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SUPERDEFORMED ROTATIONAL BANDS IN THE PRESENCE OF Y_{44} DEFORMATION

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ABSTRACT : The observation of $\Delta I = 4$ staggering in the rotational spectra of superdeformed nuclei suggests the occurrence of Y_{44} deformations in the nuclear shape with associated C_{4v} point-symmetry for the rotational Hamiltonian. We have investigated the general class of Hamiltonians with such symmetry. In addition, we require the axially symmetric terms to favour rotation about an axis that is perpendicular to the long axis of nuclear shape. The $\Delta I = 4$ staggering can indeed result from the tunneling between the four equivalent minima that occur in the plane perpendicular to the superdeformation symmetry axis, but the occurrence of this effect is a subtle matter depending sensitively on the axially symmetric terms in the Hamiltonian.

In the spectroscopy of superdeformed nuclei, rotational sequences have been observed in which states with $\Delta I = 4$ exhibit a systematic energy displacement with respect to the complementary sequence of states with spins lying half way in between ($[1],[2]$). The occurrence of such a term in the rotational energy suggests that the strongly prolate spheroidal deformation has been slightly perturbed by a term with symmetry Y_{44} with respect to the symmetry axis. In the presence of such a term the rotational energy can acquire a fourth order term proportional to $(I_1^2 - I_2^2)^2$ reflecting the C_{4v} symmetry of the shape. If such a system were to rotate about the fourfold symmetric long axis ($I_3 \approx I$) the occurrence of $\Delta I = 4$ structure in the rotational sequences would be an immediate and direct consequence of the Y_{44} deformation [3]. However, the very stability of the superdeformed shapes results from the large moment of inertia associated with rotation about directions perpendicular to the 3-axis ($I_3 \ll I$). Thus, as we show in the present note, the occurrence of $\Delta I = 4$ rotational structure in the superdeformed yrast region is a rather subtle phenomena which depends in an important manner on the axially symmetric terms in the rotational Hamiltonian [4].

We study the Hamiltonian

$$H = AI_3^2 + B_1(I_1^2 - I_2^2)^2 + B_2(I_1^2 + I_2^2)^2 \quad (1)$$

where we have omitted the I_3 -independent term proportional to $I(I+1)$ that describes the regular I dependence of the energy of the yrast states. The first term in (1) describes the strong preference of the leading order rotational energy for an orientation of $\vec{I}$ perpendicular to the prolate symmetry axis; this implies that we are interested in the region of parameter space where A is positive and large. The B_2 term is the axially symmetric part of the 4th order rotational energy; in order that the angular momentum remain perpendicular to the 3-axis we must require $2B_2I^2 < A$. We can, without loss of generality, confine the parameter space to $B_1 > 0$, provided we study both positive and negative values of B_2 .

It is useful to view the Hamiltonian (1) as defining an energy function on the sphere $|\vec{I}| = \text{constant}$; the positions of the maxima, minima, and saddle points of this function are illustrated in fig. 1, and are seen to depend on the value of the parameter $2(B_1 + B_2)I^2/A$. Quite generally there are four minima in the equatorial plane and in the yrast states the angular momentum will be localized in the neighborhoods of these minima. This structure gives rise to quartets of states, the individual member of which can be labeled by the quantum number ($A1, A2, B1, B2$, and E) of the C_{4v} point symmetry group. The fact that the intrinsic shape carries C_{4v} symmetry implies that the C_{4v} quantum number of a state in the spectrum is determined by the intrinsic configuration and all the states of a given rotational band will carry this same quantum number. (The argument here is identical to that which leads to the condition $K = \Omega$ for the rotational bands associated with an axially symmetric intrinsic shape [5]). Thus, of the four states associated with the minimum of (1) only a single C_{4v} combination is appropriate for generating the rotational band associated with a particular intrinsic state.

To illustrate the variety of yrast structures that can result from the Hamiltonian (1), we have directly diagonalized this Hamiltonian for several different values of the param-

eters (A , B_1 , and B_2) all of which are chosen so that the classical motion corresponds to rotation about an axis in the equatorial plane. (see Table 1 and fig. 2). It is seen that for some values of the parameters the splitting of the quartets implies a regular $\Delta I = 4$ alternation in the energies of the quantum states labelled by the quantum numbers $A1$ or $B1$ (see $H2$), for other values ($H3$) no alternation occurs, while for still others ($H1$) the tunnelling between minima generates an erratic pattern as a function of I , sometimes shifting a given symmetry up and sometimes down without any obvious regularity.

In order to understand these behaviors we examine the semi-classical evaluation of the tunnelling matrix element connecting adjacent minima of the rotational Hamiltonian. For this purpose we introduce canonically conjugate variables (I_3, ϕ) on the spherical surface $|\vec{I}| = I$,

$$\begin{aligned} I_1 &= \sqrt{I^2 - I_3^2} \cos \phi \\ I_2 &= \sqrt{I^2 - I_3^2} \sin \phi \\ H &= AI_3^2 + (I^2 - I_3^2)(B_1 \cos^2 2\phi + B_2) \end{aligned} \quad (2)$$

The tunneling matrix element in the WKB approximation is obtained by evaluating the action

$$S_{12} = \int_{\min 1}^{\min 2} I_3(\phi) d\phi. \quad (3)$$

where the momentum as a function of angle, $I_3(\phi)$, is obtained by solving the equation

$$\begin{aligned} H(I_3, \phi) &= E_o \\ E_o &= B_2 I^4 \end{aligned} \quad (4)$$

where E_o is the energy of the classical minimum of the Hamiltonian (2).

The interesting regions of parameter space, determine two different types of behavior for the solutions to (3) and (4).

a) if :

$$\frac{2B_2 I^2}{A} < 1 - \sqrt{\frac{B_1}{B_1 + B_2}} \quad (5)$$

then : $I_3(\phi)$ is pure imaginary for all values of ϕ in between the two minima and the resulting tunnelling amplitude $\exp(iS_{12})$ is a real number without any I dependent phase.

b) if :

$$1 > \frac{2B_2 I^2}{A} > 1 - \sqrt{\frac{B_1}{B_1 + B_2}} \quad (6)$$

then : the angular momentum $I_3(\phi)$ becomes complex (acquires a real part) while ϕ is intermediate between the two minima and the tunnelling amplitude involves a factor $\cos(\Re S_{12})$ which can change sign as a function of I and thus give rise to staggering in the yrast energies [6]. The region of the parameter space satisfying (6) is shown as a shaded area in fig.3. The fact that the Hamiltonian $H3$ defines a trajectory lying entirely in the region below the shaded area of fig.3 explains the absence of I -dependence in the pattern of quartet splittings for this Hamiltonian.

For Hamiltonians of the type of $H2$ (with $B_2 = 0$), the real part of the integral (3) can be simply evaluated

$$\Re S_{12} = \begin{cases} \frac{\pi}{2}(I - \sqrt{\frac{A}{4B_1}}) & \text{for } I > \sqrt{\frac{A}{4B_1}} \\ 0 & \text{for } I < \sqrt{\frac{A}{4B_1}} \end{cases} \quad (7)$$

The alternating sign of the tunnelling amplitude for $\Delta I = 2$ implied by (7) leads to the $\Delta I = 4$ alternating order of the $A1$ and $B1$ energies exhibited by the Hamiltonian $H2$ in fig.2.

The phase shift $\frac{\pi}{4}(A/B_1)^{1/2}$ in the tunnelling amplitude (7) has been confirmed by comparison with the direct diagonalization of the Hamiltonian (see fig.4), and it is found that A/B_1 , should be replaced by $(A/B_1) - \frac{3}{2}\hbar^2$ in order to agree with the exact quantal results. The fact that the phase of the tunnelling depends on the parameters A and B_1 provides a mechanism that may explain the inversion of the $\Delta I = 4$ staggering that has sometimes been observed in the experimental data. Another feature of the Hamiltonian $H2$ that should be noted is the fact that rotational bands with C_{4v} quantum number E (the two dimensional representation) should not exhibit $\Delta I = 4$ staggering (this state is unshifted by the tunnelling) while the intrinsic structures $A1$ and $B1$ should give rise to staggering with opposite sign.

Concerning the Hamiltonian $H1$, the value of $\Re S_{12}$ in the limit of large I can be written

$$\Re S_{12} \approx \frac{\pi}{4} I \sqrt{2} \cos\left(\frac{\pi}{8}\right). \quad (8)$$

The occurrence of the irrational number $\sqrt{2}\cos(\pi/8)$ in the coefficient of πI in (8) explains the irregular variation in the quartet splittings of the Hamiltonian $H1$.

In the present note we have focused on the connection of the $\Delta I = 4$ staggering and the structure of the rotational Hamiltonian. The values of the Y_{44} deformation and the resulting B_1 term in the Hamiltonian is a separate issue that must be understood in terms of the intrinsic structure of the superdeformed states. As a first step in establishing this connection we have studied the Y_{44} moments of cranked high j intruder orbitals. We find that indeed these orbitals carry appreciable Y_{44} moments and that the lowest and the second-lowest intruder orbitals carry moments of opposite sign. This fact may be related to the empirical observation [1] that in the $A = 150$ region $\Delta I = 4$ staggering occurs for

configurations with a single $N = 7$ intruder but not for configurations with two particles in $N = 7$ orbitals.

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References

- [1] B. Haas et al, Phys. Rev. C **42** (1990) R1817 ;
S. Flibotte et al, Phys. Rev. Lett. **71** (1993) 4299.
- [2] B. Cederwall et al, submitted for publication (private communication from F.S. Stephens).
- [3] I.M. Pavlichenkov, Phys. Reports **226** (1993) 173.
- [4] After the present work was completed, we received from Dr. Flibotte an abstract of an article by S. Flibotte and I.M. Pavlichenkov in which they exhibit a fit to the experimental data of ref. [1] on the basis of the formalism of ref. [3]. This analysis involves a Hamiltonian with C_{4v} symmetry but is based on a solution with the angular momentum aligned along a direction near to the long axis of the superdeformed shape ($|I_3| \approx I$). Such a solution seems inappropriate for reasons described in the text and is in any case very different from the structures considered in the present analysis.
- [5] A. Bohr and B. Mottelson, Nuclear Structure, Vol II, pp7 ff, Benjamin, New York (1975)
- [6] It should be noted that the conditions for $\Re S_{12} \neq 0$ are quite different from the condition in fig.1 that determines the qualitative structure of the classical energy surface. This reflects the fact that the tunnelling takes place in a phase space with complex dimensions and thus reveals features of the dynamics that are invisible in the real phase space of classical mechanics.

Table 1

Hamiltonian	A	B_1	B_2	$\Delta I = 4$ structure in yrast band
H1	100	1	- 1/2	irregular
H2	100	1	0	regular oscillation
H3	100	1	-1	no oscillation

Figure captions

Figure 1 : Classical phase space for C_{4v} rotational Hamiltonian.

Provided the minima of the rotational Hamiltonian lie in the equator (i.e. $2B_2I^2 < A$), there are two possibilities for the arrangement of maxima, minima, and saddle points : (a) for $2(B_1 + B_2)I^2 > A$; and (b) for $2(B_1 + B_2)I^2 < A$.

Figure 2 : Energy splittings within yrast quartets for three different C_{4v} rotational Hamiltonians.

The figure shows the relative positions of the states within the yrast quartets as a function of I ; the states of the representation E are unshifted by the tunnelling, while the absolute value of the splitting between $A1$ and $B1$ is given above each quartet (the number in parenthesis denotes the power of ten). The figure is appropriate for even A nuclei (i.e. integer I). For odd A , the quartets are split into two doublets and the C_{4v} classification must be replaced by the spinor representations of the double point group C'_{4v} ; the splitting patterns are similar except that unshifted E states are absent and the $A1, B1$ states are replaced by the two-fold degenerate states $E1$ and $E2$.

Figure 3 : Parameter space for C_{4v} rotational Hamiltonian.

In the shaded region the tunnelling matrix element between equatorial minima acquires an I dependent real phase, which is a necessary condition for obtaining $\Delta I = 4$ structure. The upper limit, corresponding to the condition $2B_2I^2 < A$, is imposed in order to ensure that the rotation is about an axis lying in the equator.

Figure 4 : Splittings of yrast quartet for three C_{4v} Hamiltonians of type $H2$ ($B_2 = 0$).

The figure is obtained by direct diagonalization of the rotational Hamiltonian and plots only the eigenvalue of the lowest $A1$ state from which a smooth averaged I -dependent term has been subtracted [1]. The effect of the I -independent phase shift in eq.(7) is clearly visible ; for the value $A = 101.5$ the $\Delta I = 4$ term completely vanishes.

Fig. 1

ENERGY-SPLITTING OF 4 LOWEST STATES

Fig. 2

Fig. 3

Fig. 4