \times 90A + B$ cross correlation function (points) compared with an inflation generated correlation function (central solid line) obtained using a potential of the form $V(\phi) = \lambda \phi^4/4!$. The upper and lower solid lines illustrate the effects of cosmic variance.

FIG. 4. The effective potential as a function of inflaton field $\phi$ for a potential of the form $V(\phi) = \lambda \phi^4/4!$. The arrows indicate the point at which the labeled angular scales exited the horizon during inflation.

FIG. 5. The COBE DMR $53A + B \times 90A + B$ cross correlation function (points) compared with an inflation generated correlation function obtained using a potential of the form $V(\phi) = \lambda \phi^2/x!$, where $x = 1000$. The upper and lower solid lines illustrate the effects of cosmic variance.

FIG. 6. Excluded regions of the $\alpha$ vs. $\beta' = (\beta - \alpha^2)/\alpha^6$ plane. The light shading denotes the region excluded at the 68% confidence limit and the dark shaded region denotes the 95% confidence limit. The thin lines show contours of optimum $\lambda$ for each $(\alpha, \beta')$ as labeled.
FIG. 7. The effective potential as a function of inflaton field $\phi$ for a polynomial effective potential with $\alpha = -0.9$ and $\beta' = 6.3 \times 10^{-4}$. The arrows indicate the values of the potential at which the labeled angular scales exited the horizon during inflation.

FIG. 8. The $COBE\ DMR\ 53A + B \times 90A + B$ cross correlation function (points) compared with an inflation generated correlation function (central solid line) obtained using a polynomial effective potential with $\alpha = -0.9$ and $\beta' = 6.3 \times 10^{-4}$. The upper and lower solid lines illustrate the effects of cosmic variance.