35

— ICRR 319-94-14

ISSN 1340-3745

SW 94 25

ICRR

ICRR-Report-319-94-14

On the Atmospheric Muon Energy Spectrum in the Deep Ocean and its Parameterization

Atsushi Okada

CERN LIBRARIES, GENEVA

P00024044

(June 1994)

To be published in Astroparticle Physics

**INSTITUTE FOR COSMIC RAY RESEARCH
UNIVERSITY OF TOKYO**

3-2-1 Midori-cho, Tanashi, Tokyo 188, Japan

On the Atmospheric Muon Energy Spectrum in the Deep Ocean and its Parameterization

Atsushi Okada

*Institute for Cosmic Ray Research, University of Tokyo,
Midori-cho, Tanashi, Tokyo 188, Japan*

Abstract

The energy spectrum of those muons in the deep ocean ($\gtrsim 1000$ m) that have been produced in the terrestrial atmosphere and have penetrated seawater is calculated by Monte Carlo simulation and presented in a simplified form for easy use.

(To be published in Astroparticle Physics)

1 Introduction

Recently, several proposals for constructing a deep underwater detector for high energy neutrino astronomy have been made [1] and some prototype detectors have already been deployed. For an underwater detector, the atmospheric muon may make a serious contribution to the background. It is often necessary in the design of the detector as well as in the data analysis to check how much contamination the atmospheric muons could cause in the detection of neutrinos.

The flux of the atmospheric muon in the deep ocean is roughly known experimentally. Also the energy spectrum and the angular distribution there can be calculated from the available data at the surface and the known energy loss processes in water. But, the presentation of such calculations in the literature so far is not very convenient when applying those results to other works. No one has ever presented the muon energy spectrum deep underwater very explicitly.

In such a calculation, the fluctuation of the muon energy loss must be taken into account, whether the method is analytical [18,19] or Monte Carlo [20,21]. The relatively recent paper of Lipari and Stanev [21] includes an interesting result on the neutrino induced muon intensity related to that fluctuation. But, here, restricting our subject to the atmospheric muon in the deep ocean, we calculate the energy spectrum at various depths and zenith angles by means of a Monte Carlo simulation, and propose a simple expression for the energy spectrum with parameters depth and zenith angle.

2 Monte Carlo simulation of muon energy loss in seawater

As standard sea water, we assume the chemical composition listed in Table 1 [2].

Table 1

Assumed chemical composition of sea water [2]. As the sum of the percent by weight just fails to reach 100%, we use normalized values for the present work.

Atomic Symbol	H	O	Na	K	Mg	Ca	Cl	S
Atomic Number (Z)	1	8	11	19	12	20	17	16
Atomic Weight (A)	1.0	16.0	23.0	39.0	24.3	40.0	35.5	32.0
Percent by weight	10.72	85.78	1.05	0.04	0.13	0.04	1.90	0.09

We consider four different processes of muon energy loss in seawater: ionization and excitation, bremsstrahlung, direct electron-pair production, and inelastic nuclear interaction.

The first process, ionization and excitation, is assumed to take place continuously along the muon track. The energy loss is given by,

$$a(E) = 0.00849 \ln\left(\frac{W}{\mu}\right) + 0.217 \quad (\text{GeV hg}^{-1}\text{cm}^2) \quad (1)$$

where $W = E/(1 + \mu^2/2mE)$, with muon energy E , muon mass μ , and electron mass m . For this process only, we simplify the composition given in Table 1 to pure water and 3.5% NaCl, as the more detailed version makes no difference to the result.

The other three processes bring about catastrophic energy losses and, as is well known, each averaged energy loss is approximately proportional to the muon energy. Then the total energy loss is expressed conventionally as,

$$-\frac{dE}{dx} = a(E) + b(E) E \quad (2)$$

where,

$$b(E) = b_{brem}(E) + b_{pair}(E) + b_{nucl}(E) \quad (3)$$

corresponding to bremsstrahlung, electron-pair production and nuclear interaction, respectively.

Omitting detailed discussion of the small differences among published papers, we use those of Petrukhin and Shestakov [3] for bremsstrahlung, Kokoulin and Petrukhin [4] for electron-pair production, and Bezrukov and Bugaev [5] for nuclear interaction, specifically, the version which takes into account the nuclear shadowing effect [6]. Those cross sections, at fixed energies in seawater (composed of atoms in Table 1), are shown in Fig.1 and the energy dependences of the $b(E)$ terms in Fig.2. But, we note that, in order to include the contribution of the orbital electrons, the factors of Z^2 for bremsstrahlung and electron-pair production in the cited papers have been replaced by $Z(Z+1.2)$ [7,8], a correction which is not negligible for light atoms.

Knock-on process [9] can occasionally yield electrons with high fractional energy, i.e. with a large ratio of transferred energy to the muon energy. For muons with relatively low energy ($\lesssim$ several hundred GeV) in a light substance like water,

the probability is rather higher than that of electron-pair production. Therefore, the knock-on process included in the above $a(E)$ may be regarded as one of the catastrophic processes, but as far as the present range of water depth is concerned, the assumption of continuous energy loss for knock-on does not lead to any significant error compared to the other uncertainties.

The method used for the present Monte Carlo simulation is as follows. The catastrophic energy losses are assumed to take place at random during the continuous energy loss. The probability p that the muon passes without a catastrophic energy loss decreases with the path length x , while the continuous energy loss keeps taking place. Then, the following two equations hold simultaneously:

$$-\frac{dp}{dx} = N \sigma_{cat}(E) p \quad (4)$$

$$-\frac{dE}{dx} = a(E) \quad (5)$$

where N and $\sigma_{cat}(E)$ are Avogadro's number and the cross section per nucleon for the catastrophic energy loss, respectively. If $a(E)$ in eq.(5) is approximated to $a(E_0)$, the value at $x = 0$ with energy E_0 , then eq.(4) can be integrated approximately using the cross section expanded with respect to x . The first order approximation gives

$$-\ln p \approx N \sigma_{cat}(E_0) x - N \left(\frac{d\sigma}{dE} \right)_0 a(E_0) \frac{x^2}{2}. \quad (6)$$

Here, x is smaller than x_c given by

$$x_c = \frac{\sigma(E_0)}{a(E_0) \left(\frac{d\sigma}{dE} \right)_0}. \quad (7)$$

By substituting a random number between 0 and 1 for p in eq.(6), we obtain x , the path length between catastrophic energy losses. However, since the approximation in eq.(6) is valid only for small x , we set $x_{max} = 0.1 x_c$ for the present calculation and, when the obtained x is larger than x_{max} , we move the muon forward by x_{max} with only continuous energy loss and try again to obtain an x in the proper range from a new random number.

Once x for the catastrophic energy loss is obtained, the continuous energy loss up to the position is calculated and the energy deposited in the catastrophic interaction is determined from another random number r , by solving the following equation for y :

$$r = \int_y^{y_{max}} \frac{d\sigma(E, y)}{dy} dy \bigg/ \int_{y_{min}}^{y_{max}} \frac{d\sigma(E, y)}{dy} dy = \int_y^{y_{max}} \frac{d\sigma(E, y)}{dy} dy \bigg/ \sigma_{cat}(E) \quad (8)$$

where y is the ratio of the transferred energy to the muon energy. Therefore, the muon energy just after the catastrophic energy loss is given by $(1 - y)E$.

In order to calculate x from eq.(6) and y from eq.(8), we prepared a numerical table of $\int_y^{y_{max}} \frac{d\sigma(E, y)}{dy} dy$ for discrete points of $\ln E$ and y , and interpolated from the table at each step of the calculation.

3 Energy spectra and parameterization

We took the muon energy spectrum at the surface (i.e. sea level) and performed a Monte Carlo simulation of the energy loss process of each generated muon in seawater until it reached a specified depth or consumed its energy before that. The muon spectrum at sea level is more uncertain than the energy loss processes. The existing experimental data are not always consistent with each other. Here we use Miyake's empirical formula [10] as the sea level spectrum. This was derived from data on underground muon intensities and was compared with data from magnet spectrometers at the surface. We believe it is still one of the best empirical formulae for the muon spectrum. The integral energy spectrum of Miyake is

$$F(E, 0, \theta^*) = \frac{1.74 \cdot 10^6}{5 E \cos \theta^* + 400} (5 E + 10 \sec \theta^*)^{-1.57} \frac{E + 15}{E + 10 + 5 \sec \theta^*} \quad (\text{m}^{-2} \text{s}^{-1} \text{sr}^{-1}) \quad (9)$$

where E is muon energy in units of GeV and θ^* denotes the zenith angle at the altitude where the muon is produced. We use the following relation between zenith angle θ at sea level and θ^* , assuming a fixed depth of 100 g cm⁻² for the muon production in the US standard atmosphere [11]:

$$\sin \theta^* = \frac{6370 \sin \theta}{6415.5 - 6.335 \ln(100 \cos \theta^*)}. \quad (10)$$

The differential muon spectra for several different depths obtained using the above formula are shown in Fig.3, and those for a few different zenith angles and a fixed depth are shown in Fig.4-a and Fig.4-b. The curves in the figures are calculated from the following integral muon energy spectrum which we propose for the first time:

$$F(E, D, \theta) = \frac{c_1 (e^{c_2 X} - c_3)^{-\beta}}{(\cos \theta^*)^{G_0(X, E)}} \frac{\{1 + E/G_1(X)\}^{1-\beta}}{\{1 + E/G_2(X)\}} \quad (\text{m}^{-2} \text{s}^{-1} \text{sr}^{-1}). \quad (11)$$

Here,

$$\begin{aligned} c_1 &= 7.07 \cdot 10^{-4}, \quad c_2 = 3.084 \cdot 10^{-4}, \quad c_3 = 0.979, \quad \beta = 2.57, \\ G_0(X, E) &= 1 - 28/(E + 0.1 X), \\ G_1(X) &= (X - 250) (1.812 \cdot 10^{-9} X^2 - 5.798 \cdot 10^{-5} X + 0.5267), \\ G_2(X) &= 4.25 \cdot 10^{-2} X + 130, \text{ and} \end{aligned}$$

$X = D \sec \theta$, with the vertical depth D in hg cm⁻². The muon energy E is again in units of GeV.

We have checked eq.(11) in the range of water thickness (more specifically slant depth) $500 \text{ m} \lesssim X \lesssim 14000 \text{ m}$ and found that eq.(11) well reproduces the Monte Carlo data within $\sim 10\%$ error in the depth and energy ranges $D \gtrsim 1000 \text{ m}$ and $3 \text{ GeV} \lesssim E \lesssim 10000 \text{ GeV}$. But, even at shallower depths, $D \approx 500 \text{ m}$, eq.(11) is still good enough, if one can tolerate $\sim 15\%$ deviation at the lower energy region $E \lesssim 200 \text{ GeV}$ at large zenith angles $\theta \gtrsim 70^\circ$.

The righthand side of eq.(11) has no particular physical interpretation as a whole, but the first factor,

$$F(0, D, 0) = c_1 (e^{c_2 D} - c_3)^{-\beta} \quad (12)$$

which equals the total vertical intensity, is analogous to the simplified intensity derived from eq.(2) by assuming constant $a(E)$ and $b(E)$ and no fluctuation of muons with range:

$$F(0, D, 0)_{simplified} \sim \left\{ \frac{a}{b} (e^{bD} - 1) \right\}^{-\beta}. \quad (13)$$

eq.(11) includes an approximation concerning the zenith angle dependence of the muon intensity at the surface, specifically, that it is proportional to $\sim (\cos\theta^*)^{-G_0(0,E)}$. A simpler approximation $G_0(0, E) = 1$ holds at very high energy where the probability of pion and kaon decay in the atmosphere is small enough compared to their interaction probability. In practice, that condition corresponds to $D \gtrsim 3000 \text{ hg cm}^{-2}$ in water.

The numerical value of the total vertical intensity $F(0, D, 0)$ in eq.(12) is almost identical to the one referred to by Babson et al. [12] which was a form converted from another empirical formula of Miyake for the underground depth-intensity relation [10]. Since Miyake's formula for the sea level muon spectrum eq.(9) is based on the underground measurements, the agreement is reasonable.

However, as the world survey of the direct underwater measurements [12] is widely spread, eq.(12) is not confirmed experimentally yet. Moreover, the data of MUTRON [13] and DEIS [14] indicate that the energy exponent of the sea level spectrum above a few TeV is ~ 2.7 rather than $\beta = 2.57$ used above, which must be considered in addition to the expected contrary tendency due to the prompt muon production at higher energy.

Inazawa and Kobayakawa [15] considered such effects. Their analytically calculated muon intensity without the prompt muon contribution does not much differ from the present one, but the ratio of their vertical intensity to eq.(12) decreases gradually with increasing depth as seen in Table 2 [16]. This is mainly due to the difference of the assumed muon intensity at the surface, although their $b_{brem}(E)$ is about 5% lower than the present one. They refer mainly to Komori and Mitsui [17] for the vertical muon intensity at sea level. The ratios of their intensity to eq.(9) are 1.15, 0.87, and 0.71 at 500 GeV, 5 TeV and 50 TeV, respectively.

Table 2

The vertical muon intensity ratio of Inazawa and Kobayakawa [15] without the prompt muons to eq.(12).

Depth (hg cm^{-2})	1500	2000	4000	6000	8000	10000	14000
Ratio ref.[15]/eq.(12)	1.28	1.21	1.07	1.00	0.96	0.91	0.89

For comparison, we also show our correction factors at several water depths in Fig.5 together with those of Inazawa and Kobayakawa [15]. The correction factor is defined as the ratio of the muon intensity obtained without fluctuation of the energy loss to that obtained with the real fluctuation. No fluctuation means, in other words, an assumption that all the energy losses in eq.(2) are continuous. The present correction factors at large depths are a little smaller than ref.[15] at $\beta = 2.57$, probably because of the difference of $b_{brem}(E)$ mentioned above.

Lastly, it should be mentioned that we have not taken account of multiple Coulomb scattering. The real zenith angular distribution of low energy muons ($\lesssim$

10 GeV) smears a little because a muon of energy E GeV has been scattered by $\sim 4/\sqrt{E}$ (degrees) on average.

4 Concluding remarks

The proposed simplified form eq.(11) of the muon spectrum in the deep ocean is useful in the ranges of depth $\gtrsim 1000$ m and energy from a few GeV to ~ 10 TeV. Of course, eq.(11) can be extended to higher energy over 10 TeV, but an uncertain factor due to the prompt muon production is introduced to the corresponding sea level muon intensity.

Although we have tried to make it as simple as possible, eq.(11) is still a fairly complicated formula. However, being an integral energy spectrum, it can be easily used for generation of random muon energies in a Monte Carlo study. In this paper, we have discussed the spectrum solely in the ocean, but we naturally believe that similar forms to eq.(11) will be useful to express also spectra in fresh water and standard rock.

Acknowledgements

This work was motivated through studies of DUMAND, an abbreviation of Deep Underwater Muon and Neutrino Detector. We owe our gratitude to the international collaborators of the DUMAND project. We are particularly grateful to Prof. Y. Ohashi (ICRR, University of Tokyo) for his having given the opportunity for the present work. We also thank Prof. K. Kobayakawa (University of Kobe) for valuable advice and discussions, and Dr. M.A. Barton (University of Queensland) for careful reading of the manuscript. This calculation was done using the computer facility of the Institute for Nuclear Study, University of Tokyo.

References

- [1] J.G. Learned, Proc. 13th European Cosmic Ray Symposium, Geneva, 1992 (CERN, Geneva, 1992).
- [2] For example, see A.A. Belyaev, I.P. Ivanenko and V.V. Makarov, Proc. 1976 DUMAND Summer Workshop, University of Hawaii, (1976) 563.
- [3] A.A. Petrukhin and V.V. Shestakov, Canad. J. Phys. **46** (1968) S337.
- [4] R.P. Kokoulin and A.A. Petrukhin, Proc. 11th Inter. Conf. Cosmic Rays, Budapest, **29** Suppl.4 (1970) 277.
- [5] L.B. Bezrukov and E.V. Bugaev, Proc. 17th Inter. Cosmic Ray Conf., Paris, **7** (1981) 90.
- [6] For example, see A. Okada et al., Fortschr. Phys. **4** (1984) 135.
- [7] A.G. Wright, J. Phys. **A6** (1973) 79.
- [8] L.B. Bezrukov and E.V. Bugaev, Proc. DUMAND 1979 Summer Workshops at Khabarovsk and Lake Baikal (1980) 228.
- [9] H.J. Bhabha, Proc. Roy. Soc. **A 164** (1938) 257.
- [10] S. Miyake, Proc. 13th Inter. Conf. Cosmic Rays, Denver, **5** (1973) 3638.
- [11] K. Maeda, Fortschr. Phys. **21** (1973) 113.

- [12] J. Babson et al.(DUMAND Collaboration), Phys. Rev. D **42** (1990) 3613.
- [13] S. Matsuno et al., Phys. Rev. D **29** (1984) 1.
- [14] O.C. Allkofer et al., Proc. 17th Inter. Cosmic Ray Conf., Paris, **10** (1981) 321.
- [15] H. Inazawa and K. Kobayakawa, Proc. 19th Inter. Cosmic Ray Conf., La Jolla, **8** (1985) 83.
- [16] K. Kobayakawa, private communication.
- [17] H. Komori and K. Mitsui, J. Phys. G **8** (1982) L197.
- [18] M. Mandò and P.G. Sona, Nuovo Cimento **10** (1953) 1275.
- [19] K. Kobayakawa, Nuovo Cimento **47** (1967) 156.
- [20] P.J. Hayman, N.S. Palmer and A.W. Wolfendale, Proc. Roy. Soc. **275A** (1963) 391.
- [21] P. Lipari and T. Stanev, Phys. Rev. D **44** (1991) 3543.

Figure Captions

Fig.1. Cross sections per nucleon in seawater for bremsstrahlung (brem), electron-pair production (pair) and inelastic nuclear interactions (nucl) at two muon energies, 100 GeV (dot-dash curves) and 10000 GeV (solid curves). The horizontal axis indicates fractional energy (y), i.e. the ratio of transferred energy to the muon energy.

Fig.2. Muon energy dependence of the energy loss coefficients $b(E)$ in seawater for bremsstrahlung (brem), electron-pair production (pair), inelastic nuclear interactions (nucl) and the total of the three. The marked points and the solid curves are obtained from the present Monte Carlo simulation and a numerical integration of transferred energy times cross section, respectively.

Fig.3. Differential energy spectra of vertical muons at various depths in the ocean: 500, 1000, 3000, 5000, 7000, 9000, and 11000 hg cm^{-2} from top to bottom. The marked points are obtained from the present Monte Carlo simulation and the dotted curves come from the parameterized eq.(11) in the text.

Fig.4-a. Differential energy spectra for various zenith angles at a depth of 4500 hg cm^{-2} in the ocean. The zenith angles are 20° , 40° , 60° , 65° and 70° from top to bottom. The marked points are obtained from the present Monte Carlo simulation and the dotted curves come from the parameterized eq.(11) in the text.

Fig.4-b. Same as Fig.4-a but at a depth of 1000 hg cm^{-2} and zenith angles of 20° , 40° , 60° , 70° and 80° from top to bottom.

Fig.5. Correction factor against seawater depth. The correction factor is defined as the ratio of the muon intensity obtained without including fluctuation of the energy loss to that obtained with the real fluctuation. The full circles are the present results for the vertical muon intensity, and the lines are from Inazawa and Kobayakawa [15]. Each line corresponds to a power spectrum at sea level with index $\beta = 2.5$, 2.7 and 3.0 , respectively.

Fig.1: Cross Sections of Muon Interactions

Fig.2: B-terms for Muon Interactions

Fig.3: Energy Spectra at Various Depths

Fig.4-a: E Spectra at Various θ (4500 hg/cm^2)

Fig.4-b: E Spectra at Various θ (1000 hg/cm^2)

Fig.5: Correction Factor