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Telephone: (01) 6518770 Telex: (21) 6199 GREEKATOM Telefax: (301) 6511215

DEM-NT-94-05

to appear in *J. Phys. G.*

The q -deformed Moszkowski Model: high-spin states

Constança Providência*, Lucília Brito*, João da Providência*

Dennis Bonatsos[§] and Debora Peres Menezes[†]

* Centro de Física Teórica - Depto de Física

Universidade de Coimbra - P-3000 - Portugal

§ Institute of Nuclear Physics, NCSR "Demokritos"

GR - 15310 Aghia Paraskevi, Attiki, Greece

† CFM - Depto de Física - Universidade Federal de Santa Catarina

Campus Universitário - Caixa Postal 476 - CEP 88049 - Florianópolis - SC

Santa Catarina - Brazil

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Abstract

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Abstract

The q -deformed Moszkowski model with cranking is studied in the mean field approximation. It is shown that the residual interaction simulated by the q -deformation is felt more strongly by states with large J_z . The high-spin states are studied and a sum rule in the rotating frame is obtained. The influence of the q -deformation on the transition between bands as well as the relation between q -deformation and temperature are also discussed.

The $su_q(2)$ algebra has been successfully used in describing both superdeformed and rotational bands in deformed nuclei [1], as well as vibrational spectra of diatomic molecules [2]. In these works the q -deformed algebra is considered as a spectrum generating algebra and the corresponding Casimir operator as a model for the hamiltonian including a class of complicated interactions. This is not the view we follow in the present work. Here we investigate the effect of replacing the $su(2)$ algebra by the corresponding deformed version in the Moszkowski model. This is an exactly soluble model with the $su(2) \times su(2)$ symmetry, in the context of which the rotational and vibrational degrees of freedom, as well as the transition from the vibrational to the rotational regime, can be schematically described [3, 4]. In particular, we have shown in ref. [5] that the usual mean-field and RPA methods based on the Peremolov coherent states are adequate to study the q -deformed Moszkowski model. In this letter we would like to complete the work described in ref. [5] and to understand how the transition between two bands will be affected when quantum algebras are used to describe the symmetries of the system instead of the normal Lie algebras. Using a sum rule approach, this effect is clearly seen in the distribution of the transition strength between the two normal modes of the system. One of these modes describes a transition along the yrast line. The other mode describes a vibration corresponding to a transition between two rotational bands. We use cranking to study the high spin states of the model in the mean-field approximation [6]. We are motivated to study the q -deformed version of the model because it has all the nice properties of the non-deformed version and, moreover, it is more flexible and shows stronger anharmonicities and a higher sensitivity to the value of J_z . The possibility of the q -deformation simulating temperature effects is briefly discussed in the context of the present model.

The Moszkowski model is a two level model, each of the levels being N -fold degenerate with N_a particles of type a and N_b particles of type b . The $su(2) \times su(2)$ hamiltonian which describes the model reads

$$H = c(J_z(a) - J_z(b)) + V(J_x^2 + J_y^2), \quad (1)$$

where c is the energy difference between the two levels, V denotes the interaction strength and $J_i = J_i(a) + J_i(b)$, $J^2 = \sum_i J_i^2$, $i = x, y, z$. The quasi-spin operators J_z , J_+ and J_- satisfy the usual $su(2)$ commutation relations. The physical behaviour of the present system is controlled by the Moszkowski parameter $\lambda = N|V|/c$; $\lambda = 0$ corresponds to the vibrational limit and $\lambda = \infty$ to the rotational limit.

The deformed Moszkowski model is defined in terms of the $su_q(2)$ quasi-spin operators J_+ , J_- and J_z which satisfy the commutation relations [7, 8]:

$$[J_+(i), J_-(j)] = [2J_z(i)]\delta_{ij}; \quad [J_z(i), J_{\pm}(j)] = \pm J_{\pm}(i)\delta_{ij} \quad i, j = a, b, \quad (2)$$

where $[x] = \frac{q^x - q^{-x}}{q - q^{-1}}$, q being the deformation parameter of the algebra. The Casimir operator is $C_q^2 = J_- J_+ + [J_z][J_z + 1]$. In terms of these operators we obtain, for the q -deformed Moszkowski hamiltonian [5, 9]

$$H_q = c(J_z(a) - J_z(b)) + V(C_q^2(a) - J_z^2(a) + C_q^2(b) - J_z^2(b) + J_+(a)J_-(b) + J_+(b)J_-(a)). \quad (3)$$

It is possible to study the rotational motion in the q -deformed Moszkowski model because the deformation does not prevent the q -deformed J_z from being a constant of motion. It should in addition be noticed at this point that while the J_+ and J_- operators of $su_q(2)$ are connected to the corresponding operators of the usual $su(2)$ by appropriate q -deforming functionals [10], the J_z operator of $su_q(2)$ exactly equals the corresponding operator of $su(2)$. Therefore it is legitimate to use J_z in the framework of the deformed model for the description of rotational effects. It should also be mentioned at this point that if we denote by $J(J+1)$ the eigenvalues of the second order Casimir operator of $su(2)$ and by $[J][J+1]$ the eigenvalues of the Casimir operator of $su_q(2)$, then the quantity J is the same in both cases [11]. Therefore it is legitimate in general to use J as the usual angular momentum for the description of rotations in the framework of deformed $su_q(2)$ models, exactly as in the case of the usual $su(2)$.

As in the non-deformed case, the following commutation relations are satisfied

$$[H_q, J_z] = [H_q, C_q^2(a)] = [H_q, C_q^2(b)] = 0,$$

and, therefore, H_q can be diagonalized in the basis $|\psi_{ab}\rangle = |\frac{N_a}{2}m_a\rangle |\frac{N_b}{2}m_b\rangle$. The q -deformed Moszkowski parameter is $\lambda_q = [N][V]/c$ and, in the mean-field approximation, the transition from the vibrational regime to the rotational one occurs at $\lambda_q = 0.5$, just as in the non-deformed case.

In order to calculate the minimum energy of the system, for a given J_z , we minimize the expectation value of H_q with a cranking constraint. In the independent-particle approximation we describe the ground-state of the system by means of the q -analogues of the coherent states [5, 7, 12-14] defined by

$$|\Psi(\alpha, \beta)\rangle = \exp_q[\alpha J_+(a) + \beta J_-(b)]|0\rangle, \quad \omega < c, \quad (4a)$$

$$|\Psi(\alpha, \beta)\rangle = \exp_q[\alpha J_-(a) + \beta J_+(b)]|0\rangle, \quad \omega > c, \quad (4b)$$

where ω is the cranking frequency. In the last equations $\exp_q(x) = \sum_k x^k/[k]!$ and the ground state $|0\rangle$ is given by $|0\rangle = |j_a, -j_a; j_b, j_b\rangle$ if $\omega < c$ and by $|0\rangle = |j_a, j_a; j_b, j_b\rangle$ if $\omega > c$. The state $|\Psi(\alpha, \beta)\rangle$ is not normalized. The Hartree Fock ground state is described by the wave function $|\Psi(\alpha_0, \beta_0)\rangle$ where the parameters α_0, β_0 ensure that the inequality

$$E_q(\alpha_0, \beta_0) \leq E_q(\alpha, \beta) \quad (5a)$$

holds under the constraint $\mathcal{J}_z(\alpha, \beta) = \mathcal{J}_z(\alpha_0, \beta_0) = j$, where

$$E_q(\alpha, \beta) = \frac{\langle \Psi(\alpha, \beta) | H_q | \Psi(\alpha, \beta) \rangle}{\langle \Psi(\alpha, \beta) | \Psi(\alpha, \beta) \rangle} \quad (5b)$$

and

$$\mathcal{J}_z(\alpha, \beta) = \frac{\langle \Psi(\alpha, \beta) | J_z(a) + J_z(b) | \Psi(\alpha, \beta) \rangle}{\langle \Psi(\alpha, \beta) | \Psi(\alpha, \beta) \rangle}. \quad (5c)$$

In the sequel we introduce the cranking frequency ω in order to impose this constraint. In Fig. 1 we compare the calculated energies of the yrast line states in the rotating system, $E_q(\alpha_0, \beta_0) - \omega \mathcal{J}_z(\alpha_0, \beta_0) + E_{corr}$, for $s = 0.0, 0.1, 0.1i$ (full-,

dashed-, short dashed-line) with the respective exact values (dots, crosses and triangles), where s is related to q by $q = \exp(s)$. In these calculations we have chosen $N = 20$, $c = 0.1$ MeV and $V/c = -0.075$. The term E_{corr} represents the correlation energy, which will be discussed below. We conclude that: i) the q -deformation gives rise to residual interactions such that, for q real, the energy of the states in the lowest band is lowered (each band is defined by a particular value of the total quasi-spin J) and, for q a phase, the opposite effect is observed; ii) the mean field description using the coherent states (4) and including the RPA correlation energy (see below eq.(11)) gives a good account for the exact values.

The time-dependent Hartree-Fock equations in the rotating system are obtained from the Lagrangian

$$\mathcal{L} = \frac{i}{2} \frac{\langle \Psi(\alpha, \beta) | \dot{\Psi}(\alpha, \beta) \rangle - \langle \dot{\Psi}(\alpha, \beta) | \Psi(\alpha, \beta) \rangle}{\langle \Psi(\alpha, \beta) | \Psi(\alpha, \beta) \rangle} - \frac{\langle \Psi(\alpha, \beta) | H_q - \omega J_z | \Psi(\alpha, \beta) \rangle}{\langle \Psi(\alpha, \beta) | \Psi(\alpha, \beta) \rangle}. \quad (6)$$

In order to obtain the RPA modes of the system, we choose, in the deformed phase, $\alpha = \alpha_0(1 + \eta)$, $\beta = \beta_0(1 + \xi)$, where α_0, β_0 correspond to the equilibrium values and, for the non-deformed phase, $\alpha = \eta$, $\beta = \xi$. In both cases, η and ξ are small time-dependent quantities.

The time evolution of the coordinates η, ξ is determined by using the least action principle, with the Lagrangian given by eq.(6). Expanding $\mathcal{L}$ in powers of η, η^*, ξ, ξ^* , and, introducing the canonical variables $\tilde{\eta}(\eta, \eta^*)$, $i\tilde{\eta}^*(\eta, \eta^*)$, $\tilde{\xi}(\xi, \xi^*)$, $i\tilde{\xi}^*(\xi, \xi^*)$ [5], we obtain, for the second order Lagrangian,

$$\mathcal{L}^{(2)} = \frac{i}{2} (\tilde{\eta}^* \dot{\tilde{\eta}} - \dot{\tilde{\eta}}^* \tilde{\eta} + \tilde{\xi}^* \dot{\tilde{\xi}} - \dot{\tilde{\xi}}^* \tilde{\xi}) - \mathcal{H}(\tilde{\eta}, \tilde{\eta}^*, \tilde{\xi}, \tilde{\xi}^*) + \omega J_z(\tilde{\eta}, \tilde{\eta}^*, \tilde{\xi}, \tilde{\xi}^*), \quad (7)$$

where

$$\mathcal{H} - \omega J_z = \frac{1}{2} \begin{pmatrix} \tilde{\eta}^* & \tilde{\xi}^* & \tilde{\eta} & \tilde{\xi} \end{pmatrix} \begin{pmatrix} A & B \\ B & A \end{pmatrix} \begin{pmatrix} \tilde{\eta} \\ \tilde{\xi} \\ \tilde{\eta}^* \\ \tilde{\xi}^* \end{pmatrix} \quad (8)$$

and A and B are two by two real and symmetric matrices. In terms of the variables O_ν, O_ν^* , ($\nu = 1, 2$)

$$O_\nu^* = X_\eta^\nu \tilde{\eta}^* + X_\xi^\nu \tilde{\xi}^* - Y_\eta^\nu \tilde{\eta} - Y_\xi^\nu \tilde{\xi},$$

the Hamiltonian defined in (8) is diagonal [15]

$$\mathcal{H} - \omega J_z = \sum_{\nu=1}^2 \Omega_\nu O_\nu O_\nu^*. \quad (10)$$

The RPA correlation energy is [15]

$$E_{corr} = -\frac{1}{2} \text{Tr} A + \frac{1}{2} (\Omega_1 + \Omega_2). \quad (11)$$

The two RPA modes can be interpreted as: i) an excitation along the yrast line between states of consecutive angular momentum; ii) a vibrational excitation between the yrast line and the first excited band. In the deformed phase the RPA frequency corresponding to the excitation described in i) is zero because the coherent state $|\Psi(\eta, \xi)\rangle$ is not an eigenstate of the total angular momentum, J_z , but $[H_q, J_z] = 0$.

The frequencies Ω_1 and Ω_2 should be interpreted in terms of the normal modes of $H - \omega J_z$. They represent excitations of the system in the rotating frame. As discussed in [16], $\Omega_j = (E_f - E_i) - \omega(J_{zf} - J_{zi})$, where E_i and J_{zi} are the energy and angular momentum of the state in the yrast line under consideration and E_f and J_{zf} are the energy and angular momentum of the respective excited state. In the normal phase the conversion into the laboratory system is straightforward since the operator J_z is diagonal in terms of the RPA bosons. We have, for $\omega < c$ and $\omega > c$, respectively

$$J_z = O_1^* O_1 - O_2^* O_2 \quad \text{and} \quad J_z = N - O_1^* O_1 - O_2^* O_2. \quad (12)$$

In the deformed phase an estimate of the change in angular momentum ($J_{zf} - J_{zi}$) is given by

$$-\frac{1}{2} (\{O_1, J_z\}, O_1^*) + \{O_1, \{J_z, O_1^*\}\}), \quad (13)$$

where $\{, \}$ stands for the Poisson brackets. In Fig. 2 the exact excitation energies for the transition between the yrast line and the first excited band with $\Delta J_z = 0$ (dots) are compared with the excitation energies in the laboratory system, $E_f(J_{zf}) - E_i(J_{zi})$. For each j , the energy gap between the two lowest energy states of adjacent bands, compared with the $q = 1$ result, is larger for $q > 1$, real and smaller if q is a phase. This behaviour is in accordance with the conclusions taken from Fig. 1. Moreover, this energy gap increases more with j for q real than it is observed for the non-deformed model and a slight fluctuation is observed for q being a phase. This is due to the fact that the $su_q(2)$ algebra contains an anisotropy in the z -direction which will increase the anisotropy already present in the Moszkowski model. This effect is felt more strongly in states with a large J_z . The RPA results compare well with the exact values.

We wish now to study the response of the system to a small perturbation. Let $G = \epsilon J_z$ represent this perturbation, ϵ being an infinitesimal constant. In terms of the variables O_ν , O_ν^* , G has the form

$$G = \sum_{\nu=1}^2 c_\nu O_\nu^* + c_\nu^* O_\nu, \quad (14)$$

with

$$c_\nu = i\{O_\nu, G\}. \quad (15)$$

We verify that the transition strength $\Omega_\nu |c_\nu|^2$ satisfies the following sum rule

$$S_1 = \sum_{\nu=1}^2 \Omega_\nu |c_\nu|^2 = -\frac{1}{2}\{G, \{\mathcal{H} - \omega J_z, G\}\}. \quad (16)$$

We represent in Fig. 3 the fraction of the transition strength in the rotating frame corresponding to the excitation along the yrast line, $\Omega_1 |c_1|^2 / S_1$, versus the total angular momentum. The strength is almost totally concentrated on this transition for small and intermediate values of j . However, for high j the transitions along the yrast line and between bands compete and, at the phase transition almost only the vibrational mode is excited. The influence of the deformation parameter s in the distribution of the transition strength is the following: an imaginary s

(short-dashed line) allows an excitation of the vibrational mode at smaller j than the non-deformed case (full line); for a real s (dashed line) we obtain the opposite effect. This behaviour is easily understood by observing that the RPA vibrational mode is pushed up in energy for s real and is lowered for s imaginary.

The present results confirm the conclusion already drawn in ref. [5], i.e. real values of s correspond to an attractive residual interaction and imaginary values of s simulate a repulsive residual interaction between the particles. This residual interaction is felt more strongly by states with large J_z . An explanation for this behaviour is the fact that the anisotropy introduced along the z -axis, through the $su_q(2)$ algebra, affects more strongly states with large total J_z . Comparing with the non-deformed model one observes that the transition between two bands, at large J_z , will occur more easily when q is a phase and the opposite happens for $q > 1$, real.

It also can be seen that the q -deformation introduces anharmonic effects in the model. This is observed in the results of the RPA calculations which show some deviations from the exact values for $s \neq 0$ (fig. 2). Models which have mainly an harmonic behaviour for $s = 0$, such as the Moszkowski model, acquire non-linear behaviour if $s \neq 0$. It would be interesting to study the importance of the contributions beyond the RPA.

It has already been suggested [17] that temperature can be thought of as a kind of q -deformation. If the results of the present work are compared with the ones obtained in refs [6,16] we conclude that the effects of temperature are similar to those arising from an imaginary q , since:

- i) The ground-state energy increases with q and T . This can be seen by comparing fig. 4 of the present work to fig. 1 of ref. [6], where the ground state energy versus the angular momentum $\langle J_z \rangle$ is depicted.
- ii) The excitation energies decrease with both q and T , as one can see by comparing fig. 2 of the present work to fig. 2a of ref. [16].

iii) The excitation of the vibration is possible at smaller values of $j = \langle J_z \rangle$ as the parameters q and T increase, as one can see by comparing fig. 3 of the present work to fig. 3a of ref. [16].

In the last comparisons it is important to take into account the relation between the cranking frequency and j , which is given in ref. [6]. There are, however, also some differences, namely the fact that as the cranking frequency increases temperature gives rise to a phase transition for a j lower than j_{max} , while in the q -deformed model the transition always takes place at $j = j_{max}$. This difference, which can be seen by comparing fig. 4 of the present work to fig. 1 of ref. [6], could be due to the fact that the one-body term in the non-deformed Moszkowski model has the underlying symmetry $su(N_a + 1) \times su(N_b + 1)$, while the one-body term of the q -deformed hamiltonian used in the present work does not possess an $su_q(N_a + 1) \otimes su_q(N_b + 1)$ symmetry [18,19].

This work has been supported by Junta Nacional de Investigação Científica e Tecnológica (JNICT) and Fundação Calouste Gulbenkian, Lisboa, Portugal.

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Figure Captions

Figure 1 - $E_q(\alpha_0, \beta_0) - \omega \mathcal{J}_z(\alpha_0, \beta_0)$ plus correlation energy, eq. (17), for $s = 0.0, 0.1, 0.1i$ (full-, dashed-, short dashed-line) and the corresponding exact values $E_0 - \omega \mathcal{J}_z$ (dots, crosses and triangles) versus $j = \mathcal{J}(\alpha_0, \beta_0)$.

Figure 2 - Excitation energies in the laboratory frame for the transition corresponding to the vibrational mode for $s = 0.1i, 0.0, 0.1$ (short dashed-, full- and dashed- lines respectively) and the corresponding exact values (dots) versus $\mathcal{J}_z$.

Figure 3 - Fraction of the energy weighted sum rule corresponding to the transition along the yrast line for $s = 0.1i, 0.0, 0.1, 0.2$ (short dashed-, full-, dashed- and dotted lines respectively) versus $j = \mathcal{J}(\alpha_0, \beta_0)$.

Figure 4 - Mean-field energy $E_q(\alpha_0, \beta_0)$ versus $j = \mathcal{J}_z(\alpha_0, \beta_0)$ for $N|V|/c = 1.4$, $N_a = N_b = 28$ and $s = 0.0, 0.03i, 0.06i$, and $0.105i$ from the bottom to the top.