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On the pomeron spectrum in the system of many reggeized gluons

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Abstract

It is shown that in the Hartree -Fock approximation the intercept of symmetric n gluonic colourless states is negative and diminishes linearly with n . The multipomeron solutions of the equation for n gluons are also discussed.

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In the perturbative approach the intercept of the BFKL pomeron composed of two reggeized gluons turns out considerably above unity [1]. There is a suspicion that contributions from $n > 2$ reggeized gluons may give rise to an even higher intercepts growing linearly with n [2]. In this note we present arguments showing that this suspicion is not justified. We obtain a lower limit on the intercept in the Hartree-Fock approximation presumably valid for large n . The intercept (minus one) indeed turns out growing with n but negative. We also comment on the general structure of the spectrum for n reggeized gluons and show that it separates into pomerons proper and their multiple exchanges.

As shown in [3] the transverse space and colour wave function ψ of n reggeized gluons in a colourless state satisfies a Schroedinger-like equation

$$H\psi = E \prod_{i=1}^n p_i^2 \psi \quad (1)$$

where p_i is the momentum of the i th gluon. The Hamiltonian H is given by a sum of pair terms

$$H = -(1/6) \sum_{i < k} T_i T_k H_{ik} \quad (2)$$

Here T_i is the colour vector of the i th gluon. In a colourless state

$$\sum_{i=1}^n T_i = 0 \quad (3)$$

The pair Hamiltonian H_{ik} acts on the wave function according to

$$H_{ik}\psi = \prod_{j=1}^n p_j^2 (\ln p_i^2 p_k^2 + 4\mathbf{C})\psi + \prod_{j=1, j \neq i, k}^n p_j^2 ((p_i^2 \ln(r_{ik}^2/4) p_k^2 + (i \leftrightarrow k))\psi + 2(p_i + p_k)^2 \psi(r_{ik} = 0)) \quad (4)$$

where $r_{ik} = r_i - r_k$ is the (transversal) distance between the gluons; $\mathbf{C}$ is the Euler constant.

The energy eigenvalue E is related to the intercept minus one Δ by

$$\Delta = -(3\alpha_s/\pi)E \quad (5)$$

so that the rightmost singularity in the complex angular momentum plane corresponds to the ground state. For $n = 2$ the solution of (1) is the BFKL pomeron with the energy and intercept

$$E_0 = -4 \ln 2, \quad \Delta_0 = (\alpha_s/\pi) 12 \ln 2 \quad (6)$$

No explicit solution has been found for $n > 2$.

The solution of (1) may evidently be found by a variational approach, as realizing the minimum value of the functional

$$\Phi = \int \prod d^2 p_i \psi^* H \psi \equiv \sum_{i < k} \Phi_{ik} \quad (7)$$

with the normalization condition

$$\int \prod d^2 p_i \psi^* \prod_{i=1}^n p_i^2 \psi = 1 \quad (8)$$

We start with an almost trivial comment about a colour separability property of Eq. (1). Suppose that a subsystem of $m < n$ gluons forms a colourless state, so that apart from (3) also

$$\sum_{i=1}^m T_i = 0 \quad (9)$$

and the whole system clusterizes into two colourless subsystems. Then Eq. (1) admits solutions of the form

$$\psi(1, \dots, m) \psi(m+1, \dots, n) \quad (10)$$

where numbers stand for colour and space variables of the corresponding gluons and it is assumed that each of the two wave functions factorizes into symmetric colour and space factors. Indeed, with (10) the colour average value of the products $T_i T_k$ coupling the two systems, that is with $i \leq m$ and $k > m$, is zero

$$\langle T_i T_k \rangle = (1/m) \langle \sum_{j=1}^m T_j, T_k \rangle = 0 \quad (11)$$

where we have used the symmetry of the colour wave function for the first m gluons and (9). Then the functional Φ breaks into a sum of two independent terms $\Phi_1 + \Phi_2$ describing the two colourless subsystems of gluons from 1 to m and from $m+1$ to n . Consequently the wave function (10) will be a solution of (1), provided each of the two factors solves the equation for m and $n-m$ gluons respectively. The energy will be given by a sum $E_1 + E_2$ of the energies for the two subsystems.

Of course the solution (10) does not satisfy the requirement of Bose symmetry. To restore it one has to symmetrize the wave function in all the n gluons. Then interference terms will appear in the energy which destroy the independence of the two systems and introduce interaction between them. However the interference terms will be proportional to the overlap between the spatial wave functions and will be negligible when the two colourless subsystems are widely separated in space, e.g. coupled to different nucleons in a nucleus. The interaction is then of a local type, of the range determined by the spatial dimension of each subsystem. The situation is thus fully analogous to the standard quantum mechanical bound states, which can be considered as independent so long as they are not in close contact.

Thus among the solutions of Eq.(1) there always are those which correspond to a set of smaller colourless subsystems which interact only due to the requirement of the Bose symmetry. In particular for even $n = 2m$ one can construct solutions which correspond to m BFKL pomerons with the energy $E_n = mE_0$ and intercept $\Delta_n = m\Delta_0$ growing with n . In the

standard Reggeon theory they correspond to an exchange of m pomerons and contribute to the m -pomeron branch cut. The Bose symmetry condition will introduce interaction between these BFKL pomerons. Since the dimension of the BFKL pomeron is small and diminishes with energy as $1/\sqrt{\ln s}$ this interaction will be practically local.

However these solutions do not exhaust all the spectrum of Eq.(1). Already for $n = 3$ there exist two independent colourless states: an odderon (with a negative signature) and a new pomeron consisting of three gluons (with a positive signature). For $n = 4$, apart from 3 solutions corresponding to combining the gluons into two BFKL pomeron pairs, there are 3 more colourless states which cannot be split into two colourless configurations. One expects that the maximal intercept (minimal energy) will occur for states which are symmetric both in colour and space (like the BFKL pomeron and the odderon). These symmetric solutions can be interpreted as new type pomerons or odderons with more intrinsic colour. According to the colour cluster separation property discussed above, the new pomerons and odderons will also appear as subsystems for larger number of gluons, that is, will take part in multipomeron exchanges. Such a picture was phenomenologically introduced in a model of colour string fusion [4], where it was supposed that supercritical pomerons exist for arbitrary high intrinsic colour.

It is then of importance to know intercepts of new pomeron or odderon states. Should they turn out positive, one indeed would have to take into account all number of exchanges of all new pomerons composed of any number of gluons.

To study the intercept of a pomeron or odderon composed of n gluons one can use the variational approach. As noticed in [5], for a wave functions symmetric both in colour and space variables, instead of the full functional Φ one can use any of its pair terms Φ_{ik} in (7) with i and k fixed. Moreover, for a symmetric colour wave function,

$$\langle T_i T_k \rangle = (2/n(n-1)) \sum_{i < k} T_i T_k = -3/(n-1) \quad (12)$$

where (3) and $T^2 = 3$ have been used. As a result, the energy E may be found via the minimal value of the functional in only spatial variables

$$\mathcal{E} = (1/2) \int \prod_{i=1}^n d^2 p_i \psi^* H_{12} \psi \quad (13)$$

where the Hamiltonian H_{12} is defined by (4) with $ik = 12$ and the function ψ should satisfy (8). The energy of the whole system of n gluons is determined by the minimal value ϵ of $\mathcal{E}$ according to

$$E_n = (1/2)n\epsilon_n \quad (14)$$

Actually the operator in $\mathcal{E}$ does not depend on n . The dependence on n enters only from extra arguments in ψ through the requirement of the symmetry in all arguments. The space

of trial wave functions then gets smaller with rising n , wherefrom one obtains

$$\epsilon_{n+1} \geq \epsilon_n \quad (15)$$

and in particular $\epsilon_n \geq \epsilon_2$. This gives a lower limit on the energy and an upper one on the intercept for n gluons in a symmetric state

$$E_n \geq (n/2)E_0, \quad \Delta_n \leq (n/2)\Delta_0 \quad (16)$$

This limit was obtained in [5] for the odderon $n = 3$.

One can try to obtain an upper limit for the energy using the variational approach. A natural description of a system of many reggeized gluons is the Hartree-Fock one. So we seek for the minimum of the functional $\mathcal{E}$ on functions ψ which factorize into a product of individual gluon wave functions. For the ground state, with ψ symmetric in all gluons, all individual gluon wave functions should evidently be the same. So we take for the wave function of n gluons in the ground state

$$\psi = \prod_{i=1}^n \psi(r_i^2) \quad (17)$$

(independent of the angles). The functional $\mathcal{E}$ for ψ of the form (17) can be transformed into a comparatively simple expression

$$\mathcal{E} = K + V \quad (18)$$

Here K and V refer to the "kinetic" energy ($\ln p^2$ terms in (4)) and "potential" energy ($\ln r^2$ terms and the last contact term in (4)), respectively, and are given by

$$K = 16\pi^2 \int_{-\infty}^{\infty} d\nu |f(\nu)|^2 (\text{Re } \psi(1 + i\nu) + C) \quad (19)$$

$$V = 8\pi^2 \int_{-\infty}^{\infty} dz_1 dz_2 \rho(z_1) \rho(z_2) |z_1 - z_2| \quad (20)$$

In these expressions $z = \ln r^2$, $\rho(z) = |\psi'(z)|^2$ and $f(\nu)$ is the Fourier transform of $\psi'(z)$:

$$\psi'(z) = \int_{-\infty}^{\infty} d\nu f(\nu) \exp i\nu z \quad (21)$$

The most important observation is that both K and V are positive. Therefore one cannot expect to find any negative energy eigenvalues (positive intercepts) in the Hartree-Fock approximation.

The condition $\delta\mathcal{E} = 0$ in principle gives the relevant Hartree-Fock equation which determines the ground state wave function for n gluons. It is a nonlocal equation due to nonlocality of the kinetic term. So instead of writing and solving it we tried to minimize $\mathcal{E}$ by direct methods taking a trial function as a sum of Gaussians in r^2 or in $z = \ln r^2$. Both choices give

practically the same minimal value ϵ , although with Gaussians in z it is achieved with much less number of terms

$$\epsilon = 0.959.. \quad (22)$$

It means that the Hartree-Fock energy (intercept) of the symmetric n gluon state is positive (negative) and grows (diminishes) linearly with n :

$$E_n^{HF} = (1/2) 0.959 n, \quad \Delta_n^{HF} = -(3\alpha_s/2\pi) 0.959 n \quad (23)$$

Of course, strictly speaking, this result only tells us that the high n pomerons and odderons with a positive intercept cannot be found in the Hartree-Fock approximation and, if exist, possess wave functions of a highly correlated structure (as the BFKL pomeron with $n = 2$). However we believe that the Hartree-Fock approximation is not a bad one for high n and correctly describes the bulk of the energy rising with n . Additional terms coming from correlations presumably have the order $1/n$ in ϵ and order unity in E . Then, if these additional terms are negative, we should expect at most a finite number of pomerons and odderons with rising intrinsic colour. This finite set of pomerons and odderons and their multiple exchanges will then determine the high-energy behaviour of soft strong interactions.

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