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Antisymmetric Tensor Field Interactions and Neutron Star Models

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Abstract

Gravity theories involving, besides the usual Einsteinian metric field, a macroscopically coupled, finite range antisymmetric tensor field $B_{\mu\nu}$ are studied as foils to the strong gravitational field predictions of general relativity. We consider the minimal class of such models among the multiparameter gravity theories recently introduced by Damour, Deser and McCarthy. In the limiting case where the range μ^{-1} of the B field is much larger than the characteristic size of the system, this class comprises two arbitrary parameters: the coupling length λ , which measures the coupling of $B_{\mu\nu}$ to the baryon rest-mass density, and a dimensionless parameter b characterizing the deviation of the “physical” metric from the Einstein value. We show that when λ is a few kilometers and b of order unity this class of models predicts strong-field deviations of order unity in the overall characteristics of neutron stars: radius, binding energy, surface red-shift and maximum mass.

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1 Introduction

The discovery of binary pulsars [1] has opened a new testing ground for relativistic gravity. In particular, it has been shown [2, 3] that pulsar timing and pulse-structure data provide a unique opportunity for testing the strong-field regime of relativistic gravity: in principle, up to 15 tests of strong-field gravity can be extracted from binary pulsar data by means of the purely phenomenological (i.e. theory independent) "parametrized post-Keplerian" formalism [2, 3]. In order to assess the theoretical significance of such phenomenological tests, it is very useful to have available strong-field foils to general relativity, i.e., alternative relativistic gravity theories exhibiting strong-field deviations from Einstein's theory. Most of the alternative gravity models that have been proposed to this effect are theoretically unsatisfactory in that they violate some standard physical requirements, notably ghost freedom. For instance, the bimetric theory of reference [4] (whose strong-field deviations have been studied in references [5, 6]) contains negative energy excitations (ghosts) [7], and the "nonsymmetric gravity theory" of references [8, 9] has been shown to contain ghosts modes and to exhibit unacceptable asymptotic behaviour [10, 11, 12]. Although the list of theoretically satisfactory gravity models which can pass the weak field (solar system) tests and exhibit strong-field deviations from general relativity is small, it is not empty. It includes a large class of tensor-scalar theories thanks to the recent discovery of non-perturbative strong-field effects [13] (note that the class of tensor-biscalar theories introduced in reference [14] and confronted with binary pulsar observations in references [3, 15] is not free of ghost excitations).

Recently, Damour, Deser and McCarthy [11] have extended the list of potential strong-field foils by introducing a multiparameter class of (ghost-free) theories containing, in addition to the usual metric field $G_{\mu\nu}$, a macroscopically coupled, finite range ¹ antisymmetric tensor field $B_{\mu\nu}$. The basic macroscopic coupling of the B field is through a derivative interaction with a general combination of fermion currents, J_μ :

$$\mathcal{L}_J = \frac{1}{2} f \epsilon^{\mu\nu\alpha\beta} B_{\alpha\beta} \partial_\mu J_\nu \quad (1)$$

For definiteness, we assume here that J_μ is the baryon number current, the current density $\mathcal{J}^\mu \equiv \sqrt{-G} J^\mu$ (where $G \equiv \det G_{\mu\nu}$) being a metric-independent object such that $\int_V d^3x \mathcal{J}^0$ is the total baryon number within the spatial volume V . The strength of the coupling of $B_{\mu\nu}$ to J_μ is measured by the constant f (which has dimension of action). Actually, it is more convenient to replace f by the length

$$\lambda \equiv \frac{f}{m_N} \approx 2(f/\hbar) 10^{-14} \text{ cm} \quad (2)$$

that measures the coupling strength of $B_{\mu\nu}$ to the baryon rest-mass density $m_N J^\mu$ (here m_N denotes some fiducial nucleon mass, say, one atomic mass unit). The other parameters entering the gravity models of reference [11] are the range μ^{-1} of the B field, and various (possibly composition-dependent) dimensionless parameters: $a, b, c, d, e, \alpha, \dots$. Among the latter, the parameters $c, d, e, \alpha, \dots$ induce explicit violations of the equivalence principle beyond the basic one caused by the coupling of $B_{\mu\nu}$ to baryon number. In this paper,

¹The range μ^{-1} of the B field must be finite to avoid consistency problems [11]. However, it can be extremely large, e.g. of cosmological magnitude.

we shall study the minimal models where $c = d = e = \alpha = \dots = 0$. For simplicity, we shall also set a to zero, and retain only the parameter b , defined by requiring that, besides the linear coupling (1), matter is coupled to the B field through a universal dependence on the “physical” metric

$$\tilde{g}_{\mu\nu} = (1 + bB_{\alpha\beta}B^{\alpha\beta})G_{\mu\nu} \quad (3)$$

It was shown in [11] that, when $\mu \neq 0$, the antisymmetric-tensor coupling (1) induces a “fifth-force” interaction, equivalent to the exchange of a finite-range vector ² coupled via $g_5 A_\mu J^\mu$, whose strength with respect to gravity, $\alpha_5 \equiv g_5^2/(4\pi G_{\text{Newton}} m_N^2)$, is related to the coupling length λ and the range μ^{-1} through

$$\alpha_5 = (\lambda\mu)^2 \quad (4)$$

Equation (4) shows that antisymmetric-field models (in contrast to vector ones), can naturally “explain” the non-observation of equivalence principle violations by giving a very large range to B , without having to assume that the basic coupling of B to matter (measured by f or λ) is small.

The purpose of the present paper is to show that if the coupling length λ is of order of a few kilometers ³, the parameter b in equation (3) of order unity and the range μ^{-1} larger than, say, 10^6 km, the only regime for which antisymmetric field models predict observationally significant deviations from

²When $\mu = 0$ the coupling (1) is equivalent to the exchange of an infinite-range scalar coupled to the divergence of J^μ . But, in the massless case, it is inconsistent to introduce the additional (nonlinear) coupling (3) [11].

³Note that $\lambda \sim 1$ km corresponds to a dimensionless coupling constant $f/\hbar \sim 10^{19} \sim m_{\text{Planck}}/m_N$.

general relativity is a strong-gravitational field regime such as that associated with neutron stars. More precisely, we shall see below that when $1 \text{ km} \leq \lambda \leq 10 \text{ km}$, and $1 \leq |b| \leq 4$, the global characteristics of neutron star models (mass, binding energy, radius, redshift) exhibit order unity deviations from the general relativistic case. On the other hand, for such values of the parameters, the predicted deviations from general relativity are negligible in the weak field regime (Newtonian and post-Newtonian approximations): the lower bound on μ^{-1} indicated above corresponds to $\alpha_5 \lesssim 10^{-10}(\lambda/10\text{km})^2$, i.e. to the differences in free-fall accelerations $\Delta a/a \sim \alpha_5 \Delta[\frac{\text{baryon number}}{m/m_N}] \lesssim 10^{-13}(\lambda/10\text{km})^2$, while fractional deviations of post-Newtonian effects in the solar system are of order $(\lambda/r)^2$, which is largest at the surface of the Earth where it reaches a level $\sim 10^{-6}(\lambda/6\text{km})^2$.

2 The field equations

The minimal model that we consider to evaluate strong-field deviations caused by the presence of the macroscopically-coupled field $B_{\mu\nu}$ is given by the Lagrangian density:

$$\mathcal{L} = \mathcal{L}_G + \mathcal{L}_{BJ} + \mathcal{L}_M(\Psi, \tilde{g}_{\mu\nu}) , \quad (5)$$

$$\mathcal{L}_G \equiv \frac{\sqrt{-G}}{4k^2} R_G , \quad (6)$$

$$\mathcal{L}_{BJ} \equiv -\frac{\sqrt{-G}}{k^2} \left(\frac{1}{12} \hat{H}_{\alpha\beta\mu} \hat{H}^{\alpha\beta\mu} + \frac{1}{4} \mu^2 B_{\mu\nu} B^{\mu\nu} \right) \quad (7)$$

where $k^2 = 4\pi G_\bullet$, and $G_\bullet$ is a bare gravitational constant, $G_{\mu\nu}$ is the "Einstein" metric, with determinant G and curvature R_G , used to raise and lower indices, and $\mathcal{L}_M$ denotes the Lagrangian of some matter fields Ψ propagating

in the "physical" metric $\tilde{g}_{\mu\nu} = (1 + bB_{\alpha\beta}B^{\alpha\beta})G_{\mu\nu}$ introduced in equation (3).

The quantity

$$\hat{H}_{\alpha\beta\mu} \equiv H_{\alpha\beta\mu} - k^2 f \eta_{\alpha\beta\mu\nu} J^\nu, \quad H_{\alpha\beta\mu} \equiv \partial_\mu B_{\alpha\beta} + \partial_\beta B_{\mu\alpha} + \partial_\alpha B_{\beta\mu}, \quad (8)$$

denotes a J -modified "field strength" of $B_{\mu\nu}$, and $\eta_{\alpha\beta\mu\nu} \equiv \sqrt{-G}\epsilon_{\alpha\beta\mu\nu}$.⁴

The parameters appearing in the Lagrangian are the coupling constants f , b and the range μ^{-1} of the $B_{\mu\nu}$ field. As explained in the Introduction, this Lagrangian is the simplest non-trivial case⁵ of a more general multiparameter Lagrangian introduced in reference [11]. To explicate the connection of the Lagrangian (5) with the form given in reference [11], we expand the square of $\hat{H}_{\alpha\beta\mu}$ to decompose $\mathcal{L}_{BJ}$ into

$$\begin{aligned} \mathcal{L}_{BJ} &= \mathcal{L}_B + \mathcal{L}_J + \mathcal{L}_{\text{contact}}, \\ \mathcal{L}_B &= -\frac{\sqrt{-G}}{k^2} \left(\frac{1}{12} H_{\alpha\beta\mu} H^{\alpha\beta\mu} + \frac{1}{4} \mu^2 B_{\mu\nu} B^{\mu\nu} \right), \\ \mathcal{L}_J &= \frac{1}{6} f \eta^{\mu\nu\alpha\beta} H_{\alpha\beta\mu} J_\nu \\ &= -\frac{1}{2} f \eta^{\mu\nu\alpha\beta} B_{\alpha\beta} \partial_\mu J_\nu + \text{divergence}, \\ \mathcal{L}_{\text{contact}} &= \frac{1}{2} \sqrt{-G} k^2 f^2 J^\alpha J_\alpha \end{aligned}$$

The term $\mathcal{L}_J$, which describes the basic linear coupling to the baryon number current J^μ , is equivalent to equation (1). The additional contact

⁴ $\epsilon_{\alpha\beta\mu\nu} = \epsilon^{\alpha\beta\mu\nu} = 1, -1, 0$ are metric independent objects; we denote the volume tensor as $\eta_{\alpha\beta\mu\nu} = \sqrt{-G}\epsilon_{\alpha\beta\mu\nu}$, $\eta^{\alpha\beta\mu\nu} = -\frac{\epsilon^{\alpha\beta\mu\nu}}{\sqrt{-G}}$.

⁵If b were zero, the interaction mediated by $B_{\mu\nu}$ would be equivalent to that due to a massive vectorial field with coupling $g_5 = k\mu f$ and would tend to zero as the range μ^{-1} tends to infinity.

term $\mathcal{L}_{\text{contact}}$ ensures that, when b tends to zero (keeping $\mu \neq 0$ fixed), the theory becomes equivalent to that of a massive vector interaction.⁶

To derive the field equations it is simpler to start from the unexpanded form (5), in which one remarks that the covariant components of the J -modified field strength have no metric dependence: $\hat{H}_{\alpha\beta\mu} = H_{\alpha\beta\mu} - k^2 f \epsilon_{\alpha\beta\mu\nu} \mathcal{J}^\nu$. This yields

$$E_{\mu\nu} = 2k^2(T_{\mu\nu}^{BJ} + T_{\mu\nu}^M) \equiv 2k^2 T_{\mu\nu}^T \quad (9)$$

$$\partial_\mu(\sqrt{-G}\hat{H}^{\alpha\beta\mu}) - \sqrt{-G}\mu^2 B^{\alpha\beta} = -bk^2\sqrt{-G}(1 + bB^{\mu\nu}B_{\mu\nu})\tilde{T}_\rho^\alpha B^{\alpha\beta} \quad (10)$$

where $T_{\mu\nu}^M \equiv -\frac{2}{\sqrt{-G}}\frac{\delta L_M(\Psi, \hat{g}_{\alpha\beta})}{\delta G^{\mu\nu}}$, $\tilde{T}_\alpha^\mu \equiv \tilde{g}_{\rho\alpha}\frac{2}{\sqrt{-\hat{g}}}\frac{\delta L_M}{\delta \hat{g}_{\mu\rho}}$ (the physical energy-momentum tensor), $E_{\mu\nu}$ is the Einstein tensor constructed from $G_{\mu\nu}$ and

$$T_{\mu\nu}^{BJ} \equiv \frac{1}{k^2}\left[\frac{1}{2}\hat{H}_{\alpha\beta\mu}\hat{H}^{\alpha\beta}_\nu - \frac{1}{12}\hat{H}^{\alpha\beta\rho}\hat{H}_{\alpha\beta\rho}G_{\mu\nu} + \mu^2(B_{\alpha\mu}B^\alpha_\nu - \frac{1}{4}B^{\alpha\beta}B_{\alpha\beta})\right] \quad (11)$$

The relationship between $\tilde{T}_\nu^\mu$ and $T_{\nu}^{M\mu}$ is given by:

$$T_{\nu}^{M\mu} = (1 + bB_{\alpha\beta}B^{\alpha\beta})^2\tilde{T}_\nu^\mu - 2b(1 + bB_{\alpha\beta}B^{\alpha\beta})\tilde{T}_\rho^\rho B^{\alpha\mu}B_{\alpha\nu} \quad (12)$$

An isolated non-rotating neutron star generates a static spherically-symmetric metric of the form:

$$ds^2 = G_{\mu\nu}dx^\mu dx^\nu = -e^{v(r)}dt^2 + e^{q(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2) \quad (13)$$

As usual, we can define a "mass" function $m(r)$ by:

$$e^{q(r)} = \frac{1}{1 - \frac{k^2 m(r)}{2\pi r}} \quad (14)$$

⁶This is most simply seen in starting from the first order formalism, equation (5.16) of [11] (with $K^{\mu\nu} = 0$), which yields the $\hat{H}_{\alpha\beta\mu}^2$ term upon reduction to second order.

Besides simplifying the field equations, the interest of introducing this mass function is that its asymptotic value as $r \rightarrow \infty$ is the total mass-energy of the system.⁷

The physical energy-momentum tensor $\tilde{T}_\nu^\mu$ is taken to be of the perfect-fluid form $\tilde{T}_\nu^\mu = (\tilde{\rho} + \tilde{p})\tilde{u}^\mu\tilde{u}_\nu + \tilde{p}\tilde{g}_\nu^\mu$.

By spherical symmetry and staticity, $J^\mu = J^0\delta_0^\mu$. Also, the only a priori non-vanishing components of $B_{\mu\nu}$ are its radial "electric" part $B_{01}(r)$ and its radial "magnetic" part $B_{01}^*(r)$ ($B_{\mu\nu}^* \equiv \frac{1}{2}\eta_{\mu\nu\alpha\beta}B^{\alpha\beta}$). However, the field equations (10) imply a homogeneous equation for B_{01} which has zero as the only solution falling off at infinity. Thus, defining $B(r) \equiv e^{-(v+q)/2}B_{01}^*(r)$ we obtain that $B_{\mu\nu} = r^2 \sin\theta B(r)(\delta_\mu^2\delta_\nu^3 - \delta_\mu^3\delta_\nu^2)$ (so that $B_{\alpha\beta}B^{\alpha\beta} = 2B^2(r)$).

The metric independent baryon current density $\mathcal{J}^\mu = \sqrt{-G}J^\mu$ can be written in terms of the locally measured (in $\tilde{g}_{\mu\nu}$ units) number density of neutrons $\tilde{n}$ as $\mathcal{J}^\mu = \sqrt{-\tilde{g}}\tilde{n}\tilde{u}^\mu$ (with $\tilde{g}_{\mu\nu}\tilde{u}^\mu\tilde{u}^\nu = -1$). This yields $J^0 = e^{-v/2}(1 + 2bB^2)^{3/2}\tilde{n}$.

As shown in [10, 11], it is crucial to the consistency of models incorporating a macroscopically coupled antisymmetric tensor field that this field be given a finite range $\mu \neq 0$. The massless limit $\mu \rightarrow 0$ of the finite range models introduces singularities in general. However, it is easily seen that, in the present simple situation of static spherically symmetric configurations, the limit $\mu \rightarrow 0$ is regular.⁸ More precisely, the (asymptotically vanishing)

⁷As $B_{\mu\nu}$ tends to zero at infinity, $G_{\mu\nu}$ and $\tilde{g}_{\mu\nu}$ coincide there so that, contrary to what happens in tensor-scalar theories, there is no ambiguity in the unit system used at infinity.

⁸One way of seeing this is that, contrary to the general case, taking the divergence of the field equation for $B_{\mu\nu}$ gives the identity $0 = 0$ instead of an equation for the divergence

solutions of the B field equation are analytic functions of μ around $\mu = 0$. As we are interested in solutions where μ^{-1} is much larger than the characteristic size of the system, it is therefore numerically allowed here to compute them by solving a simplified system in which one sets $\mu = 0$ from the start.

The field equations (9) and (10) together with $\nabla_\nu^G T_T^{\mu\nu} = 0$ yields the following first order differential system (a prime denotes $\frac{d}{dr}$):

$$B' = e^{q/2}[e^{-v/2}D - k^2 f \tilde{n} \beta^{3/2}] - \frac{2B}{r} \quad (15)$$

$$D' = bk^2 e^{(v+q)/2} \beta B (\tilde{\rho} - 3\tilde{p}) \quad (16)$$

$$m' = 4\pi \rho_T r^2 \quad (17)$$

$$v' = \frac{k^2}{2\pi r^2} (m + 4\pi r^3 p_r) e^q \quad (18)$$

$$p_r' = -\frac{k^2}{4\pi r^2} (\rho_T + p_r) (m + 4\pi r^3 p_r) e^q + 4\frac{\chi}{r} \quad (19)$$

where we have used the auxiliary field variable $D \equiv \sqrt{-G} \hat{H}^{123}$ and defined the following quantities:

$$\beta \equiv 1 + 2bB^2 \quad (20)$$

$$\rho_T \equiv -T^{T_0}_0 = \beta^2 \tilde{\rho} + \frac{e^{-v} D^2}{2k^2} \quad (21)$$

$$p_r \equiv T^{T_1}_1 = \beta^2 \tilde{p} + \frac{e^{-v} D^2}{2k^2} \quad (22)$$

$$\chi \equiv \frac{1}{2} (T^{T_2}_2 - T^{T_1}_1) = b\beta B^2 (\tilde{\rho} - 3\tilde{p}) \quad (23)$$

Given the equations of state relating $\tilde{\rho}$ and $\tilde{p}$ to the number density $\tilde{n}$, these equations can be integrated numerically. The details of this procedure are explained in the next section.

of B as in equation (5.13) of reference [11].

3 The numerical results

In order to integrate numerically the differential system obtained in the last section, it is necessary to impose some boundary conditions at the center and at the surface of the star, and to choose a specific equation of state to model a neutron star⁹. The numerical integration is started at the center of the star, $r = 0$, with the initial conditions $m(0) = 0$, $v(0) = 0$, $B(0) = 0$, $D(0) = D_c$ and $\tilde{n}(0) = \tilde{n}_c$. The vanishing of $v(r)$ at the origin is a choice of coordinate time scale (differing from the natural time scale at infinity). The vanishing of $B(r)$ at the origin follows from the regularity there of the spatial vector $B_{0i}^*(r)$. In the limit (considered here) where $\mu = 0$, the J -modified field strength component $D = \sqrt{-G}\hat{H}^{123}$ is, from equation (16), constant outside the star. As it must vanish (for all values of μ) at infinity, it must vanish (when $\mu = 0$) everywhere outside the star. Thus, in the numerical calculations, the central value $D(0) = D_c$ was iteratively chosen so as to ensure the simultaneous vanishing of D , $\tilde{\rho}$, $\tilde{p}$ and $\tilde{n}$ when r reaches the coordinate radius of the surface of the star, say R_* . The vanishing of $D(r)$ outside the star implies that all the source terms in the right-hand-side of equations (15,17,18,19) vanish when $r \geq R_*$. Therefore the exact solution of our differential system outside the star reads:

$$B(r) = B_*(R_*/r)^2 ,$$

⁹In assuming the existence of a usual local equation of state relating $\tilde{\rho}$ and $\tilde{p}$ to $\tilde{n}$, we are making the approximation that the direct "fifth-force-like" baryon-baryon interaction due to the $\mathcal{L}_J$ coupling is small compared to the effect of the metric coupling of the local neutronized matter moving in $\tilde{g}_{\mu\nu}$.

$$D(r) = 0 ,$$

$$ds^2 = -e^{v_\star}(1 - \frac{k^2 m_\star}{2\pi r})dt^2 + \frac{dr^2}{1 - \frac{k^2 m_\star}{2\pi r}} + r^2(d\theta^2 + \sin^2\theta d\varphi^2)$$

where $B_\star \equiv B(R_\star)$, $v_\star \equiv v(R_\star)$ and $m_\star \equiv m(R_\star)$. In particular we see that $m_\star$ gives the total mass-energy of the system, in contrast with the total baryonic mass given by:

$$M \equiv m_N \int \sqrt{-\dot{g}\dot{n}\dot{u}^0} d^3x = 4\pi m_N \int_0^{R_\star} dr \beta^{3/2} c^{q/2} r^2 \dot{n} \quad (24)$$

To model the structure of a neutron star, we have followed reference [13] in considering the polytropic equation of state

$$\tilde{p} = K n_0 m_N \left(\frac{\tilde{n}}{n_0}\right)^\Gamma \quad (25)$$

$$\tilde{\rho} = m_N \tilde{n} + K \frac{n_0 m_N}{\Gamma - 1} \left(\frac{\tilde{n}}{n_0}\right)^\Gamma \quad (26)$$

where $n_0 = 0.1(\text{Fermi})^{-3}$, $K = 0.0195$, $\Gamma = 2.34$ and $m_N = 1.66 \times 10^{-24}\text{g}$.

The values of the parameters Γ and K have been adjusted to fit the global characteristics of (general relativistic) neutron star models computed using a realistic equation of state (equation of state II of reference [16]).

The numerical results are shown in Table 1 and Figure 1.

Table 1 shows the values of the maximum mass, m (in units of solar masses, $M_\odot$), the fractional binding energy, $e \equiv \frac{M-m}{m}$ (M denoting the baryonic mass, equation (24)), the surface strength of the antisymmetric field, $B \equiv B_\star$, and the surface red-shift, $z \equiv (\frac{\tilde{g}_{00}^\star}{\tilde{g}_{00}(\infty)})^{-1/2} - 1 = (1 + 2bB_\star^2)^{-1/2}(1 - \frac{k^2 m}{2\pi R_\star})^{-1/2} - 1$, for neutron star models computed for different values of the coupling constants λ and b . The values of these quantities in general relativistic models (assuming the equation of state (25)) are $m = 1.95M_\odot$, $e = 0.145$ (i.e. $M = 2.28M_\odot$), $B = 0$ and $z = 0.457$.

When $b > 0$, the maximum mass m increases with b for fixed λ and increases with λ for fixed b . It reaches $m = 2.91M_{\odot}$ for $\lambda = 10$ km and $b = 4$, which is 49.2% greater than the maximum mass predicted by general relativity. On the other hand, z and e are usually a little bit smaller than their corresponding values in general relativity.

When $b < 0$, the maximum mass m decreases with increasing $|b|$ (for fixed λ), reaches a minimum and, then, increases with $|b|$. As in the previous case, m can reach significantly higher values than in general relativity. Its variation with respect to λ for fixed b is qualitatively the same. The values of e and z , contrary to the case $b > 0$, increase with b for fixed λ and increase with λ for fixed b . They can be markedly different from their corresponding values in general relativity. For instance, for $\lambda = 6.4$ km and $b = -4$ we obtain $z = 1.04$ and $e = 0.403$ which are, respectively, 128% and 178% greater than the values obtained in general relativity.

No values for m , e , z and B for $\lambda = 10$ km and $b = -4$ are shown in Table 1 because our numerical algorithm did not converge in that case. When $b < 0$ and λ and $|b|$ are large enough, the function $\tilde{p}(r)$ is no longer a monotonically decreasing function of r for some values of $\tilde{n}_c$. Further study is needed to know whether this unusual behaviour of $\tilde{p}(r)$ signals some limit in the physical applicability of the models considered here.

Figure 1 shows the changes in the gravitational equilibrium configurations of neutron stars for $\lambda = 10$ km with $b = 1$ and $b = -1$. A part of the general relativity equilibrium configurations is also shown, for comparison.

4 Conclusions

We have studied the minimal class of theories containing a macroscopically coupled finite-range antisymmetric tensor field $B_{\mu\nu}$ in a recently introduced multiparameter class of potential strong-field foils to general relativity. We have studied this class in the limit of a very large (though, in principle, finite) range μ^{-1} , and for values of the coupling parameters $1\text{km} \leq \lambda \leq 10\text{km}$ and $1 \leq |b| \leq 4$. For such values, this class of theories predict negligible deviations from General Relativity in the weak-field regime (Newtonian and post-Newtonian approximations). On the other hand, by studying neutron star models (with a $\Gamma = 2.34$ polytropic equation of state) we have explicitly shown that they exhibit significant deviations from General Relativity in the strong-field regime. For instance, when $\lambda = 6.4\text{km}$ and $b = -4$, the maximum mass and maximum fractional binding energy of a neutron star are, respectively, 35% and 178% higher than in General Relativity. It would be interesting to pursue this line of work by studying the observable consequences of such theories in binary pulsar systems which constitute our best experimental laboratories of the strong-field regime of relativistic gravity.

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Table 1

Values of a neutron star maximum mass m (in units of solar masses), fractional binding energy, $e \equiv \frac{M-m}{m}$ (M denoting the baryonic mass), antisymmetric field surface strength B and red-shift $z \equiv (1 + 2bB^2)^{-1/2}(1 - \frac{k^2 m}{2\pi R})^{-1/2} - 1$ (R = coordinate radius) for different values of the coupling constants λ and b , and for a polytropic equation of state $\Gamma = 2.34$ with $K = 0.0195$. The corresponding values in General Relativity ($\lambda = 0 = b$) are $m = 1.95, z = 0.457, e = 0.145$, and $B = 0$.

$\lambda = 3 \text{ km}$	b	1	2	4	-1	-2	-4
	m	1.97	1.98	2.02	1.92	1.90	1.92
	z	0.455	0.430	0.434	0.481	0.488	0.586
	e	0.143	0.147	0.142	0.147	0.151	0.175
	B	-0.0785	-0.0698	-0.063	-0.0941	-0.100	-0.125

$\lambda = 6.4 \text{ km}$	b	1	2	4	-1	-2	-4
	m	2.06	2.17	2.31	1.90	2.09	2.63
	z	0.391	0.407	0.424	0.563	0.775	1.04
	e	0.143	0.144	0.143	0.164	0.233	0.403
	B	-0.164	-0.142	-0.133	-0.228	-0.242	-0.227

$\lambda = 10 \text{ km}$	b	1	2	4	-1	-2
	m	2.27	2.55	2.91	2.23	2.95
	z	0.426	0.413	0.381	0.920	0.846
	e	0.144	0.150	0.149	0.273	0.480
	B	-0.250	-0.223	-0.169	-0.380	-0.340

Figure 1

Fractional binding energy e versus baryonic mass M in units of solar masses for the neutron star models of Table 1 computed within general relativity and antisymmetric field theories with coupling constants $\lambda = 10$ km, $b = 1$ and $\lambda = 10$ km, $b = -1$. The dashed curves represent energetically less favorable sequences of equilibrium configurations obtained when the central density increases beyond the value corresponding to the maximum mass.