β^- decay of ^{158}Tb

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The β^- decay energy of ^{158}Tb has been measured to be $Q_{\beta^-} = 929 \pm 16$ keV. The beta shape is treated within the modified B_{ij} approximation. Q_{β^-} has been combined with atomic mass differences known from mass spectroscopy to infer the electron capture decay energy of the nucleus ^{158}Tb , $Q_{\text{EC}} = 1221 \pm 17$ keV. When additional information from similarly decaying rotational nuclei is used, the recent suggestion that ^{158}Tb may allow one to measure effects of a finite neutrino mass seems improbable.

I. INTRODUCTION

Effects of a finite neutrino mass m_ν are minute. In order to become observable in electron capture decay, ideally, the available decay energy W_{nls} should equal $m_\nu c^2$. Then both the capture ratios of different atomic subshells (nls) as well as the shape and intensity of the internal bremsstrahlung (IBEC) are most strongly affected. The quest for nuclei with such properties has so far led to the three rare earth nuclei ^{158}Tb ,¹ ^{157}Tb ,² and ^{163}Ho ,³ which indeed possess branches with extremely small decay energies. Raghavan¹ recently reported a feeble 1s electron capture branch of ^{158}Tb to a highly excited level (at 1187 keV) in the daughter atom ^{158}Gd , thus establishing $W_{1s} = 156 \pm 17$ eV (for $m_\nu c^2 = 0$). The magnitude of the total atomic mass difference Q derived from this information, $Q = 1237.5$ keV, however, cannot be reconciled with the value based on nuclear reactions, namely, $Q = 1217.6 \pm 4.3$ keV.⁴

In the present paper we want to show that this mass difference can be measured in yet another way. We will exploit mass relations between several other pairs of nuclei, whose mass differences are accurately known (see Sec. II) and will conclude that the energy release in beta (minus) decay of ^{158}Tb to ^{158}Dy allows the inference of the questioned mass difference.

In Sec. III we report on a remeasurement of Q_{β^-} , although published data on this beta decay seem already sufficient to reject the lower Q_{EC} value. We then proceed, however, to show in Sec. IV that this conclusion is subject to change if we include nonallowed shape factors in the analysis of the beta spectra.

II. MASS RELATIONS NEAR ^{158}Tb

If we neglect the effects of a massive neutrino the electron capture Q value of ^{158}Tb is a direct measure of the

mass difference of the two neutral atoms ^{158}Tb and ^{158}Gd .

The same mass difference can be ascertained in various other experiments. In order to elucidate some of them we refer to Fig. 1, which depicts a part of the chart of the nuclei. The above-mentioned EC process is indicated by a broad arrow. Obviously knowledge about the proton separation energy of the mother atom ^{158}Tb and the neutron separation energy of the daughter atom ^{158}Gd (indicated by a vertical and horizontal straight line, respectively) allows the computation of Q . These separation energies in turn can be measured readily, for the isotope ^{157}Gd is a stable one. The most powerful techniques to perform these experiments involve single proton transfer reaction like (α, t) or $(^3\text{He}, d)$ in conjunction with neutron capture (n, γ) or (d, p) reactions. For further details the reader is referred to Refs. 4, where a Q value for electron capture of ^{158}Tb $Q = 1217.6 \pm 4.3$ keV is derived. Recently the same quantity was measured⁵ utilizing a radioactive ^{158}Tb

FIG. 1. Part of the chart of nuclei around $A=158$. Accurately measured atomic mass differences are indicated: Arrows denote electron capture transitions, straight lines nuclear reactions, and bent lines mass spectroscopy. For details see the text.

target, with the preliminary result $Q = 1218.5 \pm 5.0$ keV. In detail the situation is more complicated, because absolute Q values can unfortunately be measured only with accuracies of the order of 10 keV. This limitation is usually overcome by extracting the differences between Q values of neighboring isotopes instead. Although this is feasible with better precision, the results become dependent upon other, tertiary data. It is worthwhile to note that the seemingly most direct ($^3\text{He}, t$) reaction has too small a cross section to be of practical use.

Mass spectroscopy is another powerful tool to link atomic masses. Atoms whose mass differences are determined by this technique with uncertainties of less than a few kiloelectronvolts are shown in Fig. 1 as connected by a bent line. From the figure we see that the mass difference Δ of the isobars ^{158}Dy and ^{158}Gd is known via the chain $^{158}\text{Dy} - ^{160}\text{Dy} - ^{160}\text{Gd} - ^{158}\text{Gd}$. We find $\Delta = 292.2 \pm 2.3$ keV.⁶ These two $A = 158$ isobars are the respective stable daughter products following β^- and EC disintegration of ^{158}Tb . Therefore the mass difference Δ can also be assessed through the difference of its weak interaction decay energies $Q_{\text{EC}} - Q_{\beta^-} = \Delta$ (for $m_{\bar{\nu}}c^2 = m_{\nu}c^2 = 0$; see the comment at the end of this section).

The beta-minus decay of ^{158}Tb leads predominantly to one excited state at $E = 98.92 \pm 0.04$ keV (Ref. 7) in ^{158}Dy . The endpoint of the associated β spectrum, $E_{\beta, \text{max}}$, has been measured to be $E_{\beta, \text{max}} = 845 \pm 10$ keV,⁸ $E_{\beta, \text{max}} = 861 \pm 20$ keV,⁹ and $E_{\beta, \text{max}} = 857 \pm 27$ keV.¹⁰ Combin-

ing these data we obtain $Q_{\text{EC}} = E + E_{\beta, \text{max}} + \Delta = 1240 \pm 9$ keV, which agrees extremely well with Raghavan's conclusion while being in disaccord with the nuclear reaction data.

If we now allow the neutrino to acquire a finite mass, the above relations have to be modified. Although it is needless to say that for the accuracies of the present experiment these alterations will be negligible, it is, however, still worthwhile to clarify the principle.

The weak interaction Q values (Q_{EC} and Q_{β^-}) differ from the respective atomic mass differences by $m_{\nu}c^2$. Since it is a firm (yet unproven) belief that the masses of the neutrino and its antiparticle are identical, one might be led to accept the equation $Q_{\text{EC}} - Q_{\beta^-} = \Delta$ independent of the possible (anti)neutrino masses. However, the endpoint of a beta spectrum is usually extracted from a straight line fitted to the data in the Kurie representation. This line will intercept the abscissa at $Q_{\beta}^{\text{exp}} = Q_{\beta^-} + m_{\nu}c^2$ (instead of Q_{β^-}), unless the fit is restricted to the last few ($= m_{\nu}c^2$) electronvolts of the spectrum. Strictly speaking, the quantity $Q_{\text{EC}} - Q_{\beta}^{\text{exp}}$ is therefore still a function of (and only of) the mass of the neutrino.

III. MEASUREMENT OF THE BETA ENDPOINT ENERGY

The ^{158}Tb was produced in the Rossendorf reactor in the double neutron capture process:

FIG. 2. The beta spectrum of ^{158}Tb observed in coincidence with a 99 keV gamma ray is shown as the histogram. The continuous curve is the result of the folding of a theoretical beta spectrum of allowed shape with the measured response function. An endpoint energy of 840 keV has been used. A partial decay scheme for ^{158}Tb is shown as an inset. Notice that the ordinate scale has been chosen proportional to the square root of the number of counts per bin (which linearizes the β spectrum almost perfectly).

After one year of cooling the Tb was separated from the irradiated material by means of a twofold ion-exchange chromatography.³ A sample of 15 μg ^{157}Tb containing 6.5% ^{158}Tb was obtained. A small fraction of the material was mass separated in the ISOLDE 60 kV off-line separator. The final source implanted into a 45 μm thick aluminum backing consisted of $N_0 = (5.3 \pm 0.6) \times 10^{13}$ atoms of ^{158}Tb . The ^{157}Tb contamination of the source would be 8×10^{10} atoms under the assumption of a typical separator enhancement factor of 10^4 . This sample was then placed in front of a 2 mm thick Si(Li) detector with an active surface area of 200 mm^2 . As more than 80% of all ^{158}Tb nuclei undergo electron capture, we expect the high-energy part of the singles spectrum to be dominated by electrons originating from the Compton scattering of the abundant 1 MeV gamma rays.

95% of the β transitions lead to an excited state at 98.92 keV in ^{158}Dy (cf. the decay scheme shown in Fig. 2). The gamma ray deexciting this level can therefore serve to tag electron events of interest. This transition is of $E2$ character and hence highly converted ($\alpha_K = 1.2^7$). For this reason also the observed dysprosium K x rays can be used in the coincidence.

In order to observe these photons efficiently a pure germanium Ge(HP) detector was put in front of the Si(Li) counter. The resolution of the photon detector is 600 eV FWHM for $E = 122$ keV. Fast coincidences were recorded using a conventional time-to-amplitude converter. The timing resolution achieved was typically 35 ns (full width at $\frac{1}{10}$ of the maximum). Pileup rejection circuits for both the Si(Li) and the Ge(HP) detectors discarded signals that occurred with pulse pair separations between 0.2 and 5.0 μs .

Two independent measurements were performed, with only the total coincidence efficiency being changed from 0.8% to 1.5%. In the energy interval from 250 to 900 keV we have thus accumulated 10^5 coincident electron events. One of the obtained β spectra is shown in Fig. 2. Coincidences between the 944 keV K conversion electrons and gadolinium K x rays served as a convenient *in situ* calibration for the Si(Li) detector.

In order to compare the measured electron pulse spectrum with a theoretical β spectrum, the response function of the detector has to be taken into account. We measured it for our detector utilizing a ^{207}Bi conversion electron source and a novel technique.¹¹ The gross behavior found agrees well with published data¹² and can be con-

veniently described by the sum of a flat pedestal (containing 65% of the intensity) and a Gaussian peak at the original energy with 14 keV (FWHM) resolution. From the same experimental data we have also confirmed^{12,13} that the efficiency of this Si(Li) diode is independent of energy.

IV. ANALYSIS

From the ^{158}Tb decay scheme^{7,14} (cf. inset in Fig. 2) we find that two first nonunique forbidden β^- branches lead to excited levels at 317.13 and 98.92 keV, with intensities of 1.1% and 17% per disintegration, respectively. No ground state feed is observed.

Our analysis of the beta spectra rests on the modified B_{ij} approximation^{15,16} since the associated transitions ($I^\pi K = 3^- 3 \rightarrow I^\pi K = 2^+ 0$) are $\Delta K = 3$ forbidden. The beta-decay matrix elements are therefore suppressed relative to their single particle values, which has two consequences: Firstly, the transitions are extremely slow ($\log ft = 11.9$) and secondly, the rank 2 matrix element $\int B_{ij}$ is expected to preponderate. As the (rank 1) matrix element Y may also contribute,^{16,17} these beta spectra are well described by shape factors $S(W) \propto Y^2 + a_1(W)$. Here W stands for the total electron energy in units $m_0 c^2$ and $a_1(W)$ is the shape factor for a first unique forbidden beta decay. In similarly decaying $3^- \rightarrow 2^+$ deformed nuclei such deviations from the ξ approximation have indeed been found.^{15,17}

Since our experiment was not aimed at *extracting* the shape factor we will perform the analysis solely with respect to the endpoint energy and treat Y^2 as a parameter. In order to estimate the resulting spread in the endpoint energies conservatively we can use the extremes, i.e., $Y^2 = 0$ and $Y^2 \gg a_1$ ($\equiv$ allowed shape), respectively.

From our data we created for each of the two experiments three electron spectra by demanding a coincident K x ray, a 99 keV or a 218 keV gamma ray, respectively. For a chosen shape factor we then calculated a series of beta spectra. Each of them was convoluted with the experimentally determined response function and subsequently compared with the experimental spectra by a combined least squares and run test.¹⁸ The endpoint energy is the only free parameter in this comparison apart from the overall normalization, which was found to agree with the source strength within $\pm 20\%$. The results are compiled in Table I.

For the cases of the 99 keV gamma ray, as well as the Dy K x ray gated spectra, we have made allowance for the additionally present weak beta branch (to the 317 keV lev-

TABLE I. Summary of coincidence results.

Coincidence with	Populated level E (keV)	Shape factor S	Endpoint energy (keV)	χ^2
γ ray	99	const	841.5 ± 1.5	1.06
γ ray	99	a_1	817.5 ± 2.0	1.09
γ ray	317	const	628 ± 7	1.1
Dy K x ray	99	const	845 ± 5	1.1
Average beta decay energy $830 + 98.9 \pm (1.5 + 2.5 + 12)$ keV				

el). In analyzing this branch we assumed the allowed shape only.

The conversion electrons of ^{207}Bi and ^{158}Tb (EC) served for the calibration. Thereby the energy loss is implicitly taken into account and the contribution to the overall uncertainty is negligibly small.

A systematic uncertainty, however, which does remain is due to the response function. To test the sensitivity of the extracted endpoint energies we return¹² to the parametrization by a single quantity, the ratio peak-area-to-total-area, r . By varying its magnitude around the experimental value, $r=0.35\pm 0.02$, we deduce a spread $\Delta Q_\beta=2.5$ keV. It is reassuring to note that one can neglect the independent knowledge on the response function and extract both parameters, r and Q_β , simultaneously through a two-dimensional fit. This results in $r=0.36$, thus further supporting our original choice.

The dominant systematic uncertainty arises from the ignorance of the beta shape. Assuming the allowed one, we obtain $Q_\beta=941\pm 4$ keV, in full accord with the published data.⁸⁻¹⁰ For nonallowed shape factors the decay energies turn out to be lower: In the extreme case of $S\equiv a_1$ by as much as 24 keV. Since so little is known for ^{158}Tb on the magnitude of Y^2 we adopt as Q_β conservatively the mean of the two extreme values and assign half of their difference as an additional systematic uncertainty, $Q_\beta=929\pm 16$ keV.

In order to infer a more precise decay energy we need to constrain Y^2 . To do so we turn to the europium isotopes, which exhibit the same $3^- \rightarrow 2^+$ transition and find $0.35 \leq Y^2 \leq 1$.^{15,17} Because of the similarity in the decay of these deformed nuclei it is conceivable that these same limits also apply to ^{158}Tb . (As a remark of unwarranted optimism we note that indeed $Y^2=0.4$ fits our data best: $\chi^2=1.025$ for 550 degrees of freedom.) An analysis of the

resulting beta decay energies yields $Q_\beta=933\pm 6$ keV, which then translates to $Q_{\text{EC}}=1225\pm 7$ keV. This result favors the lower value, $Q_{\text{EC}}=1217.6$ keV, which was brought forward by the nuclear reaction experiments. In fact, we find a probability of only 4% for having decay energies in excess of 1237 keV (based on a normal probability density function).

When we, however, return to the conservative estimate for Q_β we derive $Q_{\text{EC}}=1221\pm 17$ keV, which has little significance with respect to the reported discrepancy.

It is important to finally comment on the magnitude of the quantity Δ . While we have adopted primary data to infer Δ it could conceivably be extracted from mass evaluations, which attempt to establish a consistent set of mass values throughout the N, Z plane. This procedure leads to $\Delta=283\pm 3$ keV,¹⁹ substantially lower than used above. The resulting decay energy, Q_{EC} , is consequently substantially lower, too, and *clearly* rejects the possibility for a K-electron capture branch to the 1187 keV level in ^{158}Gd .

In summary, we have demonstrated that the magnitude of the beta endpoint energy of ^{158}Tb has a bearing on the recent claim that the electron capture Q value is sufficiently large to populate the 1187 keV level in ^{158}Gd . While early published data on Q_β seemed to sustain this claim, we have shown that this support cannot be upheld once proper account of the nonallowed beta shape is taken. Therefore we conclude by calling for a direct and independent remeasurement of the electron capture decay, which should clarify the possible future use of the isotope ^{158}Tb in neutrino mass measurements.

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