NASA LHEA 93-25
SCW 9425

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COMPTONIZATION MODELS AND SPECTROSCOPY
OF X-RAY AND GAMMA-RAY SOURCES

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ABSTRACT

We present the results of comparisons of the Generalized Comptonization model (Titarchuk 1994) with Monte-Carlo calculations. The new analytical model, which takes into account the relativistic effects and Klein-Nishina corrections, would be very effective diagnostic for Gamma and X-ray sources. We show that new technique, in particular, makes possible a more detailed spectral data analysis where the typical temperature and energy range is up to 10^3 keV and Thomson plasma optical depths down to 0.1. We also give computational confirmation that the shape of upscattered emergent spectra is determined by only two parameters, namely, plasma cloud temperature and exponential photon escape rate which is related to the average number of scatterings suffered by photon in the plasma cloud.

Subject heading: radiative transfer - methods: analytical - radiations mechanisms: Compton and inverse Compton - relativity - Comptonization

I. INTRODUCTION

In previous work, the analysis of Compton scattering model has been simplified by considering limited ranges for photon and electron energies, assuming a non-relativistic approximation. The problem was also dealt with by use of numerical techniques (such as Monte-Carlo calculations see e.g. Pozdnyakov et al. 1983). Comptonization model (Sunyaev & Titarchuk 1980, hereafter ST80) presents analytic form of emergent spectrum by assuming nonrelativistic photon energies and plasma temperatures for the diffusion approximation of photon scattering in plasma clouds with optical depth τ_0 much greater than 1. A proper description for photon scattering in plasma cloud of arbitrary optical depth and relativistic temperature requires relativistic corrections or even a full relativistic consideration. Several attempts to develop relativistic approach have been undertaken (Zdziarski 1986, Prasad et al. 1988, Nagirner et al. 1992).

Titarchuk 1994 (hereafter T94) generalizes ST80 model and presents the analytical description of the theory of spectral formation of X-ray and gamma-ray sources in plasma cloud of arbitrary optical depth, where the effects of Comptonization and Klein-Nishina corrections are important. In developing advanced techniques for calculating detailed spectra of astrophysical sources, there is a necessity to take into account the scattering of photons of arbitrary energies (including those of high energy) off arbitrary energy electrons. We show by Monte-Carlo calculation that analytical Generalized Comptonization model T94 is applicable to a wide range of plasma temperatures and energies ($10^{-3} - 2 \times 10^3$ keV) and plasma cloud optical depth as low as 0.1.

II. RADIATIVE TRANSFER IN A HOT PLASMA, EMISSION SPECTRUM

As is known, for non-relativistic scattering of photons by a thermal plasma of temperature T_e , the mean change in the photon frequency is given by $\delta\nu/\nu = (4kT_e - h\nu)/m_e c^2$. When $kT_e \gg h\nu$, the shape of the radiation spectrum is determined, in the diffuse approximation, by the sole parameter

$$y = \frac{kT_e}{m_e c^2} \frac{3\tau_0^2}{\pi^2}, \quad (1)$$

τ_0 being the optical depth in Thomson scattering. For moderate values of y , this results in a power law spectrum $F_\nu \propto \nu^{-\alpha}$ with spectral index $\alpha = \sqrt{9/4 + 1/y} - 3/2$. At frequencies $h\nu \gg kT_e$, the spectrum has a Wien tail. When $y \gg 1$, the entire spectrum has a Wien shape.

Observations usually demonstrate power-law spectra with a spectral index $\alpha \sim 1$, and a cut-off indicating a high temperature $T_e \gtrsim 30\text{-}50$ keV. In this case, Eq. (1) and the above relation between α and y would give a moderate $\tau_0 \lesssim 1$, which violates the diffusion approximation. Under these circumstances, Eq. (1) is inadequate to determine quantitatively the effects of Compton scattering and the relationship between α and y is no longer valid. In order to treat the Comptonization process in this particular regime, one has to carry the radiation transfer equation beyond the diffusion approximation by solving the full radiative transfer equation. In this case, relativistic corrections are required and a novel parameter replacing y can be determined by the first eigenvalues of the kinetic equation for the transfer radiation problem. Titarchuk (1994) presents the general solutions of the radiative transfer in isothermal plasma clouds.

The problem of spectral and angular distribution of photons undergoing scatterings in a hot plasma cloud, where the hard photon spectrum is formed by those photons subject to much more scatterings than the average number of scatterings $\bar{u}$, has been analyzed by ST80 and Sunyaev & Titarchuk (1985, hereafter ST85). The solution of this problem is closely connected with the distribution law of number of scatterings. The exponential tail of the distribution over the collision number is a typical feature of the problem of photon escape from a limited region of space. The probability that a photon undergoes u scatterings, where $u \gg \bar{u}$, is given by the asymptotic relation:

$$P(u) = A(\bar{u}, \tau_0) \exp(-\beta u), \quad (2)$$

where τ_0 is the optical depth of the plasma cloud and the normalization constant $A(\bar{u}, \tau_0)$ depends on the distribution of low-frequency photon sources inside the plasma cloud (ST80, 85). For parameter β , asymptotic forms of its higher-order approximation were derived (T94) for disk and sphere geometries with both small and large optical depths:

$$\beta = \frac{\pi^2}{12(\tau_0 + 2/3)^2} (1 - e^{-1.35\tau_0}) + 0.45e^{-3.7\tau_0} \ln \frac{10}{3\tau_0}, \quad \text{for a disk} \quad (3)$$

$$\beta = \frac{\pi^2}{3(\tau_0 + 2/3)^2} (1 - e^{-0.7\tau_0}) + e^{-1.4\tau_0} \ln \frac{4}{3\tau_0}. \quad \text{for a sphere} \quad (4)$$

The photons which undergo many more scatterings than average produce the typical radiation field (ST85) which can be described by the transport approximation (see e.g. Pomraning 1973). The transport cross section of scattering by electrons at rest is given by

$$\sigma_{tr} = \int (1 - \cos \vartheta) d\sigma_c(\nu \rightarrow \nu') = \sigma_T \varphi(\nu) \simeq \sigma_T (1 + 2.8\nu(\Theta)z - 0.44z^2)^{-1} \quad (5)$$

where $d\sigma_c(\nu \rightarrow \nu')$ is the differential cross section of Compton scattering averaged over Maxwellian distribution; ν and ν' are the frequencies of photons before and after the scattering; $z = h\nu/m_e c^2$; ϑ is the scattering angle; σ_T is Thomson cross section and $\Theta = kT_e/m_e c^2$ is the dimensionless plasma temperature. The factor $v(\Theta) = 1 - 1.1\Theta$ and the function φ is approximated by Grebenev & Sunyaev (1988) and T94 analytically with an accuracy of better than 2% in the range $h\nu < 1$ MeV. The time evolution of the photon energy is determined by the Doppler and recoil effects derived by Prasad et al. 1988 and may be approximated with an error of no greater than 1% in the range $kT_e < 100$ keV by the formula (see Cooper 1971)

$$\eta(z, \Theta) = z^2(1 + 4.6z + 1.1z^2)^{-1}(1 + f_0(\Theta)), \quad (6)$$

where $f_0(\Theta) = 2.5\Theta + 1.875\Theta^2(1 - \Theta)$.

The Comptonization problem for plasma clouds with even smaller optical depth reduces to solving the stationary equation for occupation number $N(x)$ (T94) given by

$$L_\nu N - \beta N = -\psi(x), \quad (7)$$

where L_ν is the energy diffusion operator (e.g., Prasad et al. 1988); $x = h\nu/kT_e = z/\Theta$ is the dimensionless photon energy; $N(x, \tau) = I_\nu c^2/2\nu^2$ is the photon occupation number in phase space and $\psi(x)$ is the primary source spectral distribution.

The homogeneous stationary equation (7), i.e. with $\psi(x) = 0$, is transformed to

$$x^2 N_1'' + x(x(1 + \delta) + 4)N_1' + (x^2\delta + 4x - \gamma)N_1 = 0. \quad (8)$$

Here $\delta = \mu'/\mu$ and $\mu = \eta(x, \Theta)/z^2$ (see eq. 6). $\gamma = \beta/(\Theta\mu\varphi)$ (as for the definition of φ , see Eq. 5). Because γ and δ depend weakly on the dimensionless energy x , for $x < x_*$, where $x_* = 0.5 + \gamma_0$ and $\gamma_0 = \beta/\Theta[1 + f_0(\Theta)]$, the solutions of eq.(8) are expressed approximately by means of the Whittaker functions. In the case of low-frequency primary sources, e.g. $\psi(x) = \delta(x - x_0)$ with $x_0 \ll 1$, the spectrum of photon emerging from the plasma cloud is described by the simple formula

$$F_\nu(x, x_0) = \frac{\alpha_0(\alpha_0 + 3)e^{-x}}{x_0} \left(\frac{x}{x_0}\right)^{-\alpha_0} \frac{\int_0^\infty t^{\alpha-1}(x+t)^{\alpha+3}e^{-t}dt}{\Gamma(2\alpha+4)} \quad \text{when } x_* > x \geq x_0. \quad (9)$$

Here $\Gamma(z)$ is the gamma function and $\alpha(x) = \zeta[\gamma(x)] = \sqrt{9/4 + \gamma(x)} - 3/2$ and $\alpha_0 = \zeta(\gamma_0)$. For given optical depth τ_0 the values of β are obtained from the Eqs (3) or (4).

In order to find the asymptotic form of the spectrum $F(x, x_0)$ for high energies $h\nu > x_*kT_e$ we present the occupation number $N(x)$ in a factorized form $N(x) = e^{-x}e^{-f(x)}$. In this case equation (7) reduces to Riccati's equation for $f(x)$ as analyzed by T94. Finally we obtain spectrum $F(x, x_0)$ for $x > x_*$ through the formula

$$F(x, x_0) = \simeq c_0 x^{3-b_1} e^{-x(1+b_0)}, \quad (10)$$

where $b_0 = (\sqrt{1 + 54.2q\Theta^2\gamma_0} - 1)/2$, $b_1 = (4b_0 + 7.4q\Theta\gamma_0)/(1 + 2b_0)$ and the coefficient c_0 is determined by the continuity condition between the two parts of the spectrum (eq.(9) and eq.(10) at $x \simeq x_*$). The factor $q(\Theta) = 1 - 1.046\Theta$. It is worthwhile noting the difference between the nonrelativistic hard tail and the relativistic one. Instead of a Wien tail $x^3 e^{-x}$ in the nonrelativistic case we have the steeper tail $x^{3-b_1} e^{-x(1+b_0)}$ in the relativistic case.

As emphasized in ST85, T94 and established by our Monte-Carlo calculations in the case of small optical depths $\tau_0 < 1$ and high temperatures $kT_e > 50$ keV photons which

undergo many more scatterings than average from the specific collimated radiation field concentrated along the maximum dimension of plasma cloud (along the plane in disk geometry and along the diameter in spherical geometry). As a result of averaging over the collimated angular distribution, the energy diffusion coefficient $\eta(z, \Theta)$ (see Eq.(6)) changes its temperature dependent factor, $1 + f_0(\Theta)$, from that in Eq.(6) to $(8 + 19\Theta)/(8 + 15\Theta)$ (T94) and the transport cross section σ_{tr} (see eq. (5)) remains almost constant and equal 1 for the whole energy range. The emergent spectrum is described by Eq.(9) where γ and γ_0 could be replaced by expressions

$$\gamma = \gamma_0(1 + 4.6z + 1.1z^2)/\Theta \quad \text{and} \quad \gamma_0 = \beta(8 + 15\Theta)/[\Theta(8 + 19\Theta)]. \quad (11)$$

DISCUSSION AND CONCLUSIONS

It is shown by T94 and confirmed by our Monte-Carlo calculation that the nonrelativistic spectra ST80 are satisfactory for temperatures $kT_e < 30$ keV and energies $h\nu < 50$ keV with optical depth of plasma cloud $\tau_0 > 2$. The comparison of the Generalized Comptonization model (T94) with the nonrelativistic model of ST80 and Monte-Carlo calculations is presented in Fig.1, which shows that in the case of plasma temperatures less than 150 keV and optical depths from 1 and higher, the new analytical solution Eqs. (9), (10) works much better than even the modified nonrelativistic approximation of ST80 and matches excellently the numerical solution of eq.(7) and the Monte-Carlo calculations. The modified nonrelativistic approximation of ST80 is obtained from ST80 nonrelativistic solution by introducing the additional temperature factor $[1 + f_0(\Theta)]^{-1}$ in the γ parameter, i.e. instead of β/Θ we have $\beta/[\Theta(1 + f_0(\Theta))]$. Equation (9) is converted into the modified nonrelativistic approximation of ST80 by replacing $\alpha(x)$ to α_0 and x_* to ∞ . For plasma

temperature higher than 150 keV and optical depth greater than 5 (Comptonization parameter, $y \gg 1$ or small power law indices, $\alpha \ll 1$) the saturated Comptonization (Bose-Einstein or Wien) spectra are formed. Thus this case is trivial.

Figs.2 display that in the optical thin case ($\tau_0 < 2$) the emergent spectra are described quite well by formula (9) with γ and γ_0 determined by Eq. (11).

To conclude: (i) Analytical Compton models with relativistic corrections included make possible the reliable and effective analysis of gamma-ray continuum spectra in a much broader range of physical conditions than possible before; The range of the input parameters which would be used in our analytical models are extremely wide: the limits of plasma cloud optical depth, τ_0 from 0.1 and higher and the limits of plasma temperature kT_e is up to 1 MeV. (ii) For the same temperature and the same escape rate the same upscattered spectral shapes are formed despite of the cloud geometries (see Fig. 3).

The proposed analytical approach will make it possible to implement the effective data analysis of the list of BHC and AGN sources obtained by such instruments as OSSE and BATSE.

L.T. wants to thank NASA and NRC for support under NRC Senior Research Associateship at the LHEA in GSFC. Also L.T. thanks Nick White for support and encouragement in all respects. XMH would like to thank Reuven Ramaty for his suggestion and valuable discussion in the development of Monte Carlo technique for relativistic Comptonization process.

REFERENCES

Cooper, G 1971, *Phys.Rev.D*, **310**, 2312

Grebenev, S.A. & Sunyaev, R.A., 1987, *Pis'ma Astron. Zh.* **13**, 1042 (*Sov. Astron. Lett.* 1987 **13**, 438)

Nagirner, D.I., Kikets, E.V. & Poutanen, Yu. J. 1991, *Tr. Astron. Obs. Leningrad Univ.* **43**, 28

Pomraning, G:1973, *Radiation Hydrodynamics*, Oxford: Pergamon Press

Pozdnyakov, L.A., Sobol, I.M., and Sunyaev, R.A. 1983 in *Astrophysics and Space Physics Reviews*, *Soviet Scientific Reviews* **2**, 189 R.A. Sunyaev ed.

Prasad, M.K., Shestakov, A.I., Kershaw, D.S., & Zimmerman, G.B. 1988, *J. Quantit. Spectrosc. Rad. Transfer*, **40**, 29

Sunyaev, R.A. & Titarchuk, L.G. 1980, *Astron. Astrophys.* **86**, 121

Sunyaev, R.A. & Titarchuk, L.G. 1985, *Astron. Astrophys.* **143**, 374

Titarchuk, L.G. 1994, *Ap. J.* in press

Zdziarski, A.A. 1986, *Ap. J.* **303**, 94

FIGURE CAPTIONS

Fig.1a The comparison of the outgoing photon spectra resulting from the Comptonization of soft photon radiation with $h\nu_0 = 10^{-3}kT_e$ for plasma sphere with the electron temperatures $kT_e = 50$ keV for the modified nonrelativistic Comptonization model ST80 (the upper solid curve) and those calculated with relativistic corrections (Eqs. 7, 8) (solid curve) with Monte Carlo calculations (histogram). Dash line presents relativistic analytical approximation (Eqs. 9, 10)

Fig.1b Same as in figure 1a. for model with $kT_e = 100$ keV.

Fig. 2a The spectra resulting from the Comptonization of soft photon radiation with $h\nu_0 = 10^{-3}kT_e$ in high temperature plasma sphere with $kT_e = 100$ keV and the optical depths $\tau_0 = 0.1 - 1$ are obtained by analytical formula (9) (solid line) with γ and τ_0 determined by Eq. (11) and by Monte Carlo technique (histogram). Dash line corresponds to Zdziarski's analytical approximation (Z86).

Fig.2b Same as in figure 2a. for model with $kT_e = 500$ keV.

Fig. 3 Comparison of the spectra resulting from the Comptonization of soft photon radiation with $h\nu_0 = 10^{-3}kT_e$ in high temperature plasma clouds with $kT_e = 250$ keV and with the same β -parameter for disk ($\tau_0 = 0.2$) and spherical geometries ($\tau_0 = 0.7577$) are obtained by analytical formula (9) (solid line) with γ and τ_0 determined by Eq. (11) and Monte Carlo technique (histogram).

Fig. 10

Fig. 18

Fig. 2a

Fig. 2 b

Fig. 3