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The calculation of the hadronic contribution to the $\alpha(M_Z^2)$

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Abstract

We calculate the hadronic contribution to the $\alpha(M_Z^2)$ using
quasi analytical approximation for the cross section of
 $e^+e^- \rightarrow$ hadrons. We find $\alpha^{-1}(M_Z^2) = 128.97 \pm 0.13$.

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For LEP1 observables at the Z-pole the largest effect from pure QED corrections is the change in the electric charge when going from $q^2 = 0$, where the fine structure constant is measured to $q^2 = M_Z^2$ [1]. The change from $\alpha \equiv \alpha(q^2 = 0)$ to $\alpha(M_Z^2)$ is related to the photon vacuum polarization function $\Pi(q^2)$ via the relation

$$\alpha(q^2) = \alpha(1 - \Pi(q^2))^{-1}, \quad (1)$$

where in the leading log approximation [1]

$$\Pi(q^2) = \Pi_{\text{lept}}(q^2) + \Pi_{\text{had}}(q^2), \quad (2)$$

$$\Pi_{\text{lept}}(q^2) = (\alpha/3\pi) \sum_i Q_i^2 (\ln(q^2/m_i^2) - 5/3), \quad (3)$$

and the hadron contribution $\Pi_{\text{had}}(q^2)$ is directly determined by the total cross-section for e^+e^- annihilation into hadrons [1]

$$\text{Re } \Pi_{\text{had}}(q^2) = -(s/4\pi^2\alpha) \text{P} \int_{4m_\pi^2}^{\infty} R(s') [s'(s - s')]^{-1} ds'. \quad (4)$$

$$\sigma_h(s) = \sigma_{\text{tot}}(e^+e^- \rightarrow \text{hadrons}) = (4\pi\alpha^2/3s)R(s). \quad (5)$$

The evaluation of the integral (4) has been performed in refs. [2,3,4] using the direct experimental data for $R(s)$. The corresponding value for $\alpha^{-1}(M_Z^2)$ is [4]

$$\alpha^{-1}(M_Z^2) = 128.87 \pm 0.12.$$

In this paper we calculate the integral (4) using quasi analytical approximation for $R(s)$ namely we take low energy contribution for $R(s)$ from experimental data and use the theoretical ansatz for high energy contribution for $R(s)$. In fact we use two slightly different methods which lead

to the very similar numbers. We find that

$$\alpha^{-1}(M_Z^2) = 128.97 \pm 0.13 \quad (6)$$

In our calculations we take $M_Z = 91.2$ GeV.

The first method of the calculation of the hadronic contribution to $\alpha(M_Z^2)$ consists in numerical account of all resonances (ρ , ω , ϕ , ψ , ν) and low energy continuum region $1 \text{ GeV} \ll E \ll 2.3 \text{ GeV}$. The continuum contribution to $R(s)$ is determined by step like approximation. In the approximation when strong coupling constant $\alpha_s = 0$ our ansatz is

$$R^{\text{cont}}(s) = 2\theta(s-s_1) + (4/3)\theta(s-s_2) + (1/3)\theta(s-s_3), \quad (7)$$

where we took $s_1 = (2.3 \text{ GeV})^2$, $s_2 = (4.4 \text{ GeV})^2$ and $s_3 = (12 \text{ GeV})^2$. The parameters s_2 and s_3 correspond to the charm and beauty thresholds correspondingly. So, in our model

$$\Pi_{\text{had}}(s) = \Pi_{\text{res}}(s) + \Pi_{1-2.3}(s) + \Pi_{\text{cont}}(s) \quad (8)$$

Here $\Pi_{\text{res}}(s)$ is the contribution of the resonances to $\Pi_{\text{had}}(s)$ and the $\Pi_{1-2.3}$ is the contribution of the continuum region between 1 GeV and 2.3 GeV. For the Breight-Wigner resonance we have approximately [3]

$$\text{Re } \Pi_{\text{res}}(M_Z^2) = 3\Gamma(\alpha M)^{-1}, \quad (9)$$

where M and Γ are the mass and leptonic width of the resonance.

We took the values for the widths and masses from the particle data [5] and have found that $\Pi_{\text{res}}(M_Z^2) = 0.05639 \pm 0.00024$, that is very closed to the value $\Pi_{\text{res}}(M_Z^2) = 0.005562 \pm 0.00018$ which has been used in ref. [2]. For the $\Pi_{1-2.3}(M_Z^2)$ we took the value $\Pi_{1-2.3}(M_Z^2) = 0.001981 \pm 0.000391$ from the ref. [3]. For the central

values of the $\Pi_{\text{res}}(M_Z^2)$, $\Pi_{1-2,3}(M_Z^2)$ and for the approximation (7) we have found that $\alpha^{-1}(M_Z^2) = 129.12$. An account of QCD corrections leads to the appearance of QCD factor $1 + \alpha_s/\pi + k_1(\alpha_s/\pi)^2 + k_2(\alpha_s/\pi)^3 + \dots$ in front of each term in (7). The coefficients k_1 and k_2 have been calculated in refs. [6] and they depend on the number of flavors. For the strong coupling constant $\alpha_s(M_Z^2) = 0.12 \pm 0.01$ we have found that QCD corrections give the contribution to $\Pi_{\text{cont}}(M_Z^2)$ equal to 0.14 ± 0.02 . To estimate the uncertainties of our model we shall assume that the uncertainty related with the choice of s_1 and s_2 is 15%.

Finally we find that

$$(\alpha(M_Z^2))^{-1} = 128.98 \pm 0.06 \pm 0.07 \quad (10)$$

The first error in (10) is the statistical error related with nonexact knowledge of the resonance leptonic widths and the (1 - 2.3) GeV low energy contribution to $R(s)$ and the second error is the error which reflects the accuracy of our model for the continuum contribution to $R(s)$.

The second method of the calculation consists in taking into account of the ρ -, w -, ϕ - resonances and low energy region $1 \text{ GeV} \ll E \ll 2.3 \text{ GeV}$ numerically in dispersion integral (4). an account of the high energy contribution to $\Pi_{\text{had}}(M_Z^2)$ is performed by taking into account nonzero c - and b -quark masses. In the approximation when the strong coupling constant $\alpha_s = 0$ our ansatz for $R_{\text{cont}}(s)$ is

$$R_{\text{cont}}(s) = 2\vartheta(s - s_1) + (4/3)\vartheta(s - 4m_c^2)(1 + 2m_c^2/s)(1 - 4m_c^2/s)^{1/2}$$

$$+ (1/3)\vartheta(s - 4m_b^2)(1 + 2m_b^2/s)(1 - 4m_b^2/s)^{1/2} \quad (11)$$

We take $s_1 = (2.3 \text{ GeV})^2$, $m_c = 1.3 \text{ GeV}$ and $m_b = 4.3 \text{ GeV}$.

Numerically we find that the contribution of the ρ -, ω -, ϕ -resonances and low energy region $1 \text{ GeV} \leq E \leq 2.3 \text{ GeV}$ to $\Pi(M_Z^2)$ is

$$\Pi_{\text{low}}(M_Z^2) = 0.00647 \pm 0.00044 \quad (12)$$

For the approximation (11) and for the central value (12) we find that $(\alpha(M_Z^2))^{-1} = 129.14$. An account of the QCD corrections for the strong coupling constant $\alpha_s(M_Z^2) = 0.12 \pm 0.01$ leads to the additional contribution to $\alpha^{-1}(M_Z^2)$ equal to 0.15 ± 0.03 . To estimate theoretical uncertainties we shall assume that the accuracy in the choice of quark masses is 15%. With this assumption we find that

$$\alpha^{-1}(M_Z^2) = 128.99 \pm 0.06 \pm 0.08 \quad (13)$$

The first error is the "experimental" uncertainty related with non accurate knowledge of low energy region and the second error reflects theoretical uncertainties of our model. So, we have found that both considered models give very similar results. It should be noted that we have used in our calculations the current masses [7] for c- and b-quarks

which are ~10% smaller than the quark masses however we did not took into account ψ - and χ -resonances explicitly, their account is supposed by the current c- and b-quark masses as it takes place in the QCD sum rule method [7]. An account of the two loop QED effects and the use of the renormalization group approach leads to the correction [8]

$$\delta\Pi(M_Z^2) = (3/4\pi)\ln[\alpha/\alpha(M_Z^2)] \approx 0.014 \quad (14)$$

So we have used two different "quasi analytical" methods to calculate $\alpha^{-1}(M_Z^2)$. Both methods give very similar results. Taking into account higher loop correction (14) we find that

$$\alpha^{-1}(M_Z^2) = 128.97 \pm 0.13$$

Our result is slightly higher than the often quoted value [4]

$$\alpha^{-1}(M_Z^2) = 128.87 \pm 0.12$$

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