LUMINOSITY REDUCTION CAUSED BY THE FULL-DETUNING LLRF SCHEME ON THE HL-LHC CRAB CAVITIES

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Abstract

The High-Luminosity LHC (HL-LHC) crab cavities (CCs) will be installed on both sides of IP1 (ATLAS) and IP5 (CMS) to compensate for the geometric luminosity reduction due to the crossing angle. To cope with the increased beam current (0.55 A DC for LHC, 1.1 A for HL-LHC), the operation of the LLRF system has been changed: rather than fully compensating the transient beam loading, we allow the phase to vary along the turn (100 ps peak-peak with 1.1 A DC). This has been implemented at LHC since July 2017. The CCs have high loaded Q (5e5) and the available RF power is insufficient to follow the bunch phase modulation. The crabbing voltage is not modulated, causing a phase error w.r.t. the individual bunch centroids, leading to transverse kicks of the centroids and an asymmetric crabbing of the bunch cores. We present an analytical model for the resulting luminosity reduction and validate with particle tracking simulations. Due to the symmetry of the bunch filling patterns for the counter-rotating beams, the peak luminosity is reduced by only 2% for nominal HL-LHC parameters at IPs 1 and 5, which is within tolerable limits.

INTRODUCTION

The HL-LHC project aims at luminosity upgrades by accepting a large crossing angle to mitigate consequential long-range beam-beam effects [1]. However, a large crossing angle reduces overlap densities of the colliding beams, resulting in a luminosity loss. To recover the geometric overlaps and realize almost head-on collisions at IPs, superconducting deflecting RF cavities (CCs) are installed up- and downstream of IP1 and IP5 to create local crabbing bumps around IPs [2]. The HL-LHC general parameters of optics version 1.2 (collision round $\beta^*$=0.20 m) [3] are summarized in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proton energy at collision</td>
<td>7 TeV</td>
</tr>
<tr>
<td>Beam intensity $N$</td>
<td>2.2 x 10^{11} ppb</td>
</tr>
<tr>
<td>Number of bunch $n_b$</td>
<td>2748</td>
</tr>
<tr>
<td>r.m.s. bunch length ($\sigma_z$)</td>
<td>9.0 cm</td>
</tr>
<tr>
<td>Transverse emittance $\epsilon_{n(x,y)}$</td>
<td>2.5 $\mu$m</td>
</tr>
<tr>
<td>Synchrotron frequency $f_s$</td>
<td>23.8 Hz</td>
</tr>
<tr>
<td>RF frequency of main/crab cavity</td>
<td>400.79 MHz</td>
</tr>
<tr>
<td>Total voltage of accelerating RF cavity</td>
<td>16 MV</td>
</tr>
<tr>
<td>Full crossing angle</td>
<td>510 $\mu$rad</td>
</tr>
<tr>
<td>CC voltage</td>
<td>3.4 MV/cavity</td>
</tr>
</tbody>
</table>

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The LHC bunch filling pattern has a complex structure with gaps for the injection and beam dump kickers rise time (Fig. 1). This results in transient beam loading at the revolution frequency (11 kHz). The current LHC has adopted a Full-Detuning LLRF algorithm to control the transient beam loading in the main accelerating RF cavities since 2017 [5,6]. The Full-Detuning algorithm keeps the cavity amplitude constant over the whole turn, while accepting a cavity phase modulation according to the periodic beam current. With this scheme, the required klystron power is independent of beam current and can be minimized by adjusting the detuning and loaded Q-value ($Q_L$) of the cavity. The present RF accelerating system will thus have a power sufficient for the HL-LHC beams [6, 7]. As the loaded $Q_L$ of the CCs is very large (5e5), their RF voltage cannot follow the bunch phase modulation estimated at 100 ps peak-peak in one turn. The CCs will be operated with a constant RF voltage (amplitude and phase). The bunch center will therefore arrive in the CC early or late (compared to the CC zero phase), resulting in an asymmetric transverse bunch distortion at IP and a
transverse displacement of the collision vertex (Fig. 2). The HL-LHC bunches are long compared to the RF period (almost 170 degrees at 4σ), and the significant distortion in bunch transverse distribution will affect the peak luminosity. In this paper, we evaluate the luminosity reduction caused by the bunch phase modulation for the HL-LHC.

ANALYTICAL MODEL

We consider the horizontal thin momentum kick from the CC around IP5. An horizontal phase space coordinate is defined by (x′, x′). We transform the phase space coordinate at the CC (xCC, x′CC) to the IP (xIP, x′IP) using the linear transfer matrix

\[
\begin{pmatrix}
    x_{IP} \\
    x'_{IP}
\end{pmatrix} =
\begin{pmatrix}
    \frac{\beta_{CC}}{\beta_{IP}} & \sqrt{\beta_{CC}} \sqrt{\beta_{IP}} \\
    -\frac{\sqrt{\beta_{IP}}}{\sqrt{\beta_{CC}}} & 0
\end{pmatrix}
\begin{pmatrix}
    x_{CC} \\
    x'_{CC} + \delta x'_{CC}
\end{pmatrix}
\]

(1)

where the betatron phase advance between the CC and the IP is \( \pi/2 \). Twiss parameter \( \alpha \) at IP is zero, \( \beta^* \) and \( \beta_{CC} \) are the \( \beta \) functions at IP and CC locations respectively, and \( \delta x'_{CC} \) is the thin momentum crab kick, \( \delta x'_{CC} = -eV \sin(kz_{CC} + \phi_{1,2}) \) where \( k \) is the wave number, \( z \) is the longitudinal position from the reference particle, \( \phi \) is the bunch phase modulation, \( E_s \) is the energy of the synchronous particle, \( e \) is the charge of the proton, and \( V \) is the crapping voltage given by

\[
V = \frac{cE_s \tan \theta/2}{e\omega \beta_{IP} \sin \mu}
\]

(2)

where \( c \) is the speed of light, \( \omega \) is the angular frequency of the CC, \( \mu \) is the betatron phase advance between the upstream CC and the IP, and \( \theta \) is the crapping angle that will be smaller than the crossing angle and be varied during the physics fill. This scheme is called partial crabbing [4]. We consider Gaussian transverse and longitudinal bunch distribution at the CC, and translate it from the CC to the IP using Eq. 1. The coordinate system is rotated from (xIP, zIP) to (x̃IP, ẑIP) at the IP by half the crossing angle. The counter-rotating bunch is rotated in the opposite direction. The bunch distribution in terms of the (x̃IP, ẑIP) coordinate system can be derived. The same distribution is taken for the bunches of both rings. Figure 3 shows the bunch distributions at the IP with equal phase modulations for colliding pairs.

The bunch distribution of the non-deflecting transverse direction at IP, taken to be \( y \), is also defined as Gaussian. An integral peak luminosity [8, 9] is then derived by overlap densities of two colliding bunches at the IP given by

\[
L = \frac{\cos^2 \frac{\phi}{2} N^2 f_{rev} \rho_b}{4\pi^5/2 \sigma_x^2 \sigma_y^2 \sigma_z^2} \int \int \int_{-\infty}^\infty d\tilde{x}_{IP} d\tilde{z}_{IP} d(\Delta t)
\]

(3)

\[
= \frac{e^{-\tilde{x}^2_{IP} + \tilde{z}^2_{IP} - \tilde{x}^2_{IP} \cos \theta \tilde{z}_{IP} \sin \theta \tilde{z}_{IP} \cos \theta}}{2\sigma_x^2 \sigma_z^2} \times
\]

\[
\cdot \left[ \frac{e^{-\tilde{x}^2_{IP} + \tilde{z}^2_{IP} - \tilde{x}^2_{IP} \cos \theta \tilde{z}_{IP} \sin \theta \tilde{z}_{IP} \cos \theta}}{2\sigma_x^2 \sigma_z^2} \right].
\]

Figure 3: Bunch distributions at IP with equal phase modulations in time for the colliding pairs with total crapping voltage of 6.8 MV.

where \( N \) is the number of particles in a bunch, \( f_{rev} \) is the revolution frequency and \( C_{1,2} \) are

\[
C_{1,2} = \left( \beta^* \beta_{CC} \frac{eV}{E_s} \sin \left( k \left( \pm \tilde{x}_{IP} \sin \theta + \tilde{z}_{IP} \cos \theta \mp ct \right) \pm \phi_{1,2} \right) \right)
\]

\[
\pm 2\sqrt{\beta^* \beta_{CC}} \sin \left( \tilde{x}_{IP} \cos \theta \mp \tilde{z}_{IP} \sin \theta \right)
\]

\[
\cdot eV \sin \left( k \left( \pm \tilde{x}_{IP} \sin \theta + \tilde{z}_{IP} \cos \theta \mp ct \right) \pm \phi_{1,2} \right).
\]

(4)

COMPARISON WITH SIMULATIONS

The single bunch tracking simulation is performed using PYTRACK to calculate peak luminosity and compare the results to the analytical model of Eq. 3 [10]. PYTRACK is a particle tracking code in 6 dimensional phase space coordinates and transports particles using linear transfer maps generated by MADX [11]. We use the HL-LHC ring optics version 1.2 in this study (Table 1).

The longitudinal and transverse momentum kicks from the CC are expressed by

\[
\Delta x' = \frac{eV}{p_z} \sin (\phi_t - k \tilde{z})
\]

(5)

\[
\Delta p_z = -k \times \frac{eV}{p_z} \cos (\phi_t - k \tilde{z})
\]

where \( \phi_t \) is the synchronous phase, \( x' = p_t/p_z \), \( p_z \) is the longitudinal momentum, \( \Delta p_z/p_z \) is the fractional momentum kick, and \( \tilde{z} = c\Delta t \) is the longitudinal position offsets with respect to the reference particle. The initial bunch consists of 10^5 macroparticles with transverse Gaussian distribution.
We consider two longitudinal distributions: Gaussian and q-Gaussian [4]. The initial bunch is injected at IP1 ($\alpha^* = 0$) and we observe the bunch distribution at IP5. The crabbing voltages for nominal (6.8 MV) and full compensation (9.6 MV given in Eq. 2) are applied for both crabbing and anti-crabbing cavities. Both crab voltages are ramped up linearly over 1000 turns, that corresponds to two synchrotron periods.

The analytical peak luminosity is computed by numerical integration of Eq. 3 using Python. For the simulation, we translate both bunches from IP5 along the longitudinal direction. Then we compute the overlapping densities at each longitudinal position. Finally we sum up overlapping densities for all longitudinal positions and calculate the peak luminosity after multiplication by the scaling factor in Eq. 3. The symmetry of the filling pattern for collisions in IP1 and IP5, the Full-Detuning algorithm causes phase errors that are identical for the two colliding bunches. We call this mode "coherent" phase modulation. It is also of interest to evaluate the luminosity degradation in case of a phase error in one beam only. This mode is called "incoherent" phase modulation in this paper.

Simulations with the q-Gaussian longitudinal profile result in luminosity larger than the analytical result using Gaussian. A q-Gaussian distribution is more localized at the center of the RF bucket than a Gaussian distribution, resulting in an increased peak luminosity.

The analytical model contains no momentum spread while the simulations include the momentum spread. This explains the smaller peak luminosity in the simulation for a Gaussian longitudinal profile than in the analytical model. To verify this, we have run simulations for the Gaussian bunch profile with and without momentum spread and observe a very good agreement between the simulation and analytical model in the absence of momentum spread (Table 2). The simulations without momentum spread were done without the main accelerating RF cavities, considering the CC transverse kicks only.

Table 2: Comparison of peak luminosity with coherent phase errors for Gaussian longitudinal bunch profile. The percentage drop in luminosity is given in parenthesis below each luminosity value.

<table>
<thead>
<tr>
<th>Offset [ps]</th>
<th>Peak luminosity [$10^{35}$cm$^{-2}$s$^{-1}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Analytical = 1.163, Gaussian = 1.160±1.1×10$^{-3}$</td>
</tr>
<tr>
<td>100</td>
<td>Analytical = 1.141 (1.89%), Gaussian = 1.128±9.1×10$^{-3}$</td>
</tr>
<tr>
<td>200</td>
<td>Analytical = 1.079 (7.22%), Gaussian = 1.068±1.2×10$^{-3}$</td>
</tr>
</tbody>
</table>

**Figure 4**: Peak luminosity for various phase modulations in time. The pink dashed line is the case of head-on collisions. The black dashed line is the luminosity without CCs. The other curves are analytical (Ana.) models of coherent (Coh.) and incoherent (Incoh.) phase modulations for Gaussian (Gaus.) and q-Gaussian (q-Gaus.) longitudinal bunch profiles. Simulation results (PYTRACK) are shown as markers.

Simulations result in luminosity larger than the analytical result using Gaussian. A q-Gaussian distribution is more localized at the center of the RF bucket than a Gaussian distribution, resulting in an increased peak luminosity.

To evaluate the luminosity reduction caused by the Full-Detuning LLRF algorithm on the HL-LHC CCs, we have derived an analytical model for the peak luminosity including the effect of phase modulations. To validate the analytical model, we have applied tracking simulations with Gaussian and q-Gaussian longitudinal bunch profiles. The analytical model is in good agreement with the simulations for both coherent (identical phase displacement of two colliding bunches) and incoherent (phase offset of one bunch only) modulations in Gaussian longitudinal bunch distributions. For the expected maximum coherent and incoherent phase errors of 100 ps, the reductions in peak luminosity are less than 2 % (coherent) and 6 % (incoherent) based on the analytical model, which is negligible for the baseline HL-LHC operation with leveling scheme [4].

**CONCLUSION**

To evaluate the luminosity reduction caused by the Full-Detuning LLRF algorithm on the HL-LHC CCs, we have derived an analytical model for the peak luminosity including the effect of phase modulations. To validate the analytical model, we have applied tracking simulations with Gaussian and q-Gaussian longitudinal bunch profiles. The analytical model is in good agreement with the simulations for both coherent (identical phase displacement of two colliding bunches) and incoherent (phase offset of one bunch only) modulations in Gaussian longitudinal bunch distributions. For the expected maximum coherent and incoherent phase errors of 100 ps, the reductions in peak luminosity are less than 2 % (coherent) and 6 % (incoherent) based on the analytical model, which is negligible for the baseline HL-LHC operation with leveling scheme [4].

**REFERENCES**


