The status of the theoretical uncertainties for LEP 1 observables associated with the corresponding comparison among different codes is briefly reviewed.

The achieved experimental accuracy at LEP 1 requires a detailed comparison of the theoretical predictions from different codes with the final goal of an estimate of the theoretical error. This is presently under investigation by the LEP 1 Precision Calculations Working Group and here a short report will be given on the status of the $Z$ parameters.

The comparison will proceed in three different phases by considering the results of five different codes, MIZA [1], LEPTOP [2], TOPAZ6 [3], WOH [1] and ZFITTER [4]. In phase 1 one has taken into account the predictions for the so called pseudo-observables, namely $\sin^2 \theta_w, \sin^2 \theta_t, \ldots$, while in a second step the $Z$ parameters have been compared, i.e. $\Gamma_Z, \Gamma_\mu, \Gamma_e, \ldots$ or $R, r_\tau, \ldots$ or $A_{FB}(\ell), \ldots$. The final step in this project will require a comparison at the level of realistic observables, like $\sigma(e^+e^- \rightarrow f\bar{f})$ and $A_{FB}(e^+e^- \rightarrow f\bar{f})$, including a realistic set-up and $f = e, \mu$. At present the first two phases have been completed. As a result of this work we have realized the fact that there are two complementary objectives. In one case the theoretical uncertainty for a given observable $O$ is estimated as

$$\Delta O = \frac{1}{2} \left( \max_{i=Codes} O_i - \min_{i=Codes} O_i \right)$$

On the other end we can just derive additional informations from each single code by adopting different options which in turns are related to different implementations of higher orders radiative corrections. In the end we propose to compare hands of predictions instead of lines of predictions, as a function of $m_{top}$ or of any other unknown parameter of the standard model. In the following we would like to present a short list of Options with some words of comments.

The scale of $\alpha$ in the final state QED corrections. This is the well known factor $1 + \frac{3}{4} Q_f^2 \alpha$. Actually this should not be an Option at all since the correct scale is $\alpha(m_Z)$ [5], i.e. the correction...
factor is

\[ 1 + \frac{a(m_Z)}{\pi} Q_f^2 \left[ \frac{3}{4} - \frac{1}{4} \frac{a_s(m_Z)}{\pi} \right] + \mathcal{O}(\alpha^2) \] (2)

The scale in the vertex EW corrections. We simply mention the three possibilities, namely \( a(0) \), \( a(m_Z) \equiv G_F \) or \( G_F \) for the leading (subleading) \( m_{\text{top}} \) corrections and \( a(0) \) otherwise. The scale in the FTJR \([6]\) corrections to \( Z \to \bar{b}b \). At the moment the common choice is \( a_s(m_{\text{top}}) \), i.e.

\[ \Gamma_{Z \bar{b}b} = \frac{\alpha_s(m_Z)}{\pi} \] (3)

\[ - \delta \left[ 1 + (3 - \pi^2) \frac{a_s(m_{\text{top}})}{\pi} \right] \] (4)

but one could also isolate the gluon radiation as

\[ \Gamma_{Z \bar{b}b} = \frac{\alpha_s(m_Z)}{\pi} \times \left[ 1 - \delta \left[ 1 - \pi^2 \frac{a_s(m_{\text{top}})}{\pi} \right] \right] \] (5)

\[ \times \left[ 1 - \delta \left[ 1 - \pi^2 \frac{a_s(m_{\text{top}})}{\pi} \right] \right] \] (6)

and the QCD factor \( 1 + a_s(m_Z)/\pi \) is not included for the asymmetry \( A_{FB}(b) \) and for \( \sin^2 \theta_W \).

The aimed accuracy requires that \( m_b \neq 0 \) also in one loop diagrams. At the moment what it is used is

\[ \Gamma_{Z \bar{b}b} = \frac{G_F m_\pi^2}{8\sqrt{2}\pi} \rho \left( g_V^2 + (1 - 6 m_b^2/m_Z^2) g_A^2 \right) \] (7)

However vertex corrections, included in \( g_{V,A} \) are usually computed for \( m_b = 0 \) and this is not fully consistent even if in the missing contributions the leading terms \( \mathcal{O}(m_b^2 m_{\text{top}}^2) \) are absent. Moreover it is an open question what to use for \( m_b \) in this case, the pole mass \( m_b = 4.7 \) GeV or the running one \( \overline{m}_b(m_Z) \)?

The physical Higgs contribution to the correction factor \( \Delta \rho_{\text{Higgs}} \) is not ultra-violet finite and only through the \( \overline{MS} \) prescription we have a finite \( \Delta \rho_{\text{Higgs}}(\overline{MS}) \). After that we are left with the option of a re-summation of such contribution, which makes the result slightly scale dependent. Otherwise we can just decide that all bosonic corrections are expanded to first order.

**Figure 2.** Predictions for \( \Gamma_{Z} \).

The scale of \( \alpha_s \) in the \( \mathcal{O}(\alpha_s) \) corrections to the vector boson self-energies \([7]\). This question is particularly relevant in view of a correct treatment of the \( \tilde{t} \tilde{t} \) thresholds \([8]\) where it has been recently suggested \([9]\) that the non-perturbative effects can be numerically recovered by allowing a relatively small scale in the perturbative expansion, i.e. \( \alpha_s(0,154 m_{\text{top}}) \). The singlet QCD contribution which is simple and unambiguous for the hadronic width but which becomes ambiguous, starting at \( \mathcal{O}(\alpha_s^2) \), for individual \( q\bar{q} \) channels. Actually some sort of agreement has been recently reached on these matter but we want to summarize the roots of the problem \([10]\). From a pragmatic point of view there is a hierarchical description where

\[ \Gamma_{\tau \bar{\tau}} = \Gamma(Z \to \tau \bar{\tau}(g) + \tau' \bar{\tau}') \] (8)

for all \( \tau' \) such that \( m_{\tau'} < m_\tau \). On the other end we could have a democratic description where the final states \( \bar{\tau} \tau + \tau' \bar{\tau}' \) are assigned for \( \frac{1}{3} \) to \( \Gamma_{\tau \bar{\tau}} \) and for the other \( \frac{1}{2} \) to \( \Gamma_{\tau \bar{\tau}'} \). The two descriptions
agree fortunately for the leading terms. In general
one could also decide that such final states should
not be assigned to any specific channels in such a
way that
\[
\Gamma_b = \sum_q \Gamma_{qq} = \sum_q \Gamma(Z \rightarrow \tau \bar{\tau})
\]  
\[
+ \sum_{q,q'} \Gamma(Z \rightarrow \tau q \tau' q') + \ldots
\]

In particular there is an $O(a_s^3)$ contribution
to $\Gamma_{\tau \tau}$ which cannot be assigned to any specific
flavour.

\[\Gamma_b = \sum_q \Gamma_{qq} = \sum_q \Gamma(Z \rightarrow \tau \bar{\tau}) (9)\]
\[+ \sum_{q,q'} \Gamma(Z \rightarrow \tau q \tau' q') + \ldots (10)\]

where $i = V, A$ and where $g_i^0$ is including the
re-summation of universal terms. In computing
partial widths or deconvoluted asymmetries dif-
ferent codes adopt different choices, i.e.
\[
g_i^2 = (g_i^0 + \frac{\alpha}{\pi} g_i^1)^2 \quad (12)\]
\[
g_i^2 = (g_i^0)^2 + \frac{2 \alpha}{\pi} g_i^0 g_i^1 \quad (13)\]

where in the first case we square numerically
and in the second one we expand consistently in
perturbation theory. This two Options lead to
sizeable effects if we consider for instance $A_{FB}$.

Another Option which should be taken into
account is relative to the factorization vs non-
factorization of final state QCD corrections. For
a given channel one can write
\[
\Gamma_{qq} = \frac{G_F m_Z^2}{8 \sqrt{2} \pi} \rho (g_V^2 + g_A^2) (1 + \delta_{QCD}) (14)\]

where $g_V, g_A$ include non-universal vertex cor-
rections. Notice that for $b$-quarks, in order to
avoid double-counting, the FTJR term must not
include $1 + a_s/\pi$. However the complete answer
at $O(a_s)$ is not known and strictly speaking the
QCD correction should only multiply the universal
terms absorbed into $g_V$ and $g_A$. Thus one
can also adopt a non-factorized width both for
$b$ and light quarks. We mention also that the
double scale in the FTJR term, while certainly
gauge-invariant for the leading $m_{top}$ corrections,
remains questionable for the sub-leading and con-
stant ones. Indeed, going back to the $O(a_s)$
corrections, we recall that the full result, even in-
cluding $m_b \neq 0$, is available
\[
a a_s m_{top}^2 [1 + \frac{K}{m_{top}^2} + O(\frac{m_{top}}{m_{top}})] (15)\]

while for vertex corrections to $Z \rightarrow \tau \bar{\tau}$ the
following results are known: for $q \neq b$ the
$O(a_s \text{ const.})$ is missing, for $q = b$ the leading
$a a_s m_{top}^2$ corrections is the well known FTJR term
while all the sub-leading (log and non-log) terms
are missing.

The present choice for the comparisons is
$1/a(m_Z)_{\text{light}} = 128.87 \pm 0.12$ [11] but unfortu-
nately the full updated analysis, including the behav-
ior of $a(p^3)$, has not yet been published.
The running of $m_c$ has been included in the numerical work but it should be noticed that the use of

$$m_m = m_c [1 - \frac{4}{3} x(m_c) + \frac{16}{9} K_c x^2(m_c)]$$

(17)

where $x = \frac{a_s}{\pi}$ gives unreliable results due to the low scale needed in $\alpha_s$. We suggest therefore to use

$$m(m_c) = \frac{m_c}{1 + \frac{4}{3} x(m_c) + K_c x^2(m_c)}$$

(18)

where $m_c = 1.5$ GeV is the pole mass.

In the light of the fact that $m_{top} \approx 1.9 m_Z$ it looks opportune to raise the question whether or not the full two-loop standard model predictions are needed and requested. There are two possible answers, namely the present experimental accuracy plus the uncertainties connected with QED strongly support the idea that we don't need a full two-loop calculation. Actually there are discrepancies among various $O(\alpha^2)$ QED Bhabha generators that should be solved before devoting any attempt towards the two-loop electroweak effects. To the contrary of that a certain requirement of internal consistency, especially illustrated by our choice of options, would suggest that we indeed need them.

In conclusion we can affirm that by comparing the available semi-analytical codes which are based, among other things, on different renormalization schemes ($\overline{MS}$ or on-shell) or the same code with different Options for radiative corrections we derive the result that

$$\Delta_{th} O \ll \Delta_{exp} O$$

(19)

where $O$ is any pseudo-observable or $Z$ parameter with the inclusion of QCD corrections. Certainly the remaining differences must be investigated both for internal consistency and for aesthetical reasons. Hopefully new calculations will contribute in a near future to make some of the Options obsolete and to unify the various treatments of radiative corrections. On the other hand a certain tendency to create a common default, by adopting a common procedure in front of alternative possibilities, should be taken with the due caution.

For cross sections and asymmetry with a realistic set-up the work is in progress among those groups with libraries which are also QED-dressers (BHM, TOPAZ0 and ZFITTER), but the greatest effort must be spent in order to reduce the theoretical error. Certainly the first step will be a detailed comparison for the forward-backward asymmetry $A_{FB}$.

Finally a very small sample of the various comparisons is shown in Figures 1-5. In order to give an idea of the situation we have decided to plot in Figure 1 the various predictions for $\Gamma_c$ as a function of $m_{top}$. The same information is displayed in Figure 2 for $\Gamma_b$. For a better understanding of the differences among the four codes we present

Figure 4. Absolute deviations for $\sin^2 \theta(c)$ from the average value, $m_{Higgs} = 1$ TeV.
in Figure 3-4) the absolute deviation of the four codes for \( \sin^2 \theta(\tau) \) from its average. In Figure 5 we present the theoretical uncertainty for the ratio \( R \) as estimated by TOPAZ0 by switching on and off the various options that we have discussed above.

Figure 5. Predictions and uncertainties for \( R \) from TOPAZ0.

I would like to thank Oreste Nicrosini, Guido Montagna and Fulvio Piccinini for the continuous collaboration. I am also grateful to Dima Bardin and Manel Martinez for stimulating discussions. Finally I would like to thank Tord Riemann for the invitation and the very pleasant atmosphere at this Conference.

REFERENCES


11 F. Jegerlehner, private communication.