Dynamical Electroweak Symmetry Breaking ¹

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Abstract

We review the status of and recent developments in dynamical electroweak symmetry breaking, concentrating on the ideas of technicolour and top quark condensates. The emphasis is on the essential physical ideas and experimental implications rather than on detailed mathematical formalism. After a general overview of the subject, we give a first introduction to technicolour, and extended technicolour, illustrating the ideas with a simple (unrealistic) model. Then we review the progress that has been made with enhancing the technicolour condensate, using the Schwinger-Dyson gap equation. The discussion includes the so-called walking technicolour and strong extended technicolour approaches. We then turn to the experimental prospects of technicolour models, including longitudinal gauge boson scattering experiments at the LHC, the detection of pseudo-Goldstone bosons and the hints about electroweak symmetry breaking which comes from precision measurements at LEP. We also discuss a low-scale technicolour model, which has experimental signatures at LEP and the Tevatron. Lastly we turn to the idea of the top quark condensate. After reviewing the basic ideas of this approach, we turn to some extensions of these ideas involving the idea of fourth family condensates, and the role of irrelevant operators, which may allow the scale of new physics in these models to be reduced to the TeV scale. Finally we discuss an explicit gauge model of the top quark condensate with a built-in GIM mechanism.

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Contents

1. Introduction
   1.1 What is dynamical symmetry breaking?
   1.2 The standard model
   1.3 Overview of dynamical electroweak symmetry breaking

2. Technicolour
   2.1 The minimal technicolour model
   2.2 Extended technicolour
   2.3 A simple extended technicolour model

3. Condensate Enhancement
   3.1 Schwinger-Dyson gap equation
   3.2 Walking technicolour
   3.3 Strong extended technicolour

4. Experimental Prospects
   4.1 Longitudinal W and Z scattering at the LHC
   4.2 Pseudo-Goldstone bosons from a single techni-family
   4.3 Precision Electroweak Measurements
   4.4 Low scale technicolour

5. Top Quark Condensates
   5.1 Four-fermion theory
   5.2 Renormalisation group approach
   5.3 A fourth family after all?
   5.4 Irrelevant operators
   5.5 TeV scale models of the top quark condensate
1 Introduction

1.1 What is dynamical symmetry breaking?

The subject of dynamical symmetry breaking has applications to many branches of physics (e.g., superconductivity, superfluidity, nucleon pairing, chiral dynamics of hadrons). This review article will not be concerned with any of these applications but instead will be concerned with the application of dynamical symmetry breaking to the problem of understanding the origin of the \textit{electroweak} part of the masses of elementary particles.\footnote{The part of the quark masses generated by their strong interactions will not concern us here.} However it may be instructive to begin by tracing the development of dynamical symmetry breaking in an area of physics other than particle physics, in order to get a feel for what dynamical symmetry breaking is about. To this end we shall briefly discuss the phenomenon of superconductivity.

The history of superconductivity began in 1908 at Leiden in Holland when Kammerlingh Onnes succeeded in liquefying helium. He not only found that it boiled at 4.2 K, but three years later discovered that the resistance of a sample of mercury became negligible quite abruptly at this temperature – i.e. it became superconducting. As the supply of liquid helium became plentiful many metallic alloys were found to exhibit the same behaviour below their individual critical temperature $T_c$. The superconducting phase transition was later successfully described by a theory of Ginzburg and Landau (1950). According to this theory a macroscopic order parameter, corresponding essentially to the wavefunction of the superconducting charges, acquires a nonzero vacuum expectation value (VEV) in the superconducting state. Although this theory was quite successful it was later superceded by a theory of Bardeen, Cooper and Schrieffer (BCS) (1962). In the microscopic BCS theory the dynamical origin of the order parameter is identified with the formation of bound states of elementary fermions, namely Cooper pairs of electrons. Each Cooper pair has zero spin and and zero momentum and together these pairs form a zero temperature condensate of bosons. At a finite temperature below $T_c$ some of the pairs are thermally excited across a small energy gap to form quasi-particle excitations, as shown in figure 1. According to BCS theory this energy gap $\Delta$ falls from its maximum value $\Delta(0)$ to zero at $T_c$ in a characteristic way, as shown in figure 2.

As we shall see shortly we have a very successful theory of particle physics known as the standard model, in which the symmetry is broken by the VEV of an order
parameter analogous to the order parameter of the Ginzburg-Landau model. The question which will concern us here is whether the standard model will one day be superceded by some dynamical theory analogous to the BCS theory in which the electroweak symmetry is broken by a condensate of fermion pairs.

1.2 The Standard Model

The standard model of particle physics is based on the gauge group,

$$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$$

(1)

The standard model involves three families of fermions, the quarks and leptons, whose left (L) and right (R) handed components transform under the gauge group as,

$$Q_L^i = \begin{pmatrix} U_L^i \\ D_L^i \end{pmatrix} \sim (3, 2, 1/6)$$
$$\begin{pmatrix} U_R^i \\ D_R^i \end{pmatrix} \sim (3, 1, 2/3)$$

(2)

$$L_L^i = \begin{pmatrix} \nu_L^i \\ E_L^i \end{pmatrix} \sim (1, 2, -1/2)$$
$$E_R^i \sim (1, 1, 1)$$

where $i = 1 \ldots 3$ labels the three families of quarks and leptons, and in my convention the electric charge generator is given by

$$Q = T_{3L} + Y$$

(3)

where $T_{3L}$ is the third component of weak isospin $SU(2)_L$ and $Y$ is the hypercharge generator.

In addition the standard model introduces a complex scalar doublet,

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} w_1 + i w_2 \\ h_0 + i u_0 \end{pmatrix} \sim (1, 2, 1/2)$$

(4)

where the VEV of the neutral scalar component $< h^0 > = v = 246$ GeV spontaneously breaks the electroweak symmetry down to electromagnetism,

$$SU(2)_L \otimes U(1)_Y \longrightarrow U(1)_Q$$

(5)
The term in the Lagrangian responsible for the $W^\pm$ and $Z^0$ masses is the kinetic term for the Higgs doublet,
\[ (D_\mu H)^\dagger (D^\mu H), \] (6)
where the covariant derivative is defined as,
\[ D_\mu = \partial_\mu + igW_\mu^a T^a + ig'Y B_\mu \] (7)
and $T^a$ ($a = 1 \ldots 3$) are the $SU(2)_L$ generators. The combination $(W_1 - iW_2)/\sqrt{2}$ is the $W^+$ field and receives a mass squared,
\[ M_{W}^2 = g^2 v^2/4 \] (8)
while the combination
\[ Z = \frac{gW_3 - g'B}{\sqrt{g^2 + g'^2}} \] (9)
receives a mass squared,
\[ M_{Z}^2 = (g^2 + g'^2)v^2/4 \] (10)
and the orthogonal combination is the massless photon field,
\[ A = \frac{g'W_3 + gB}{\sqrt{g^2 + g'^2}} \] (11)
The weak mixing angle is defined by $\tan \theta_W = g'/g$ so that we have the mass relation
\[ M_{W}^2/M_{Z}^2 = \cos^2 \theta_W \] (12)
This mass relation has been tested experimentally to better than 1% accuracy.

The terms in the Lagrangian responsible for the fermion masses are the so called Yukawa couplings,
\[ -g_{ij}D_{iR}H^\dagger Q_{jL} - h_{ij}C_{iR}\tilde{H}^\dagger Q_{jL} - f_{ij}E_{iR}H^\dagger L_{jL} + h.c. \] (13)
where $\tilde{H} = i\sigma_2H^*$, where $\sigma_2$ is the second Pauli matrix, is the charge conjugate Higgs doublet. When the Higgs doublet receives its VEV, fermion masses result. Thus the Higgs mechanism achieves two things simultaneously: it provides $W, Z$ masses and fermion masses. According to the Higgs mechanism, the three pseudoscalar degrees of freedom $w_i$ are eaten by $W^\pm_L, Z^0_L$ and the shifted $h_0$ field remains in the spectrum as a physical neutral scalar particle – the Higgs boson. More details concerning the Higgs boson can be found in the review by Cahn (1989).

Experimentally the standard model is in remarkably good shape. All the fermions of three families of quarks and leptons have been seen experimentally, including the
recent tentative discovery of the top quark with a mass of around 174 GeV by Abe et al (1994). The standard model is perfectly consistent with high precision electroweak measurements which are sensitive to radiative corrections (Ellis et al 1992). However the Higgs boson has so far not been detected experimentally, with LEP putting a lower limit on its mass of about 60 GeV. This lack of experimental observation of the Higgs boson is one of the main motivations for considering models of dynamical electroweak symmetry breaking.

Theoretically the standard model is on less secure ground. The standard model involves elementary scalar fields, and these are associated with quadratic divergences at the one-loop level. Of course such divergences are renormalised away and the renormalised theory is finite and well behaved. The problem arises when the standard model is embedded into some larger structure involving a mass scale \( M \) much larger than the electroweak scale characterised by the VEV \( v \). For example \( M \) might be identified with a scale of grand unification with \( M \approx 10^{16} \) GeV. In such a framework the Higgs sector responsible for breaking the larger gauge symmetry at the scale \( M \) and the Higgs sector responsible for breaking electroweak symmetry at the scale \( v \) cannot be kept distinct, and communicate through one-loop radiative corrections. The hierarchy of mass scales can then only be maintained at the one-loop level by fine-tuning the basic Higgs parameters of the theory to an accuracy of about 24 decimal places in this example. Such fine-tuning arises because of the quadratic nature of the scalar divergences. Furthermore the fine-tuning must be re-done at every order of perturbation theory. There are two solutions to this so-called hierarchy problem. The first is to introduce an \( N = 1 \) supersymmetry which tames the quadratic divergences into benign logarithmic divergences, due to the fermionic partners to the scalars. The second, which is the subject of the present review, is to banish elementary scalars from the theory, and to replace their effect by fermion dynamics—the dynamical symmetry breaking approach.

### 1.3 Overview of dynamical electroweak symmetry breaking

Although the standard model provides a remarkable description of all known experimental data, there is as yet no direct evidence for the existence of the Higgs boson. Of course one may argue that the eaten Goldstone bosons (GB’s) of the Higgs mechanism have already been detected, since they may (in a limit which we shall make precise later) be identified with the longitudinal (L) polarisation states of the W and Z bosons. However the key question of electroweak symmetry breaking is whether
these eaten GB’s are composite or elementary. Such questions concerning compositeness must always be related to some momentum or distance scale, since systems which appear to be elementary at some scale may in fact appear to be composite at some higher momentum (shorter distance) scale. We shall chiefly be interested in whether the Higgs sector turns out to be composite on the TeV scale, which will be probed by the next generation of colliders. We emphasise that even if compositeness effects are revealed on the TeV scale, this does not imply that the Higgs mechanism is inoperative. It simply means that the Higgs mechanism is implemented dynamically, i.e. that there is dynamical electroweak symmetry breaking (DEWSB). In other words, there are no elementary scalars and their effect is replaced by the dynamics of fermions. Usually this implies some strong interactions between the fermions in order for a composite fermion-antifermion bound state to be formed which will play the role of the Higgs scalar fields.

The simplest example of DEWSB is to invent a new strong confining gauge force called technicolour (TC) with which to bind new fermion-antifermion pairs into composite systems whose behaviour resembles that of the Higgs field. TC was reviewed by Farhi and Susskind (1981). Some of the material discussed by Farhi and Susskind is condensed into sections 2.1, 2.2 and 4.2 of the present article. The remaining sections of the present article will focus on more recent developments. In the TC approach the new fermions are called technifermions, and the whole theory is modelled on the known behaviour of quarks in quantum chromodynamics (QCD) – but scaled up to the TeV scale. It turns out that TC by itself is not sufficient to provide fermion masses. One way forward is to embed the TC gauge group into a larger gauge group known as extended technicolour (ETC) (section 2.2). However we shall see it is not an easy task to describe the quark and lepton mass spectrum without running into phenomenological problems. These problems include the flavour-changing neutral current (FCNC) problem, problems with massless Goldstone bosons (or pseudo-Goldstone bosons (PGB’s) which are too light). These problems have thwarted attempts to construct ETC models, and to date there is no accepted standard ETC model in the literature. Some recent model building attempts are discussed in section 2.3. There are alternatives to ETC which can lead to fermion masses. For example one may reintroduce the scalar doublet in order to communicate electroweak symmetry breaking to the quarks and leptons via Yukawa couplings (Simmons 1989, Samuel 1990, Kagan and Samuel 1991). However this appears to be somewhat of a retreat since the motivation for technicolour is to banish elementary scalars. However one may envisage a natural scenario involving both technicolour and supersymmetry (Dimopoulos and
Raby 1981). We shall not discuss this possibility any further here.

Recently ETC has staged a comeback due to a lot of exciting progress with the above problems. In section 3 we shall discuss the recent progress in dealing with the FCNC problem of ETC, namely the ideas of walking TC and strong ETC. Both these ideas result in the technifermion $T$ condensate receiving a high momentum enhancement, while the pion decay constants $F_\pi$ which depend on low momentum physics (Pagels and Stokar, 1979) are almost unchanged. This is important since the quark and lepton masses and PGB masses depend upon the value of the condensate, while the $W, Z$ masses depend upon $F_\pi$. Condensate enhancement may therefore increase fermion masses without increasing gauge boson masses. As we shall see, the condensate enhancement is given by,

$$<\bar{T}T>_{ETC} \sim \Lambda_{TC}^3 (M_{ETC}/\Lambda_{TC})^{\gamma}$$

where $\Lambda_{TC} \sim 500$ GeV is the TC confinement scale, $M_{ETC}$ is the ETC breaking scale and $\gamma$ is the anomalous dimension of $\bar{T}T$. The condensate is enhanced from its naive value of $\gamma = 0$ to values of up to $\gamma = 1$ for walking theories or up to $\gamma = 2$ for strong ETC. It turns out that the fermion masses $m_f$ are enhanced by a factor of $(M_{ETC}/\Lambda_{TC})^{\gamma}$. If we take $M_{ETC} \sim 500$ TeV to avoid problems with FCNC's then the enhancement factor is $M_{ETC}/\Lambda_{TC} \sim 10^3$ and it is clear that the enhancements in fermion masses can be large. For example, if we take the naive value of the anomalous dimension $\gamma = 0$ then it turns out that $m_f \sim 0.5$ MeV. This value is fine for the electron mass, but is clearly too small to account for the quark masses, especially the top quark. However if we have $\gamma = 1$ then we gain a factor of $10^3$ and $m_f \sim 500$ MeV. This is large enough for the strange quark mass. If we wish to account for the top quark mass we need $\gamma$ to approach its maximum value of 2, and it turns out that this may be achieved in so-called strong ETC theories.

As its name suggests, strong ETC theories are theories in which the ETC gauge coupling $\alpha_{ETC}$ is quite strong at the scale $M_{ETC}$ at which ETC is broken. Thus heavy ETC boson exchange can play an important role in DEWSB at low energies. We shall review strong ETC theories in section 3.3 where we shall see that such theories involve some degree of fine-tuning and have light scalar bound states consisting of tightly bound technifermion-antitechnifermion systems, bound by short-range strongly coupled heavy ETC boson exchange. Such theories have some difficulties obtaining a sat-

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4Since the most severe bound from FCNCs comes from the $K_L - K_S$ mass difference and hence $\Delta S = 2$ operators, one may argue that the relevant fermion mass we should strive to achieve is the strange quark mass.
isfactory top-bottom mass splitting without over-infecting the $\rho$-parameter. One may study these theories in the four-fermion approximation in which the exchange of ETC bosons gives rise to effective four-technifermion operators. In this approximation the breaking of electroweak symmetry is driven dominantly by the four-technifermion interaction, and resembles the Nambu-Jona-Lasinio (NJL) (1961) model.

In section 4 we discuss some of the experimental prospects for TC models. We begin in section 4.1 with a discussion of longitudinal gauge boson scattering at the LHC, which is the classic signature of the minimal TC model of section 2.1. In section 4.2 we briefly describe some of the physics of PGB’s associated with a single family of technifermions. This discussion is deliberately brief firstly because it is well documented elsewhere, and secondly because TC models with large numbers of technifermions are somewhat under seige at the moment from the comparison of theoretical estimates of the contribution of radiative correction involving TC to electroweak parameters which have been recently measured quite accurately at LEP. As discussed in section 4.3, there are increasingly strong constraints on additional fermions which couple to the $W$ and $Z$ boson from precision electroweak measurements. Theories which involve a complete family of technifermions consisting of three coloured doublets of techniquarks $(U^{TC}, D^{TC})_{r,\beta}$ plus a doublet of technileptons $(N^{TC}, E^{TC})$ are severely challenged by recent data. There are solutions to these problems that one can envisage so perhaps the question is not entirely clear cut. Finally a low scale TC model is discussed in section 4.4. In this model the TC confinement scale $\Lambda_{TC}$ is reduced from 500 GeV to 50-100 GeV, leading to spectacular TC signals at LEP and the Tevatron.

Another example of DEWSB is to suppose that the top quark is subject to some short-range strong interaction which causes “Cooper pairs” of top-antitop condensates to form, breaking the electroweak symmetry. This is the subject of section 5. This mechanism could also be applied to a fourth family, providing the fourth neutrino is heavy enough so that it would not have been produced in $Z$ decays and so would not contribute to the accurately measured $Z$ width. These models confront the problem of the large top quark mass in a very direct way by doing away with TC altogether and simply postulating some four-fermion operators of the form

$$(\lambda/\Lambda^2)(t_L^R t_R f_R f_L)$$

where $t_L^R$ are top quark fields, $\Lambda$ has the dimensions of mass and is some sort of...
ultraviolet cut-off (which may represent the scale at which we expect the onset of new physics such as heavy gauge boson exchange as in the example of strong ETC above), and $\lambda \sim 8\pi^2/3$ is a strong dimensionless coupling. If the value of $\lambda$ is carefully adjusted the four top quark operator becomes transmuted into a theory which resembles the Higgs sector of the standard model with a composite Higgs boson $H^0 \sim \tilde{t}t$ coupling only to top quarks. In section 5.3 we discuss the prospects of a fourth family after the LEP measurement of the $Z$ width, and in section 5.4 we shall discuss to what extent such models predict the Higgs boson and top quark masses, due to the effects of the so-called “irrelevant operators” which may change the naive predictions based on the (quasi-) infra-red fixed point of these models. Finally in section 5.5 we discuss a TeV scale model of the top quark condensate with a built-in Glashow-Iliopoulos-Maiani (GIM) (1970) mechanism.

Section 6 concludes the paper.

2 Technicolour

2.1 The Minimal Technicolour Model

The subject of dynamical electroweak symmetry breaking (DEWSB) has a long and distinguished history going back to the early work of Nambu and Jona-Lasinio (1961). A series of pioneering papers followed which extended these ideas to the realm of gauge theories. Eventually the idea of technicolour (TC) was introduced by Susskind (1979) and Weinberg (1979) as a mechanism for DEWSB. The early development of TC is nicely traced in the collection or reprints by Farhi and Jackiw (1982).

Following Susskind (1979) let us first consider a toy model which consists of the standard model with the Higgs doublet $H$ removed, and with the fermion sector paired down to just one family of quarks (discarding the leptons, and forgetting about the question of triangle anomalies). In this toy model the quarks ($u, d$) remain massless down to the QCD confinement scale $\Lambda_{QCD} \approx 250$ GeV, whereupon the quarks and gluons become confined into hadrons with typical masses of a GeV. What about the pions? Susskind realised that in this toy model something very interesting happens to the pions: they behave just like the exact would-be Goldstone bosons of the Higgs model and get eaten by the $W_L, Z_L$. In other words the $W, Z$ acquire mass in much the same way as in the usual Higgs mechanism. This is such an important result which underpins much of what follows, that it is worth understanding exactly how
this happens.

First of all let us switch off the $SU(2)_L \otimes U(1)_Y$ electroweak interactions, so that the QCD Lagrangian is invariant under separate $SU(2)$ chiral rotations of the left-handed quark doublet $(u, d)_L$ and the right-handed quark doublet $(u, d)_R$, and so respects a global chiral symmetry,

$$SU(2)_L \otimes SU(2)_R$$

(15)
together with some $U(1)$ symmetries which, although important in some other respects, will not play a role in the following analysis and so will be ignored. The QCD Lagrangian is invariant under the chiral symmetry because it contains no mass terms for the quarks. However we know from our experience of hadron dynamics that the QCD vacuum does not respect the full chiral symmetry which is said to be spontaneously broken down to isospin symmetry,

$$SU(2)_L \otimes SU(2)_R \rightarrow SU(2)_{L+R}$$

(16)
In fact this is also an example of dynamical symmetry breaking because the order parameter of the symmetry breaking is the VEV of a quark-antiquark condensate,

$$\langle \bar{q}q \rangle_{\text{vac}} \neq 0$$

(17)
One can imagine a simple picture of a quark-antiquark pair with the combined quantum numbers of the vacuum attracted by gluon exchange, with a stronger attraction at long distances due to the asymptotic freedom of the QCD coupling constant. When the force of attraction becomes greater than a certain critical value, corresponding to a certain critical QCD coupling constant, the effective potential energy of the ground state is lowered by producing $q\bar{q}$ pairs. A phase transition then occurs and it becomes energetically favourable for the vacuum to fill up with $q\bar{q}$ pairs, and the VEV of the bilinear operator $q\bar{q}$ becomes different from zero, as above. It is manifestly clear that this mass-type operator does not respect the chiral symmetry, but isospin symmetry is preserved by the vacuum.

According to Goldstone’s theorem we would expect three massless pseudoscalars to accompany the above chiral symmetry breaking, and phenomenologically these are identified with the pions. In our toy model the pions would be exactly massless (with electroweak interactions switched off). The following current algebra results are then exact in this limit. The triplet of axial currents may be defined either in terms of the quark fields $q = (u, d)$ or pion fields $\pi^a$ as follows,

$$j_\alpha^\mu = f_\pi \partial^\mu \pi_\alpha = \bar{q} \gamma_\mu \gamma_5 \frac{\tau^a}{2} q$$

(18)
where $\tau^a (a = 1 \ldots 3)$ are the Pauli matrices and $f_\pi$ is called the pion decay constant because of the matrix element

$$< 0 | j_5^\mu | \pi_6 >= i f_\pi p^\mu \delta_{ab} \tag{19}$$

These results can be written equivalently as

$$j_5^{\mu+} = \frac{\sqrt{2}}{2} f_\pi \partial^\mu \pi^+ = \bar{d} \gamma_{\mu} \gamma_5 u \tag{20}$$
$$j_5^{\mu-} = \frac{\sqrt{2}}{2} f_\pi \partial^\mu \pi^- = \bar{u} \gamma_{\mu} \gamma_5 d \tag{21}$$
$$j_5^{\mu0} = f_\pi \partial^\mu \pi^0 = \frac{1}{2} (\bar{u} \gamma_{\mu} \gamma_5 u - \bar{d} \gamma_{\mu} \gamma_5 d) \tag{22}$$

and

$$< 0 | j_5^\mu | \pi^\pm > = \sqrt{2} i f_\pi p^\mu \tag{23}$$
$$< 0 | j_5^\mu | \pi^0 > = i f_\pi p^\mu \tag{24}$$

where the charged pions are

$$\pi^\pm = \frac{\pi_1 \mp i \pi_2}{\sqrt{2}} \tag{25}$$

and the charged currents are

$$j_5^{\mu\pm} = j_{51}^{\mu} \pm ij_{52}^{\mu} \tag{26}$$

Experimentally $f_\pi \approx 93 \text{ MeV}$, from the charged pion lifetime.

The above current algebra results assume that electroweak symmetry is switched off, so that in this limit the pions appear in the physical spectrum as massless states. When electroweak interactions are switched on Goldstone’s theorem (which assumes the absence of gauge symmetries) no longer applies. Instead we shall see that, as in the standard model, we have a Higgs mechanism with the massless pions combining with the massless gauge bosons to form massive gauge bosons. Using the above results it is straightforward to show that the kinetic terms for the quarks,

$$\bar{q}_L D_\mu \gamma^\mu q_L + \bar{u}_R D_\mu \gamma^\mu u_R + \bar{d}_R D_\mu \gamma^\mu d_R \tag{27}$$

lead to the following pion-gauge boson derivative couplings,

$$\frac{g}{2} f_\pi W^+_\mu \partial^\mu \pi^+ + \frac{g}{2} f_\pi W^-_\mu \partial^\mu \pi^- + \frac{g}{2} f_\pi W^0_\mu \partial^\mu \pi^0 + \frac{g'}{2} f_\omega B^+_\mu \partial^\mu \pi^0 \tag{28}$$

The derivative couplings bring down a factor of $p^\mu$, the four momentum of the pion and hence that of the gauge boson with which it mixes. In a frame in which $p^\mu = (E, 0, 0, p)$ the polarisation vectors are $e_T^\mu = (0, a, b, 0)$ (for the transverse T vector)
and \( \epsilon'_L = (c, 0, 0, d) \) (for the longitudinal L vector), from which it is obvious that all the mixing takes place in the L direction only.

Using these results it is now straightforward to see how the gauge boson masses arise. Consider the full propagator for the \( W^\pm \) bosons given by summing all the diagrams shown in figure 3. Dropping all the indices for simplicity, the full propagator is given by the geometric series,

\[
\text{full propagator} = \frac{1}{p^2} + \frac{1}{p^2} (g f_{\sigma \pm}/2)^2 \frac{1}{p^2} + \ldots
\]

\[
= \frac{1}{p^2} \left[ 1 + \frac{(g f_{\sigma \pm}/2)^2}{p^2} + \ldots \right]
\]

\[
= \frac{1}{p^2} \left[ 1 - \frac{(g f_{\sigma \pm}/2)^2}{p^2} \right]^{-1}
\]

\[
= \frac{1}{p^2 - (g f_{\sigma \pm}/2)^2}
\]

From above it is clear that the pole in the gauge boson propagator is shifted from zero to \( p = g f_{\sigma \pm}/2 \) which implies that the \( W^\pm \) has acquired a mass,

\[
M_W = g f_{\sigma \pm}/2
\]

Note that two powers of momentum from the derivative coupling in the numerator have cancelled with \( \frac{1}{p^2} \) from the massless pion propagator. If the pion were not exactly massless then this cancellation would not take place, and the gauge boson would remain massless. A similar calculation for the \( W^0 \) and \( B \) propagators, including the mixing shown in figure 4, leads to the mass squared mixing matrix for \( W^0 \) and \( B \),

\[
\begin{pmatrix}
M_{W^0}^2 & M_{W^0 B}^2 \\
M_{W^0 B}^2 & M_B^2
\end{pmatrix}
= \frac{f_{\sigma}^2}{4} \begin{pmatrix}
g^2 & g g' \\
g g' & g'^2
\end{pmatrix}
\]

The eigenvalues of the matrix are

\[
M_Z^2 = (g^2 + g'^2) f_{\sigma}^2/4, \quad M_A^2 = 0
\]

with corresponding eigenvectors,

\[
Z = \frac{g W_3 - g' B}{\sqrt{g^2 + g'^2}}
\]

\[
A = \frac{g' W_3 + g B}{\sqrt{g^2 + g'^2}}
\]
These results are quite similar to those discussed earlier in the standard model. In fact the gauge boson mass ratio comes out exactly right

\[ \frac{M_W}{M_Z} = \frac{f_{\pi^\pm}}{f_{\pi^0}} \cos \theta_W \]

since isospin symmetry guarantees that

\[ f_{\pi^\pm} = f_{\pi^0} \]

The only problem is that the pion decay constant \( f_{\pi} \approx 93 \) MeV is too small to account for the \( W, Z \) masses. In the standard model \( f_{\pi} \) is replaced by \( v = 246.2 \) GeV. If we could find some way of boosting \( f_{\pi} \) by a factor of 2,650 then the \( W, Z \) masses would have their correct values. This is exactly what TC is: a scaled-up version of QCD with a pion decay constant equal to \( F_{\pi} = v \).

According to TC there are no elementary Higgs scalars, instead the Higgs mechanism is implemented by the TC sector. The TC sector consists of a QCD-like gauge group called TC whose coupling grows strong at around \( \Lambda_{TC} \sim 500 \) GeV, together with some technifermions which carry TC and are confined. The TC dynamics is just a scaled up version of QCD so that we expect chiral symmetry breaking and light pions in TC as in QCD. Three of the technipions will get eaten by the Higgs mechanism and their degrees of freedom are replaced by the longitudinal (L) components of the \( W, Z \).

We shall now discuss the simplest TC model of Susskind (1979) and Weinberg (1979) in a little more detail. In this minimal TC model the gauge group of the world is

\[ SU(N)_{TC} \otimes SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \]

where we have added to the standard model gauge group a new confining QCD-like gauge group \( SU(N)_{TC} \) which is asymptotically-free and confines at \( \Lambda_{TC} \sim 500 \) GeV. The number of technicolours \( N \) is left as a free parameter \(^6\). The minimal set of technifermions is the anomaly-free doublet,

\[ \begin{pmatrix} p_L \\ m_L \end{pmatrix}^\alpha \sim (N, 1, 2, 0) \\
\begin{pmatrix} p_R \alpha \\ m_R \alpha \end{pmatrix} \sim (N, 1, 1/2, 1/2) \]

\(^6\)Any remaining technipions will hopefully gain sufficient mass from explicit symmetry breaking effects to render them too heavy to be produced.

\(^7\)It is conventional but not necessary to take the TC group to be a unitary gauge group. Orthogonal TC gauge groups are perfectly adequate alternatives.
Table 1: Comparison of minimal TC and QCD with one quark doublet.

<table>
<thead>
<tr>
<th>QCD</th>
<th>TC</th>
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<tbody>
<tr>
<td>SU(3)C Quarks</td>
<td>SU(N)TC Techniquarks</td>
</tr>
<tr>
<td>$&lt;\bar{u}u + \bar{d}d &gt; \sim \Lambda_{QCD}^3$</td>
<td>$&lt;\bar{p}p + \bar{m}m &gt; \sim \Lambda_{TC}^3$</td>
</tr>
<tr>
<td>$SU(2)_L \otimes SU(2)<em>R \rightarrow SU(2)</em>{L+R}$</td>
<td>Chiral Symmetry $\rightarrow$ (techni-)isospin</td>
</tr>
<tr>
<td>3 QCD pions</td>
<td>3 Technipions</td>
</tr>
<tr>
<td>$f^\pm = f_\pi^0 \approx 93MeV$</td>
<td>$F^\pm = F_\pi^0 \approx 246GeV$</td>
</tr>
<tr>
<td>$\Lambda_{QCD} \approx 200MeV$</td>
<td>$\Lambda_{TC} \approx 500GeV$</td>
</tr>
</tbody>
</table>

where $a = 1\ldots N$ labels TC. Each techniquark carries TC but not ordinary colour, and the left-handed techniquarks form an $SU(2)_L$ doublet just like ordinary quarks. The electric charge generator is given by $Q = T_L^a + Y$ so that the plus ($p$) and minus ($m$) techniquarks have charges given by $Q = \pm 1/2$, respectively. The electric charges sum to zero, a requirement of anomaly freedom. Needless to say the quarks and leptons defined earlier do not carry technicolour.

There is an obvious parallel between the technidoublet above and the doublet of lightest quarks in QCD, as shown in Table 1.

In the idealised world of Table 1 consisting of a doublet of massless quarks and a doublet of massless techniquarks, the 3 pions and 3 technipions are massless up to electroweak effects. When $SU(2)_L \otimes U(1)_Y$ forces are switched on, 3 of the pions will be eaten by the Higgs mechanism and 3 pions will remain in the spectrum as physical states. The 3 eaten pions are mainly technipions, and the 3 physical pions are mainly QCD pions,

$$|\text{eaten pion} > = \frac{F_\pi |\text{technipion} > + f_\pi |\text{QCD pion} >}{\sqrt{F_\pi^2 + f_\pi^2}}$$

$$|\text{physical pion} > = \frac{F_\pi |\text{QCD pion} > - f_\pi |\text{technipion} >}{\sqrt{F_\pi^2 + f_\pi^2}}.$$ 

In the approximation $F_\pi \gg f_\pi$, the gauge boson masses are given by,

$$M_{W^\pm}^2 = \frac{g^2 F_\pi^2}{4}$$

$$\begin{pmatrix} M_{W^+_\nu}^2 & M_{W^0_B}^2 \\ M_{W^+_e B}^2 & M_{B^0_B}^2 \end{pmatrix} = \frac{F_\pi^2}{4} \begin{pmatrix} g \gamma & gg' \\ gg' & g^2 \end{pmatrix}$$
In the limit of exact techni-isospin $SU(2)_{L+R}$ we have $F_{\pi^+} = F_{\pi^0}$ and these results are identical to those of the standard model if we identify $F_\pi = v$ where $v$ is the vacuum expectation value (vev) of the Higgs doublet, and $g$ and $g'$ are the $SU(2)_L$ and $U(1)_Y$ gauge couplings. When the mass-squared matrix is diagonalised we obtain a massive $Z^0$ gauge boson and a massless photon. The gauge boson masses are thus given by

$$M_{W^\pm} = \frac{gF_\pi^+}{2}$$

$$M_{Z^0} = \frac{gF_{\pi^0}}{2 \cos \theta_W}.$$  \hspace{1cm} (45)

We arrange $F_\pi = 246$ GeV in order to give masses of the correct magnitude. This is the reason why we want a large confinement scale $\Lambda_T \sim 500$ GeV.

### 2.2 Extended Technicolour

The minimal TC model just described provides a neat way of breaking $SU(2)_L \otimes U(1)_Y$ without using the Higgs doublet $H$. In providing masses for the $W$ and $Z$ TC is unsurpassed in its elegance. The successful $W/Z$ mass ratio is a consequence of the global $SU(2)_{L+R}$ techni-isospin symmetry (the analogue of ordinary isospin symmetry in QCD). However $H$ is not only is responsible for gauge boson masses, but also provides fermion masses via the Yukawa couplings which break the chiral symmetries of the fermion sector. So far we have not mentioned how quark and lepton masses may be achieved in TC theories. Now we shall discuss this question.

In the standard model the Higgs doublet has Yukawa couplings to fermions $f$ of the form $\bar{f}_{iR} H f_j^L$. In minimal TC the Higgs doublet $H$ is replaced by the technifermion doublet $T = (p, m)$ which have no couplings to quarks and leptons, except through chirality conserving gauge interactions. Therefore quark and lepton masses are not generated in the minimal TC model. It is at this point that many people choose to give up on TC theories since they do not possess the economy of the single Higgs doublet$^8$. However TC enthusiasts claim that the economy of the Higgs doublet is illusory, and in reality the arbitrary Yukawa couplings of the standard model just emphasise the fact that we have absolutely no understanding of the origin of fermion masses. The fact that the fermion masses come out to be zero in minimal TC could even be construed as a good thing, since this simple fact forces one to introduce theories of fermion masses from the outset. The fact that it will turn out to be exceedingly difficult to correctly account for the fermion mass spectrum, without running into

---

$^8$Higgs kills two birds with one stone by providing both gauge boson masses and fermion masses.
phenomenological difficulties of one sort or another, can again be interpreted either positively or negatively. TC sceptics regard these difficulties as hints that Nature does not like TC theories. TC believers regard these difficulties as challenges which must and can be overcome. There are so many approaches to the fermion mass problem in TC theories that this subject has become model-dependent in the extreme. In this subsection we shall concentrate on the general strategy known as extended technicolour, bearing in mind that there are other approaches to the problem. Later on we shall consider some of the other approaches (e.g. the top quark condensate approach in section 5).

One solution to the problem of fermion masses in TC is to extend the TC sector in such a way as to allow the technifermons to talk to the fermions (quarks and leptons) and so allow the dynamically generated technifermon mass to be fed down to the fermions by some sort of radiative correction. This strategy is called extended technicolour (ETC) and was first discussed by Dimopoulos and Susskind (1979) and Eichten and Lane (1980). The general strategy of ETC theories is clear: one embeds the TC gauge group $G_{TC}$ into a larger ETC gauge group $G_{ETC}$ which is broken somehow at a scale $M_{ETC}$ down to $G_{TC}$,

$$G_{ETC} \xrightarrow{M_{ETC}} G_{TC} \otimes \cdots$$

(47)

where $M_{ETC} > \Lambda_{TC} \sim 500 GeV$. The heavy ETC gauge bosons of mass $M_{ETC} \sim 1 - 1000$ TeV, corresponding to the broken ETC generators, can in general couple fermions (f) to technifermons (T), fermions to fermions, and technifermons to technifermons, as shown in figure 5. The ETC coupling of technifermons to fermions, allows the radiative diagram in figure 6. This diagram allows the feed-down of the dynamically generated technifermon mass to the quarks and leptons, and leads to a fermion mass $m_f$ crudely given by,

$$m_f \sim \frac{<\mathcal{T}T>_{M_{ETC}}}{M^2_{ETC}}$$

(48)

where a naive estimate of the technifermon condensate, based on simple dimensional analysis is,

$$<\mathcal{T}T>_{M_{ETC}} \sim \Lambda_{TC}^3.$$ 

(49)

We can improve on the naive estimates above of the fermion mass in ETC, and in section 3 we shall do exactly that. But for now we shall be content with the simple estimates above.
However according to figure 5 the ETC bosons also couple fermions to fermions which leads to flavour changing neutral currents (FCNC). The most severe constraint (Eichten and Lane, 1980) comes from $\Delta S = 2$ operators $\sim (1/M_{ETC}^{-1})\bar{s}d\bar{s}d$ which mediate $\bar{s}d \leftrightarrow \bar{d}s$ mixing due to the diagram in figure 7. The $K_L - K_S$ mass difference leads to the constraint $M_{ETC} > 500 TeV$. This in turn leads to a bound on the fermion mass $m_f < 0.5 MeV$ from equations 48, 49. The FCNC problem is simply that such fermion masses are unrealistically small. Such problems arise in part because ETC theories have set themselves the ambitious task of explaining all the quark and lepton masses and quark mixing angles without the aid of elementary scalar fields.

Apart from the FCNC problem, ETC theories face the problem of ensuring that there are no massless Goldstone bosons (GB’s) in the spectrum. In the minimal TC model, in the limit that electroweak interactions were switched off, there are exactly three massless GB’s. When electroweak interactions are switched on these get eaten by the $W, Z$, according to the Higgs mechanism, leaving no GB’s in the physical spectrum. However many ETC models give rise to a low-energy effective TC theory which involves more than one doublet of technifermions, and hence has a larger chiral symmetry than in the minimal model. When this larger chiral symmetry is broken, there will be more than three broken generators leading to more than three GB’s, according to Goldstone’s theorem. The solution to this potential disaster is to ensure that the original chiral symmetry of the TC theory is not exact, but is in fact explicitly broken in some way, so that we do not end up with exactly massless GB’s but instead rather light pseudo-Goldstone bosons (PGB’s). The PGB’s of course may be problematic unless they are sufficiently heavy, but at least we have a strategy for solving this difficulty.

The simplest example of PGB’s are the ordinary pions of QCD. The pions are not exactly massless since the original chiral symmetry is explicitly broken by two effects. Firstly the quarks $u, d, \ldots$ have explicit masses (sometimes called current masses or intrinsic masses) which clearly do not respect the chiral symmetry. Secondly the quarks carry electric charges which violate the isospin symmetry. The mass difference between the charged and neutral pion is entirely due to electromagnetic effects. The mass of the neutral $\pi^0$ on the other hand is totally due to the effect of the explicit quark masses which break the chiral symmetry explicitly. The value of the pion masses may be estimated using either current algebra techniques or chiral lagrangian techniques which are equivalent to the current algebra techniques but much nicer to work with. Chiral lagrangians are discussed by Georgi (1984). The well known result
is that the neutral pion mass squared varies as,

\[ m_{\pi0}^2 \sim (m_u + m_d)f_{\pi} \]  \hspace{1cm} (50)

where \( m_u, m_d \) are the explicit quark masses, and as before \( f_{\pi} = 93 \) MeV is the pion decay constant. The difference between the squares of the masses of the charged and neutral pions varies as,

\[ m_{\pi+}^2 - m_{\pi0}^2 \sim \alpha f_{\pi}^2 \]  \hspace{1cm} (51)

where \( \alpha = 1/137 \) is the fine structure constant.

In TC theories with more than one doublet of technifermions, the PGB’s which carry colour or electroweak quantum numbers will receive masses which may be estimated by scaling up the charged-neutral pion mass difference in equation 51 by using \( F_{\pi} = 246 \) GeV and replacing \( \alpha \) by the appropriate coupling factor. The electrically neutral PGB’s may or may not receive mass from electroweak effects. However in ETC theories there are also ETC gauge boson effects to take into account, as these may also explicitly violate the chiral symmetry of the low-energy TC theory. The precise details will depend on the particular ETC model under consideration, and in fact the requirement of no extra GB’s will put powerful constraints on the ETC theory as emphasised by Eichten and Lane (1980). Assuming the ETC theory has the correct interactions to break the chiral symmetry of the TC theory, the resulting masses from ETC effects are estimated to be

\[ m_{\pi}^2 \approx \frac{<TT>}{F_\pi^2 M_{ETC}} \]  \hspace{1cm} (52)

where \( P \) represents a PGB of TC. Such mass contributions may be the only source of mass for the neutral PGB’s, and may be an important source of mass for the electrically charged but colour singlet PGB’s, which must be sufficiently heavy to not have been produced at the LEP \( e^+e^- \) collider.

### 2.3 A simple ETC model

So far we have been deliberately vague about the details of the ETC theory. This is because, as mentioned, ETC theories are highly model dependent. There are about as many different ETC models as there are people who have ever worked on ETC. There is certainly no leading candidate ETC model, and all ETC models that have ever been proposed has difficulties of one sort or another. Nevertheless it is important to discuss explicit detailed models, since only then can one gain an appreciation of what the
issues are and what the kinds of problems are that one may expect to face. With this in mind I shall now discuss a very basic prototype ETC model first introduced by the present author in 1989. The ETC model involves a complete family of technifermions, i.e. a set of technifermions with the quantum numbers of a complete quark and lepton family, but which carry an additional TC quantum number. It is a common feature of ETC models that they yield a single complete techni-family in the low energy TC theory. However this is by no means a necessary consequence of ETC, and there are many other ETC models which do not yield a techni-family. Another feature of the model we are about to describe is that it is a multi-scale ETC model. In other words, the ETC gauge group does not break down to TC at a single energy scale, but rather it breaks sequentially over a hierarchy of scales. The hierarchy of ETC scales is a simple reflection of the hierarchy of fermion masses between families.

Consider the following ETC theory (King 1989a),

\[
SO(10)_{ETC} \otimes SU(3)_C \otimes SU(2)_L \otimes U(1)_Y
\]

(53)

\[
Q^E_{ETC} = \left( \begin{array}{c} U^E_{ETC} \\ D^E_{ETC} \end{array} \right)^\alpha \sim (10,3,2,1/6)
\]

\[
U^E_{ETC} \sim (10,3,1,2/3)
\]

\[
D^E_{ETC} \sim (10,3,1,-1/3)
\]

(54)

\[
L^E_{ETC} = \left( \begin{array}{c} N^E_{ETC} \\ E^E_{ETC} \end{array} \right)^\alpha \sim (10,1,2,-1/2)
\]

\[
E^E_{ETC} \sim (10,1,1,1)
\]

\[
N^E_{ETC} \sim (10,1,1,0)
\]

where \( \alpha = 1 \ldots 10 \) is an \( SO(10)_{ETC} \) index. Apart from the index \( \alpha \) these fermions have the quantum numbers of a family of quarks and leptons. Let us assume that the ETC symmetry is broken sequentially at three ETC scales as shown below,

\[
SO(10)_{ETC} \overset{M^{ETC}_1}{\rightarrow} SO(9)_{ETC} \overset{M^{ETC}_2}{\rightarrow} SO(8)_{ETC} \overset{M^{ETC}_3}{\rightarrow} SO(7)_{TC}
\]

(55)

The final symmetry group \( SO(7)_{TC} \) is assumed to be unbroken, and is identified as a TC gauge group which confines at the TC scale \( \Lambda_{TC} \). At each stage of symmetry breaking, the ETC representation decomposes into a smaller ETC representation plus a TC singlet, according to,

\[
10 \overset{M^{ETC}_1}{\rightarrow} 9 \oplus 1
\]

(56)

\[
9 \overset{M^{ETC}_2}{\rightarrow} 8 \oplus 1
\]

(57)

\[
8 \overset{M^{ETC}_3}{\rightarrow} 7 \oplus 1
\]

(58)
Each of the TC singlets so produced is identified with a family of quarks and leptons. Thus the low-energy TC theory consists of three families of quarks and leptons plus one techni-family,

\[ SO(7)_{TC} \otimes SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \]  
(59)

\[
Q^T_{i, \alpha} = \begin{pmatrix} U^T_{i, \alpha} \\ D^T_{i, \alpha} \end{pmatrix} \sim (7, 3, 2, 1/6) \]  
(60)

\[
L^T_{i, \alpha} = \begin{pmatrix} N^T_{i, \alpha} \\ E^T_{i, \alpha} \end{pmatrix} \sim (7, 1, 2, -1/2) \]  
(61)

where \( \alpha = 1 \ldots 7 \) is an \( SO(7)_{TC} \) index, and \( i = 1 \ldots 3 \) labels the three families of quarks and leptons.

There is no Higgs doublet in this model, and electroweak symmetry is broken by the TC condensates,

\[
< U^T_{i, \alpha} U^T_{\bar{i}, \bar{\alpha}} > \neq 0 \\
< D^T_{i, \alpha} D^T_{\bar{i}, \bar{\alpha}} > \neq 0 \\
< E^T_{i, \alpha} E^T_{\bar{i}, \bar{\alpha}} > \neq 0 \\
< N^T_{i, \alpha} N^T_{\bar{i}, \bar{\alpha}} > \neq 0
\]  
(62, 63, 64, 65)

Quark and lepton masses arise from the graphs in figure 8. The resulting fermion masses according equation 48 are thus, dropping the ETC and TC superscripts, and the L and R subscripts, for brevity,

\[
m_t \sim \frac{< U >}{M_3^2}, \quad m_s \sim \frac{< U >}{M_2^2}, \quad m_u \sim \frac{< U >}{M_1^2} \]  
(66)

\[
m_b \sim \frac{< D >}{M_3^2}, \quad m_s \sim \frac{< D >}{M_2^2}, \quad m_d \sim \frac{< D >}{M_1^2} \]  
(67)

\[
m_{\tau} \sim \frac{< E >}{M_3^2}, \quad m_{\mu} \sim \frac{< E >}{M_2^2}, \quad m_e \sim \frac{< E >}{M_1^2} \]  
(68)
The ETC bosons all commute with the standard model gauge group and so are electrically and colour neutral. The mass scale of each quark and lepton family is set by the ETC scale at which that particular family was born as a TC singlet. Thus the family mass hierarchy may be associated with the three ETC scales $M_i$. Within a particular family, the large mass splittings which may occur are much harder to account for. For instance the top-bottom mass splitting is at odds with our natural expectation that

$$ m_{\nu_t} \sim \frac{<\tilde{N}N>}{M_3^2}, \quad m_{\nu_b} \sim \frac{<\tilde{N}N>}{M_2^2}, \quad m_{\nu_e} \sim \frac{<\tilde{N}N>}{M_1^2} \quad (69) $$

Indeed the first and last equalities are necessary in order to preserve techni-isospin symmetry which is vital for the correct $W/Z$ mass relations. The techniquark condensates $<\tilde{U}U>=<\tilde{D}D>=<\tilde{E}E>=<\tilde{N}N>$, for example on the grounds that techniquarks carry colour as well as technicolour, whereas technileptons only carry technicolour, so the bottom-tau splitting may in principle be explainable, as we mention in section 3.2. Also the smallness of the neutrino masses may be accounted for by assuming that the original right-handed ETC-neutrino fields $N^R_{ETC}$ are given a Majorana mass. This will then result in both the right-handed technineutrinos $N^R_{ETC}$ and ordinary right-handed neutrinos $\nu_{\text{IR}}$ having the same Majorana mass. After the technineutrino condensates form small physical neutrino masses may then result as a consequence of a see-saw mechanism. Majorana technineutrino masses also lead to negative contributions to the $S$ and $T$ parameters, as we discuss in section 4.3. Note that Majorana masses are only allowed in the context of orthogonal gauge groups, e.g. if we had chosen the ETC gauge group to be $SU(10)_{ETC}$ rather than $SO(10)_{ETC}$ then we would not have been allowed to assume any Majorana masses. Finally quark mixing angles are all zero in this model.

Clearly the model has some problems, but is useful as a simple example of an ETC model. As shown by King (1989b) and King and Mannan (1992) it is possible to fix-up the model by introducing two ETC gauge groups

$$ SO(10)_{ETC1} \otimes SO(10)_{ETC2} \quad (71) $$

9This is because for $SU(N)$ groups the product $N \otimes N$ does not contain a singlet whereas for $SO(N)$ groups it does. This in turn is because for $SU(N)$ $N$ is a complex representation, while for $SO(N)$, $N$ is real.
The low-energy TC theory is the same, but the undesirable fermion mass relations may all be cured. The PGB spectrum, and the question of FCNC’s in this model are also discussed in the above paper. The analysis of King and Mannan (1992) involves a numerical solution to the full set of coupled gap equations and involves the techniques to be discussed in section 3.

We should emphasise that there are many other ETC models in the literature which use the condensate enhancement techniques about to be discussed. For example Guidice and Raby (1992) have attempted to construct a grand unified ETC model. By contrast Sundrum (1993) has constructed an ETC model valid up to 150 TeV in which electroweak symmetry is broken by a single technidoublet. Recently Appelquist and Terning (1994) have attempted to construct an ETC model in which the ETC gauge group is broken by a tumbling mechanism. Tumbling is essentially the statement that chiral gauge groups can become strong and form condensates in the most attractive channel (MAC) which lead to the gauge group being broken to a smaller gauge group (Dimopoulos et al 1980). This is a nice idea because it means that one does not have to re-introduce scalars in order to break the ETC gauge group.

3 Condensate Enhancement

3.1 Schwinger-Dyson Gap Equation

We have seen in equation 48 that the fermion mass is proportional to the technifermion condensate $<\mathcal{T}\mathcal{T}>_{M_{ETC}}$ and that consequently if one wishes to enhance fermion masses one must enhance the condensate as in equation 14. Similarly, the ETC contribution to PGB masses in equation 52 is also proportional to the condensate, and may be similarly enhanced.

Fermion masses arise from the feed-down of the dynamically generated technifermion mass as shown in figure 6. One may approximate the loop calculation in figure 6 by writing the ETC gauge boson propagator as $g^{\mu\nu}/(-M^2_{ETC})$ (i.e. neglect the momentum $p$ dependence compared to the heavy boson mass $M_{ETC}$) and then introduce an ultraviolet cut-off $M_{ETC}$ into the Euclidean space loop integral. The result is, after performing the angular integrations,

$$m_f \approx g_{ETC}^2 \frac{N}{M_{ETC}^2} \int_0^{M_{ETC}^2} dp^2 \, p^2 \frac{\Sigma(p^2)}{p^2 + \Sigma^2(p^2)}$$

(72)

where $g_{ETC}$ is the ETC gauge coupling constant evaluated at the scale $M_{ETC}$, $N$ is
the number of technicolours carried by the technifermions, and $\Sigma(p^2)$ is the running technifermion mass in Euclidean space. $\Sigma(p^2)$ is the self-generated mass of the technifermions and is the direct analogue of the dynamically generated quark mass in QCD. In QCD we have good reasons for believing that $\Sigma(0) \approx 300\text{MeV}$ (a third of the proton mass) and that $\Sigma(p^2)$ decreases for large $p^2$ like $1/p^2$ (times a log) (Lane 1974, Politzer 1976). It is conventional to define the condensate as,

$$<\bar{T}T>_{\text{ETC}} = \frac{N}{4\pi^2} \int_0^{M_{\text{ETC}}^2} dp^2 p^2 \frac{\Sigma(p^2)}{p^2 + \Sigma(p^2)}$$

so that the fermion mass is given by

$$m_f \approx \frac{g_{\text{ETC}}^2}{M_{\text{ETC}}^2} <\bar{T}T>_{\text{ETC}}$$

(73)

Since gauge couplings are typically of order unity this result agrees with equation 48. Furthermore if we assume that $\Sigma(0) \approx \Lambda_{\text{TC}}$ and falls like $1/p^2$ then up to factors and a log we reproduce the dimensional analysis estimate of the condensate in equation 49. It is important to know how the technifermion dynamical mass function $\Sigma(p^2)$ falls at large momentum in order to estimate the condensate reliably. For example if $\Sigma(p^2) \sim \Lambda_{\text{TC}}^2/p$ then the condensate goes as, $<\bar{T}T>_{\text{ETC}} \sim \Lambda_{\text{TC}}^2 M_{\text{ETC}}$ and is enhanced by a factor of $(M_{\text{ETC}}/\Lambda_{\text{TC}})$, corresponding to the enhancement mentioned in equation 14 with $\gamma = 1$. This is precisely what happens in walking TC theories. In order to discuss this effect quantitatively we need to understand how to estimate the behaviour of $\Sigma(p^2)$. We shall shortly introduce a gap equation which will enable $\Sigma(p^2)$ to be determined. But before doing so, we shall write down the Pagels-Stokar (1979) formula for the pion decay constant $F_\pi$,

$$F_\pi^2 = \frac{N}{(2\pi^2)} \int_0^{M_{\text{ETC}}^2} dp^2 p^2 \Sigma(p^2) \frac{\Sigma(p^2) - 1/2p^2 d\Sigma(p^2)/dp}{(p^2 + \Sigma(p^2)^2)^2}$$

(75)

It is clear that while the condensate in equation 73 is sensitive to the high $p$ region of the integration, the pion decay constant in equation 75 is relatively insensitive to this region. Thus quark and lepton masses may be enhanced without changing the $W$ and $Z$ masses very much.

The technifermion mass function $\Sigma(p^2)$ may be estimated from the Schwinger-Dyson gap equation in ladder approximation. We shall follow the approach of Appelquist et al (1986, 1987, 1988a) where the gap equation is given by,

$$\Sigma(p^2) = \frac{3C_2(R)}{4\pi} \int_0^{M_{\text{ETC}}^2} dk^2 k^2 \alpha_{\text{ETC}}(\text{max}(k^2, p^2)) \frac{\Sigma(k^2)}{k^2 + \Sigma^2(k^2)}$$

(76)

where $C_2(R)$ is the quadratic Casimir of the technifermions in the TC representation\footnote{If the technifermions are in the fundamental representation of $SU(N)$ then $R = N$.}.
\( R \), \( \alpha_{TC}(max(k^2, p^2)) \) is the running TC coupling evaluated at the maximum of \( k^2 \) and \( p^2 \). The gap equation above is valid in the Landau gauge where the fermion wavefunction renormalisation \( Z(p^2) = 0 \). The angular integrations have been performed in Euclidean space, and the ETC scale has been inserted as an ultraviolet cut-off (physically it corresponds to the onset of new physics beyond TC, i.e. ETC). The above gap equation is represented diagrammatically in figure 9.

The justification for the use of the gap equation is discussed by Appelquist and Wijewardhana (1987). One should be aware of the well known criticisms of the use of this equation to estimate such quantities as the condensate. Essentially the gap equation is an uncontrolled truncation of the full Schwinger-Dyson gap equations. It is valid only in one gauge and so is not manifestly gauge invariant. Finally it involves a running coupling constant inside the integrand, which purists may object to. Nevertheless we shall use the gap equation to give us insights into the dynamics which are not possible to achieve without it. Despite all its failings, it is undoubtedly a really useful equation.

In order to solve the gap equation we need some ansatz for the running TC coupling constant \( \alpha_{TC}(p^2) \). Since the lower limit of integration extends into the non-perturbative region, we need to model the coupling in this region somehow. The ansatz used is the following. The technifermions are assumed to condense at a scale \( \mu = 2\Sigma(0) \) so that, for \( p > \mu \), \( \alpha_{TC} \) runs with all the fermions present, while for \( \Lambda_{TC} < p < \mu \) it runs with no fermions present. The scale \( \Lambda_{TC} \) represents the TC confinement scale, below which the coupling is taken to be some constant value \( \alpha_{TC}^0 \). By studying the effective potential (Peskin 1982) one may estimate that the critical coupling constant at which the technifermions condense is \( \alpha_{TC}^c = \pi/3C_2(R) \). If one takes this estimate seriously then one may arrange that in the above ansatz \( \alpha_{TC}(\mu) = \alpha_{TC}^c \). With the ansatz for \( \alpha_{TC} \) specified then one may proceed to solve the gap equation by recursive numerical techniques. The results turn out to be remarkably sensitive to the behaviour of the coupling constant in the high momentum region. Above the chiral symmetry breaking scale \( \mu = 2\Sigma(0) \) the TC coupling evolves according to the one loop result,

\[
\alpha_{TC}(p^2 > \mu^2) = \frac{\alpha_{TC}(\mu^2)}{1 + b\alpha_{TC}(\mu^2)\log(p^2/\mu^2)}
\]

where

\[
b = \left( \frac{11}{3} C_2(G) - \frac{4}{3} T(R)n_f \right)
\]

and \( C_2(G) \) is the quadratic Casimir of the adjoint representation of the technigluons.
$G, T(R)$ is the index of the technifermion representation $R$ and $n_f$ is the number of Dirac technifermons. More generally the beta function is given by,

$$\beta_{TC} = p \frac{d\alpha_{TC}}{dp} = -b\alpha_{TC}^2 - c\alpha_{TC}^3 \ldots$$  (79)

### 3.2 Walking Technicolour

Holdom’s original observation back in 1981 was that if the TC theory was born at a fixed point $\alpha_{TC} \neq 0$, $\beta_{TC} = 0$ then the condensate could be enhanced as in equation 14 with $\gamma \sim 1$. This assumption of a fixed point TC theory was later considered by Yamawaki et al (1986a, 1986b). Condensate enhancement (without $F_\pi$ being changed) is also possible for asymptotically free (ASF) theories in which $\beta_{TC} \approx 0$ and the coupling is slowly running (walking) (Holdom 1985, Appelquist et al 1986, 1987). Walking TC theories rely on either a large number $n_f \gg 2$ of technifermions in the fundamental representation $R = N$ (type C theories) or $n_f = 2$ technifermions in a very large representation $R \gg N$ (type A theories), or some other combination (type B theories), in order to arrange that $b \approx 0$. Care must be taken to ensure that $b > 0$ (to maintain ASF), and $c\alpha_{TC}^3 < b\alpha_{TC}^2$ (for good convergence of the series in equation 79). Numerical solutions of equation 76 are depicted in figure 10. These numerical solutions for $\Sigma(p^2)$ are inserted into equation 73 in order to estimate the condensate. Detailed numerical studies (Appelquist et al 1988a, King and Ross 1989) confirm the simple depictions in figure 10, and demonstrate that the values of $F_\pi$ obtained from the Pagels-Stokar result are insensitive to high-p physics.

The gap equation may also receive contributions from other sources as shown in figure 11. Each of these diagrams in principle must be added to the right-hand side of figure 9. The next order technigluon corrections in figure 11a beyond the ladder approximation have been studied by Appelquist et al (1988b) and Holdom (1988a) and tend to decrease the critical coupling $\alpha_{TC}^r$ by a few per cent for type A walking theories, and by about 20% for type C walking theories. The question of gauge dependence and higher order corrections has also been studied by Kamli and Ross (1992), who conclude that there are quite large uncertainties in the estimation of the condensate. Another important correction for techniquarks $Q$ (absent for technileptons $L$) is QCD gluon exchange (Holdom, 1988b) in figure 11b. This can result in condensate splitting $<\bar{Q}Q> / <\bar{L}L> \sim 10$, which may split the quark and lepton masses by a similar factor if one assumes that quarks receive mass from techniquarks, and leptons from technileptons as in the simple model of section 2.3
for example. ETC boson exchange corrections in figure 11c have been studied in the
four-fermion approximation by Appelquist et al (1989) and Holdom (1989), and in
the full theory by King and Ross (1990) and give important condensate enhancement
if the ETC coupling constant $\alpha_{ETC}$ is sufficiently strong. This effect (called strong
ETC) will be discussed in section 3.3. Finally another important contribution to the
technifermion condensate is top quark exchange, mediated by ETC bosons as shown
in figure 11d and discussed by Appelquist and Shapira (1990) and King and Mannan
(1992). This effect is interesting because the top quark is treated as a dynamical
fermion on the same footing as the techniquarks. For example, in the orthogonal
ETC model of King and Mannan (1992) there are no ETC bosons which connect
techniquarks to techniquarks, but there are ETC bosons which connect top quarks
to techniquarks. These ETC bosons allow the techniquark mass to be fed-down to
the top quark mass, but because the top quark is so heavy the top quark mass also
contributes to the techniquark mass by a similar diagram. Thus there is a sort of
feed-back effect which can enhance both the techniquark and the top quark masses.
This is in fact a simple example of a top quark condensate, about which we shall say
more in section 5.

3.3 Strong Extended Technicolour

Walking TC can at best lead to $\Sigma(p^2) \sim 1/p$ and hence $\gamma \approx 1$. As discussed in
the introduction this amount of condensate enhancement is welcome but insufficient
to account for heavy quark masses, especially the top quark mass. The ultimate
condensate enhancement arises from $\Sigma$ which is flat out to $M_{ETC}$, corresponding to
$\gamma \sim 2$ in equation 14. Such condensate enhancements would be certainly sufficient to
account for the top quark mass, if they can be achieved without ill effects.

As mentioned, condensate enhancements up to $\gamma = 2$ can result from the idea
is strong ETC? It is a correction to the gap equation coming from the diagram in
figure 11c. Thus there is a second term on the rhs of equation 76 which is similar to
the original term but with $\alpha_{TC}$ replaced by $\alpha_{ETC}$, the Casimir replaced by a dif-
ferent Casimir associated with the embedding of $G_{TC}$ into $G_{ETC}$ and with $\max(k^2, p^2)$
replaced by $\max(k^2, p^2, M_{ETC}^2)$ which approximates the exchange of the heavy ETC
boson as a step function, where the upper limit of integration now extends to infinity.
Strongly coupled heavy ETC gauge boson exchange was first studied by King and
Ross (1990). The effect of heavy ETC boson exchange may also be approximated by
making a four fermion approximation and cutting off the upper limits of integration at \( M_{ETC} \), and this approximation was originally made by Appelquist \textit{et al} (1989) and Holdom (1989). In all these approaches the physical effect is the same: for sufficiently strong ETC couplings \( \alpha_{ETC} \) the exchange of the heavy boson can dominate the physics of electroweak symmetry breaking, with technigluon exchange being of secondary importance. The result of this is that electroweak symmetry is broken dominantly by physics associated with a high energy scale \( M_{ETC} \). For sufficiently large \( \alpha_{ETC} \) the model will resemble the NJL model and all technifermion masses \( \Sigma \) will be of order \( M_{ETC} \). This is undesirable since it implies that the \( W, Z \) masses will similarly be \( 0(M_{ETC}) \).

We are interested in values of \( \alpha_{ETC} \) which are strong but not too strong. What happens to \( \Sigma \) as we increase the value of \( \alpha_{ETC} \) from zero up to some strong value? For \( \alpha_{ETC} = 0 \), \( \Sigma \sim 1/p^{1-2} \) as discussed in section 3.2. As \( \alpha_{ETC} \) is increased the shape of \( \Sigma \) changes in a subtle way at first by developing a tail for \( p \approx M_{ETC} \) which enhances the condensate somewhat. Then at some strong value of \( \alpha_{ETC} \) there is a dramatic change in the solution, and \( \Sigma \) becomes quite flat for larger values of \( \alpha_{ETC} \) and the value of \( \Sigma(0) \) begins to rise as quite a sensitive function of \( \alpha_{ETC} \) as it approaches its NJL solution.

The above effect has been observed in all the approaches mentioned above but it is most simply studied in a theory in which \( \alpha_{TC} \) does not run and the effect of ETC boson exchange is approximated by the four fermion approximation. Apart from a Casimir this toy ETC theory is just quenched QED which has been studied extensively by Bardeen \textit{et al} (1986, 1990b), Appelquist \textit{et al} (1988c) Kondo \textit{et al} (1989), Dagotto \textit{et al} (1990) and Curtis and Pennington (1993). For quenched QED one may plot a criticality curve as in figure 12 which shows the separation between the broken chiral symmetry phase of the theory and the unbroken phase of the theory as a function of the gauge coupling \( \alpha \) and the four fermion coupling \( \lambda \). The dimensionless \( \lambda \) is normalised so that for \( \alpha = 0 \) chiral symmetry is broken for \( \lambda > 1 \) (the NJL point). In the pure gauge limit \( \lambda = 0 \) chiral symmetry is broken for \( \alpha > \alpha^c = \pi/3 \). On the criticality curve one has,

\[
\lambda = \frac{1}{4} (1 + \sqrt{1 - \alpha/\alpha^c})^2
\]

The anomalous dimension increases along the curve from \( \gamma = 1 \) (recall that this is quenched QED so that the coupling does not run) to \( \gamma = 2 \) according to,

\[
\gamma = 1 + \sqrt{1 - 3\alpha/\pi}
\]
In such theories a phenomenologically acceptable top quark mass has a price associated with it. Firstly it seems unlikely that one can account for the mass ratio $m_t/m_b$ without overinfecting the $\rho$ parameter (Appelquist et al 1989, Holdom 1989). Secondly the required condensate enhancements can only be achieved at the expense of some fine-tuning.

The fine-tuning of $\alpha_{ETC}$ (or equivalently $\lambda$) is in turn associated with the appearance of light scalar bound states (Chivukula et al 1990). As Miransky (1991) has emphasised light bound states are always associated with finely tuned theories with $\gamma \approx 2$. Miransky’s argument can be phrased very simply in terms of the physical dimension of the operator $\bar{\psi}\psi$ which approaches the dimension of a scalar field $\phi$ as $\gamma$ approaches 2,

$$D_{\bar{\psi}\psi} = 3 - \gamma \rightarrow 2 \quad 1 = D_{\phi}$$

(82)

The fine-tuning is associated with how close to the critical line one’s parameters are chosen to be. The scalar fields appear as heavy, broad resonances, depending on the parameters of the theory (Appelquist et al 1991).

This phenomenon is the basis for the top quark condensate model to be discussed in section 5.

4 Experimental Prospects

4.1 Longitudinal W and Z scattering at the LHC

We now discuss the experimental signatures of TC, beginning in this section with the minimal TC model. As we saw earlier, the three would-be GB’s of this model are eaten by the $W$, $Z$ and become their $L$ components. There is no Higgs boson in this model, but there is a broad scalar resonance at the TeV scale, which is the TC analogue of the Higgs boson. In fact the entire techni-hadron spectrum at the TeV scale is just a scaled-up version of the ordinary QCD hadron spectrum as indicated in table 2. The technimesons have the same quantum numbers as the mesons of QCD but with isospin being replaced by techni-isospin. We have simply scaled up the masses by a factor of 2650, assuming a QCD-like TC theory based on $SU(3)_{TC}$. Note that in addition to these technimesons, there will also be TeV scale technibaryons. The lightest technibaryon may be absolutely stable and is a candidate for the dark matter of the Universe (Nussinov 1985).
Table 2: Comparison of meson and technimeson spectrum in minimal TC and QCD with one quark doublet.

<table>
<thead>
<tr>
<th>meson/technimeson</th>
<th>$J^{PC}$</th>
<th>$I^G$</th>
<th>meson mass (MeV)</th>
<th>technimeson mass (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^+/\pi^+_TC$</td>
<td>0$^-$</td>
<td>1$^-$</td>
<td>139.6</td>
<td>eaten</td>
</tr>
<tr>
<td>$\pi^0/\pi^0_{TC}$</td>
<td>0$^+$</td>
<td>1$^-$</td>
<td>135.0</td>
<td>eaten</td>
</tr>
<tr>
<td>$\eta^0/\eta^0_{TC}$</td>
<td>0$^+$</td>
<td>0$^+$</td>
<td>549</td>
<td>1.5 TeV</td>
</tr>
<tr>
<td>$f'/f'_{TC}$</td>
<td>0$^{++}$</td>
<td>0$^+$</td>
<td>1400</td>
<td>3.7 TeV</td>
</tr>
<tr>
<td>$\rho^+/\rho^+_{TC}$</td>
<td>1$^-$</td>
<td>1$^+$</td>
<td>770</td>
<td>2.0 TeV</td>
</tr>
<tr>
<td>$\rho^0/\rho^0_{TC}$</td>
<td>1$^-$</td>
<td>1$^+$</td>
<td>770</td>
<td>2.0 TeV</td>
</tr>
<tr>
<td>$\omega^0/\omega^0_{TC}$</td>
<td>1$^{--}$</td>
<td>0$^-$</td>
<td>782</td>
<td>2.1 TeV</td>
</tr>
</tbody>
</table>

If minimal TC is correct then there will be no light Higgs boson, but instead a “mini-desert” up to the TeV scale where all the technimesons are. To test the minimal TC model we therefore must wait for LHC energies at which we can hope to find evidence for some of the technimesons in table 2. In fact the best way to study minimal TC is to study longitudinal $W$ and $Z$ scattering processes such as:

$$W_L W_L \rightarrow W_L W_L$$
$$W_L Z_L \rightarrow W_L Z_L$$

Since the L components of the gauge bosons correspond to technipions, this amounts to technipion-technipion scattering. More precisely, there is an equivalence theorem proved to one-loop order and discussed by Chanowitz and Gaillard (1985) (based on earlier work as discussed by these authors). According to the equivalence theorem, in the ultra-relativistic limit the longitudinal gauge boson scattering amplitudes calculated in unitary gauge are equal to the corresponding GB scattering amplitudes calculated in renormalisable gauge. At finite energies $E_i$, there are corrections as shown below

$$M(W_L(p_1)W_L(p_2)\ldots) = M(\pi(p_1)\pi(p_2)\ldots) + 0(M_W/E_i)^2$$

Armed with the equivalence theorem, low energy pion-pion scattering results can be applied to technipion-technipion scattering and hence to longitudinal gauge boson scattering. The most convenient and systematic way of discussing this is to use the chiral lagrangian (see Georgi 1984). For example to lowest order the chiral lagrangian yields,

$$M(W_L^+W_L^- \rightarrow Z_L Z_L) = \frac{s}{F^2}$$

which leads to a cross-section which rises with the energy squared $s$ and eventually violates partial wave unitarity for $\sqrt{s} \approx 1.7$ TeV. The above result has been extended
to order $0(s^2/F_x^4)$ by several authors, for example Dawson and Willenbrock (1989). In the minimal standard model, the amplitude is cut off by s-channel Higgs exchange, which cancels the bad high energy (unless the Higgs mass is very large). In TC the amplitude is cut off by the s-channel technimeson resonances, but since these masses are very large the scattering amplitude must necessarily be quite strong.

There has been an enormous amount of work on trying to determine exactly how the scattering amplitude is cut off in TC theories and consequently what the precise signature of TC would be at the LHC. The big question is how to distinguish the TC signature from a very heavy Higgs boson in the standard model. The issues are quite complicated and rather than discuss them here we refer the reader to two examples of recent literature. The first example is the so called BESS model of Casalbuoni et al (1987, 1991) which assumes a triplet of new vector resonances (the technirhos). The second example is the chiral lagrangian approach of Dobado et al (1991). As discussed by Pauss (1990) LHC studies indicate that providing the (charged) technirho is not much heavier than about 1.5 TeV the WZ channel would, after suitable cuts, exhibit a peak above the background in both the BESS and the DHT approaches, assuming an LHC of $\sqrt{s} = 16$ TeV, with an integrated luminosity of $10^5 \, pb^{-1}$, as shown in figure 13.

### 4.2 Pseudo-Goldstone bosons from a single techni-family

In this section we consider the single techni-family scenario, and briefly discuss the resulting pseudo-Goldstone boson phenomenology. A more detailed discussion can be found in Farhi and Susskind (1981) who were the first to suggest such a model. Consider a TC model based on the gauge group,

$$SU(N)_{TC} \otimes SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$$  \hspace{1cm} (86)

and with a single techni-family,

$$Q_L^{TC\alpha} = \left( \begin{array}{c} U_L^{TC\alpha} \\ D_L^{TC\alpha} \end{array} \right) \sim (N,3,2,1/6)$$
$$U_R^{TC\alpha} \sim (N,3,1,2/3)$$
$$D_R^{TC\alpha} \sim (N,3,1,1/3)$$
$$L_L^{TC\alpha} = \left( \begin{array}{c} N_L^{TC\alpha} \\ E_L^{TC\alpha} \end{array} \right) \sim (N,1,2,-1/2)$$
$$E_R^{TC\alpha} \sim (N,1,1)$$
$$N_R^{TC\alpha} \sim (N,1,0)$$  \hspace{1cm} (87)
The technifermions which carry technicolour and QCD colour are referred to as techniquarks, while the technifermions which carry technicolour but not QCD colour are called technileptons. Note that the right-handed technineutrino $N_{R}^{TC}$ is required by anomaly cancellation, and cannot be given a Majorana mass without breaking $SU(N)_{TC}$. If we had chosen the TC gauge group to be $SO(N)_{TC}$ then the right-handed technineutrino would not be required by anomaly cancellation, and it could be given a Majorana mass without breaking $SO(N)_{TC}$. This is simply because the $N$ representation of $SU(N)$ ($SO(N)$) is complex (real). The quarks and leptons transform as usual and are technicolour singlets

$$Q_{L}^{i} = \left( \begin{array}{c} U_{L} \\ D_{L} \end{array} \right) ^{i} \sim (1, 3, 2, 1/6)$$

$$U_{R}^{i} \sim (1, 3, 1, 2/3)$$

$$D_{R}^{i} \sim (1, 3, 1, -1/3)$$

$$L_{L}^{i} = \left( \begin{array}{c} \nu_{L} \\ E_{L} \end{array} \right) ^{i} \sim (1, 1, 2, -1/2)$$

$$E_{R}^{i} \sim (1, 1, 1)$$

$$\nu_{R}^{i} \sim (1, 1, 0)$$

(88)

where $\alpha = 1 \ldots N$ is an $G_{TC}$ index, and $i = 1 \ldots 3$ labels the three families of quarks and leptons.

In the limit that QCD and electroweak interactions are switched off, the TC sector of the model respects a large chiral symmetry

$$SU(8)_{L} \otimes SU(8)_{R}$$

(89)

since there are eight technifermions arranged in the vectors,

$$(U^{red}, U^{blue}, U^{green}, D^{red}, D^{blue}, D^{green}, E, N)_{L,R}$$

(90)

Electroweak symmetry is broken by the condensates,

$$< U^{TC} U^{TC}_{L} > = < D^{TC} D^{TC}_{R} > = < F^{TC} F^{TC}_{L,R} > = < N^{TC} N^{TC}_{R} > \neq 0$$

(91)

Since the techniquark condensates have QCD colour, there are really eight separate condensates above, which break the chiral symmetry down to $SU(8)_{L+R}$. The electroweak symmetry is now broken by the equivalent of four separate technidoublets (red, blue, green doublets of techniquarks and a doublet of technileptons) and so the technipion decay constant required to give the correct $W, Z$ masses is now,

$$F_{\pi} = \frac{246}{\sqrt{4}} GeV = 123 GeV$$

(92)
According to Goldstone's theorem we would expect $8^2 - 1 = 63$ massless GB's, one for each broken generator. Three of these will get eaten by the Higgs mechanism, leaving 60 GB's in the physical spectrum.

According to our previous discussion, we would not expect these to be massless GB's but light PGB's due to the explicit chiral symmetry breaking effects which we have so far ignored. The PGB mass spectrum was estimated by Peskin (1980) and Preskill (1981). These authors also studied the whole question of vacuum alignment, i.e. the problem of which condensates form and which symmetries are broken in TC theories. For example it is possible in principle for a techniquark to condense with a technilepton, thereby breaking QCD gauge symmetry. Clearly this would be undesirable, and as it turns out is disfavoured by QCD gluon exchange which acts as a small perturbation which breaks the degeneracy of the technifermion condensates. The condensates which respect QCD correspond to a slightly lower vacuum energy than the condensates which break QCD, and since the condensates are otherwise degenerate this is enough to tip the balance in favour of QCD-conserving condensates. The condensates which we have assumed in equation 91 are in fact the ones which are preferred by these kinds of arguments. The same formalism which is used to study the question of vacuum alignment also yields the estimates of PGB masses shown in figure 14 taken from Peskin (1980). As mentioned, these mass estimates are increased in walking TC theories since condensate enhancement also enhances PGB masses.

In figure 15 the PGB spectrum is shown for an orthogonal TC gauge group $SO(N)_{TC}$ in which the symmetry breaking for a single technifamily is $SU(16) \rightarrow SO(16)$. An example of an orthogonal TC model was given in section 2.3. There are $(16^2 - 1) - (16 \times 15)/2 = 135$ broken generators, leading to 132 PGB's (plus 3 eaten would-be GB's).

Note that in both figures 14 and 15 the class I PGB's corresponding to the colour singlet states receive mass from electroweak and ETC perturbations and so are not expected to be massless. Indeed Eichten and Lane (1980) estimate such PGB's to have masses up to 40 GeV from ETC effects. Due to condensate enhancement, much larger masses are also possible.

The 60 PGB's of the $SU(N)_{TC}$ model consist of the following states,

$\bar{Q}\gamma_5 \Lambda^a \tau^a Q$
$\bar{Q}\gamma_5 \Lambda^a Q$
$\bar{Q}_L \gamma_5 \tau^a L$
\[
\begin{align*}
Q_{\gamma_5} L \\
L_{\gamma_5} \tau^a Q_i \\
L_{\gamma_5} Q_i \\
Q_{\gamma_5} \tau^a Q + L_{\gamma_5} \tau^a L \\
Q_{\gamma_5} \tau^\pm Q - 3 L_{\gamma_5} \tau^\pm L \\
Q_{\gamma_5} \tau^3 Q - 3 L_{\gamma_5} \tau^3 L \\
Q_{\gamma_5} Q - 3 L_{\gamma_5} L
\end{align*}
\] (93)

where \( Q = (U, D), \ L = (N, E), \ \lambda^a (\alpha = 1 \ldots 8) \) are the Gell-Mann colour matrices and \( \tau^a (a = 1 \ldots 3) \) are the Pauli isospin matrices.

The experimental signatures of the PGB’s were studied by Ellis \textit{et al} (1981), Dimopoulos (1980), and many other authors at around this time. A more recent discussion can be found in Eichten \textit{et al} (1986). We divide our brief discussion into searches for colour octet PGB’s, colour triplet PGB’s (i.e. leptoquarks) and colour singlet PGB’s (which resemble charged Higgses).

As it turns out, it is relatively straightforward to find the colour octet PGB’s at the LHC, but much harder (but not impossible) at the Tevatron. Consider the colour octet neutral state \( P^0_8 = (U \bar{U} + D \bar{D})_8/\sqrt{2} \). This is a techni-isospin singlet and can be produced singly in hadronic collisions with a cross-section \( \frac{dx}{dy} \approx 1(10^{-2}) \text{nb} \) at the LHC (Tevatron) (for rapidity \( y = 0 \)). The \( P^0_8 \) can decay back into \( gg \) or into \( t \bar{t} \) if kinematically allowed. The first signal at the Tevatron may be an enhancement of the top quark production cross-section, as discussed by Eichten \textit{et al} (1986) and Appelquist and Triantaphyllou (1992b). In fact the CDF cross-section for \( t \bar{t} \) production does appear to be somewhat higher than standard model expectations (Abe \textit{et al} 1994).

The colour triplets are examples of leptoquarks. There are many of them in the spectrum, consisting of colour triplet combinations such as \( U \bar{N}, \ U \bar{E}, \ D \bar{N}, \ D \bar{E} \) and their antiparticles. At the LHC or HERA they are copiously pair produced and tend to decay into heavy quarks and leptons with relatively background-free signatures. For example a typical signature of a leptoquark pair might be \( t \bar{t} \tau \bar{\tau} \) which has a particularly low background.

The colour singlets are similar to charged and neutral Higgs bosons. The best place to look for them is the clean environment provided by the LEP collider, although they should also be seen at the LHC. The neutral PGB’s do not have a tree-level coupling to
the $Z$ boson, however, which should enable it to be distinguished from neutral Higgs bosons. They couple to gauge bosons via the triangle anomaly, with technifermions running round the loop, as discussed in some detail by Ellis et al (1981). The charged PGB’s couple to the photon and $Z$ by tree-level couplings which resemble those for charged Higgs, and like charged Higgs tend to decay into the heaviest fermions around.

Finally note that the coloured PGB’s may re-scatter into eaten technipions, and hence may enhance the rates of longitudinal gauge boson scattering, as observed by Bagger, Dawson and Valencia (1991).

### 4.3 Precision Electroweak Measurements

The subject of this section is very specialised and rapidly changing, as new data becomes available as well as new theoretical variables with which theory can be compared to experiment. Yet this is also a subject of immense importance to TC theories, because it has been widely reported that the recent data disfavour TC theories, a claim that has been disputed by several authors.

The discussion of radiative electroweak corrections is commonly restricted to the oblique corrections (Kennedy and Lynn 1989) (i.e. the vacuum polarisation graphs only) which can be very conveniently parametrised in terms of three parameters $S$, $T$ and $U$ (Peskin and Takeuchi 1990, 1992). One can argue that present data do not restrict the $U$ parameter very much which can thus be ignored.

The S parameter is defined by,

$$S \equiv 16\pi \frac{d}{dq^2} [\Pi_{33}^{new\ physics}(q^2) - \Pi_{33}^{new\ physics}(q^2)]_{q^2=0}$$

The T parameter is defined by,

$$\alpha T \equiv \frac{e^2}{\sin^2 \theta_W M_W^2} [\Pi_{11}^{new\ physics}(0) - \Pi_{33}^{new\ physics}(0)]$$

where $\Pi_{ij}^{new\ physics}$ are the gauge boson self-energies, with indices $i,j = 1,3$ referring to $SU(2)_L$ currents and $j = Q$ referring to the electromagnetic current, $M_W$ is the $W$ mass, $e$ is the electromagnetic charge, and $q$ is the momentum flowing through the gauge boson propagator. In writing the self-energies we have assumed that the total self-energy can be divided into two parts,

$$\Pi_{ij}(q^2) = \Pi_{ij}^{standard\ model}(q^2) + \Pi_{ij}^{new\ physics}(q^2)$$
and so the quantities $S$, $T$ are only concerned with the new physics contributions, relative to some definition of the standard model (i.e. some top and Higgs mass must be assumed). In order for these definitions to be useful, it is necessary to assume that the new physics consists of new particles whose mass is very much greater than the $W$, $Z$ masses. The point is that the physical measured quantities such as the $Z$ mass and width, the electroweak coupling constants, the $W$ mass, the various forward-backward asymmetries, and so on are measured at $q^2 = M_Z^2$ or $q^2 = M_W^2$ or $q^2 = 0$ and so involve quantities like $\Pi_{ij}^{\text{new physics}}(M_Z^2)$, $\frac{d}{dq^2}\Pi_{ij}^{\text{new physics}}(q^2)|_{q^2=M_Z^2}$, and similar expressions with $M_Z \to M_W$. Since these values of $q^2$ are assumed small compared to the masses of the new particles running round the loops, it is assumed that

$$\Pi_{ij}^{\text{new physics}}(M_Z^2) \approx \Pi_{ij}^{\text{new physics}}(0) + M_Z^2 \frac{d}{dq^2}\Pi_{ij}^{\text{new physics}}(q^2)|_{q^2=0}$$  \hspace{1cm} (97)$$

$$\frac{d}{dq^2}\Pi_{ij}^{\text{new physics}}(q^2)|_{q^2=M_Z^2} \approx \frac{\Pi_{ij}^{\text{new physics}}(M_Z^2) - \Pi_{ij}^{\text{new physics}}(0)}{M_Z^2}$$  \hspace{1cm} (98)$$

and similar expressions with $M_Z \to M_W$. These approximations correspond to the assumption that the self-energy functions have a linear slope over the small (compared to new physics masses) $q^2$ region $0 - M_{W,Z}^2$. With this approximation, all the physical quantities of interest may be re-parametrised in terms of $S$, $T$ (and $U$ which we have ignored).

The $T$ parameter is essentially just the contribution to $\delta \rho$ from new physics, $\alpha T \approx \delta \rho^{\text{new}}$. Crudely, $T$ is a measure of the isospin splitting of the new fermion doublets, while $S$ is a measure of the number of new fermion doublets. Detailed fits of all the electroweak data lead to ellipses in the $S,T$ plane representing the allowed contribution from new physics at, say 90% c.l., with the origin $S=\alpha T=0$ corresponding to the standard model at some reference value of $m_t, m_H$ (figure 16). These fits tend to favour negative values of $S$ and $T$ such as $S \approx -1, T \approx -0.5$, with positive values such as $S \approx 1, T \approx 1$ disfavoured by the data. The question is what does TC predict for $S$ and $T$? The answer is far from certain. The problem is that $S$ and $T$ are difficult to estimate non-perturbatively. Various authors have attempted to do so however using various techniques (Peskin and Takeuchi 1990, 1992, Cahn and Suzuki 1991, Golden and Randall 1990, Holdom and Terning 1990, Appelquist and Triantaphylou 1992a, Sundrum and Hsu 1991, Chivukula et al 1992a). It turns out that $T$ (or equivalently $\delta \rho$) is highly model dependent.

The $S$ parameter is of more interest due to its relative model independence. On the other hand $S$ is very difficult to estimate in strongly coupled theories such as TC.
Peskin and Takeuchi (1990, 1992) estimated $S$ in TC from a scaled-up QCD dispersion relation (assuming techni-isospin conservation and parity) and concluded that $S \approx 1.6$ for a technifamily and $S \approx 0.5$ for a single doublet (assuming $N = 4$ in both cases). The perturbative result which essentially counts the number of electroweak doublets is $S \approx NN_D/(6\pi)$ where $N$ is the number of TC’s and $N_D$ is the number of doublets (e.g. $N_D = 4$ for a technifamily). The QCD estimates are about a factor of 2 larger than the perturbative estimates would yield. Indeed several authors have pointed out that in walking TC or strong ETC where $N$ is flattened then the estimates for $S$ and $T$ should resemble more closely the perturbative estimates (see for example Appelquist and Triantaphyllou 1992a). But even in the perturbative limit the $SU(4)_T$ technifamily will still contribute $S = 0.85$ which is on the fringes of the present experimentally allowed range. The only conclusion from this is that the techni-family is being severely challenged by this data. I would go even further and say that unless there are any get-outs, the techni-family is effectively ruled out by this data. However as we shall see there are get-outs available, so the situation is not entirely clear cut.

I wish to emphasise once again that estimates of $S$ are made using a variety of approaches, which are not all mutually consistent. In the midst of all the confusion, it seems to me that one approach is particularly elegant, namely the quasi-perturbative methods developed by Holdom et al (1989,1990) and Terning (1991). These methods draw their inspiration from the Pagels-Stokar approach which calculates pion decay constant using an integral sum rule in which the only input is $\Sigma(p^2)$ (and its first derivative). In the presence of isospin breaking one may extend the Pagels-Stokar result in order to calculate $F_{\pi^\pm}$ and $F_{\pi^0}$ from $\Sigma_U \neq \Sigma_D$. One then computes,

$$\alpha T = \delta \rho^{\text{new}} = M_{\pi^\pm}^2 / M_{\pi^0}^2 - 1 = F_{\pi^\pm}^2 / F_{\pi^0}^2 - 1$$  \hspace{1cm} (99)$$

In a similar spirit one may derive a formula for $S$ (valid in the isospin limit) as an integral sum rule in terms of $\Sigma(p^2)$ and its higher derivatives. In these approaches the gauge boson loop corrections are approximated by loops of technifermions with a running self-mass function $\Sigma(p^2)$, which may be estimated from a gap equation. Such estimates effectively “mock-up” the corrections from heavy technirho exchange as well as the other heavy vector resonances. They do not include contributions from light PGB’s which must be included separately and will therefore give additional contributions ( Peskin and Renken 1983, Golden and Randall 1991, Holdom and Terning 1990). Luty and Sundrum (1993) have even suggested that under some circumstances the PGB contribution to $S$ could turn out to be negative.

Several authors have considered sources of new physics which can yield negative
contributions to $S, T$. One example is the observation that Majorana neutrino masses can lead to negative $S, T$ (Gates and Terning 1991). As observed by Gates and Terning, a Majorana mass for the technineutrino would also be expected to lead to similar effects. Using the non-local effective theory methods, and assuming an orthogonal $SO(N)_{TC}$ gauge group, one can obtain a negative contribution to $S$ and $T$ by giving the techni-neutrino a Majorana mass (Evans et al 1993b, 1993c). In $SU(N)_{TC}$ a technineutrino mass is of course forbidden by the gauge symmetry. This may help with models such as the chiral ETC model (King 1989b, King and Mannan 1992) which are based on $SO(7)_{TC}$ and so allow a right-handed technineutrino mass term.

More recently, Burgess et al have gone beyond the approximations in equations 97, 98 and considered the case that the new particle masses are not far above the $Z$ mass. Whereas Peskin and Takeuchi were able to parametrise all the low energy data in terms of three parameters $S, T, U$, Burgess et al showed that by relaxing the above approximations, all the data could be parametrised in terms of six parameters $S, T, U$ as before, plus three additional parameters $V, W, X$, which correct the simple approximations. For example the $V$ parameter is defined as the correction to equation 98,

$$
\alpha V = \frac{d}{dq^2} \Pi_{ZZ}^{new\ physics}(q^2)|_{q^2=M_Z^2} - 
\left[ \Pi_{ZZ}^{new\ physics}(M_Z^2) - \Pi_{ZZ}^{new\ physics}(0) \right] \frac{1}{M_Z^2} 
$$

(100)

$V, W, X$ all vanish if the self energy functions are linear functions of $q^2$. Burgess et al made fits to $S, T$ from the most recent electroweak data, first in the case that $V, W, X$ all vanish (as was assumed by Peskin and Takeuchi 1990, 1992) and then allowing $V, W, X$ to vary. The result, which is quite dramatic, is shown in figure 16. Clearly the inclusion of $V, W, X$ weakens the bounds on $S, T$ considerably. It turns out that the $V$ parameter is the most important from the point of view of relaxing the constraints on $S, T$. With $V$ differing significantly from zero, we have weaker limits on $S, T, V$ like,

$$
S < 2.5, \ T < 1.3, \ V < 2.0 
$$

(101)

Evans (1993) has made a detailed study of $V, W, X$ in TC models, and considered two possible sources of new particles with light (not much heavier than the $Z$) masses, namely light technifermions, and light PGB’s. He found that the effect of the PGB’s was typically small, and that $V = 0$ to good approximation unless the technifermions
had dynamical masses less than about 150 GeV. In the next section we shall consider exactly such a low scale TC scenario.

So far we have restricted our attention to oblique radiative corrections, i.e. corrections to gauge boson propagators coming from loops of heavy new particles. What about non-oblique corrections, coming from vertex corrections for example? In general, it turns out that vertex and box diagrams are suppressed relative to the oblique corrections, essentially due to small Yukawa couplings which enter in these diagrams.

However in certain cases the vertex corrections can be important for TC models, the prime example being the $Zb\bar{b}$ vertex. There are two sources of corrections to this vertex in TC models, namely from ETC gauge boson exchange (Chivukula et al 1992b, 1994) and from PGB exchange (Xiao et al 1994). The ETC gauge boson exchange contribution is shown in figure 17 in the case where the ETC gauge group commutes with the standard model gauge group, so that the ETC gauge boson is colour and electrically neutral. Such diagrams alter the left-handed gauge coupling $g_L$ by an amount,

$$\delta g_L \sim \frac{m_t}{16\pi F_\pi} \frac{e}{\sin \theta_W \cos \theta_W}$$

(102)

where we have assumed that the top mass is given by

$$m_t \approx 4\pi \frac{F_\pi^3}{M_{ETC}}$$

(103)

The experimentally relevant quantity is the ratio,

$$\Delta_R = \frac{\delta(\Gamma_b/\Gamma_{hadrons})}{\Gamma_b/\Gamma_{hadrons}}$$

(104)

(wher hadrons do not include $b$ quarks) which LEP can measure to a precision of at least 2%. The standard model radiative corrections to $\Delta_R$ are about $-2\%$. The ETC corrections are

$$\delta \Delta_R \sim -3.7\% \left( \frac{m_t}{100 \text{ GeV}} \right)$$

(105)

Thus this ETC model is ruled out since the top quark is much heavier than 100 GeV. PGB contributions to $\Delta_R$ can range from 0 to -10\% depending on the exact PGB spectrum (Xiao et al 1994). As usual there are ways out. For example, Evans (1994) points out that if the ETC scale is boosted by strong ETC effects then the constraint may be avoided. This requires fine tuning of several per cent, however. Alternatively, one may suppose that the top quark does not gain its mass from ETC gauge boson exchange, but instead gains it mass from the top quark condensate. Once again this motivates the low scale TC scenario in the next section.
4.4 Low scale technicolour

So far we have assumed that the TC confinement scale is $\Lambda_{TC} \sim 500$ GeV so that in order to probe the TC dynamics one must wait for the LHC. In this section we discuss the phenomenology of an $SU(2)_{TC}$ technicolour model proposed by King (1993) with a low technicolour confinement scale $\Lambda_{TC} \sim 50 - 100$ GeV. Such a low technicolour scale may give rise to the first hints of technicolour being seen at LEPI and spectacular technicolour signals at LEPI II and the Tevatron.

How do we achieve a low TC scale and still break electroweak symmetry strongly enough? The answer is that we use the idea of strong ETC as discussed in section 3.3 to enhance the TC condensate sufficiently to give the correct $W$ mass. In other words we rely on some four-technifermion operators which help to break electroweak symmetry, so that TC does not have to do all the work itself, thus allowing the TC scale to be reduced. There may be dominant technifermions which are subject to the strong operators and sub-dominant technifermions which are not. The sub-dominant technifermions will condense at a TC low scale leading to the striking experimental signatures advertised above. There is one other feature of the model worthy of note. We shall assume that ordinary fermions (quarks and leptons) do not acquire masses from the ETC mechanism, but instead generate their own masses from heavy quark and lepton condensates (e.g. the top quark condensate discussed in section 5). Our motivation for this is the desire to account for the top mass without running into the phenomenological difficulties that ETC faces.

Consider the gauge group

$$SU(2)_{TC} \otimes SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$$

where we have added to the standard model gauge group a new confining QCD-like gauge group $SU(2)_{TC}$ which is asymptotically-free and confines at $\Lambda_{TC} \sim 50 - 100$ GeV. The technifermions in this model may be written,

$$T_L = \begin{pmatrix} P_L \\ M_L \end{pmatrix} \sim (2, 1, 2, 0)$$

$$P_R \sim (2, 1, 1, 1/2)$$

$$M_R \sim (2, 1, 1, -1/2)$$

$$t^i_L = \begin{pmatrix} p_L^i \\ m_L^i \end{pmatrix} \sim (2, 1, 2, 0)$$

$$p_R^i \sim (2, 1, 1, 1/2)$$

$$m_R^i \sim (2, 1, 1, -1/2)$$

39
where we have denoted the dominant technifermions by upper case letters, and sub-dominant technifermions by lower case letters labelled by $i = 1, \ldots, n_F - 1$ where $n_F$ is the number of lepton families. Henceforth we shall assume for simplicity that $n_F = 3$.

To begin with let us consider the dominant technidoublet $T = (P, M)$. We assume operators of the form $G_T(T_L T_R)(\bar{T}_R T_L)$. These operators are associated with a dimensional scale $\Lambda \sim 1$ TeV. For example in section 5.5 we shall discuss a model in which the four-fermion operators will arise from heavy (1 TeV) gauge boson exchange. There are similar contact terms of the form $G_r(\bar{\tau}_L \tau_R)(\bar{\tau}_R \tau_L)$ which induce a tau lepton condensate. Since we assume a tau condensate then $G_T \approx G_r$ must therefore be strong, leading to a condensate $< \mathcal{P} P + \mathcal{M} M >\neq 0$. If the operator respects isospin then the pattern of symmetry breaking expected is just

$$SU(2)_L \otimes SU(2)_R \rightarrow SU(2)_{L+R}$$ (109) yielding a triplet of technipions $\Pi_{TC}^{\pm 0} \sim \bar{T} \sigma^{\pm \gamma_5} T$, where $\sigma^\alpha$ are the Pauli matrices. In such a theory the technipion decay constant may be much larger than $\Lambda_{TC}$ since chiral symmetry breaking is driven mainly by the above contact operator. We assume $F_{TC} \sim 245$ GeV. The technipions associated with the dominant technidoublet get eaten, and the remaining technihadrons have masses set by the enhanced dynamical masses of the technifermions, and are in the LHC range.

Now let us extend our discussion to include one of the sub-dominant technidoublets in this model. There are now two technidoublets $T = (P, M), t = (p, m)$, which have identical quantum numbers. The set of operators in this case are of the form

$$G_T(T_L T_R)(\bar{T}_R T_L), \quad G_t(\bar{T}_L t_R)(\bar{t}_R t_L), \quad G_{tt}(\bar{T}_L T_R)(\bar{t}_R t_L)$$ (110) where $G_t, G_{tt} \ll G_T$. In this case the theory naturally splits into two parts, a high scale TC sector associated with the technifermions $T$ which form condensates driven by the operators associated with the scale $\Lambda \sim 1$ TeV, as discussed above, and a low scale TC sector associated with technifermions $t$ which form condensates driven by TC interactions at the scale $\Lambda_{TC} \sim 50 - 100$ GeV. The low scale TC sector again has an approximate global symmetry as before, but now there is a vacuum alignment problem which depends on the relative strength of the contact term and the TC gauge forces. The TC gauge forces tend to favour the chirally invariant condensate $< \bar{T}_L t_L + \bar{t}_R T_R >\neq 0$, while the contact terms prefer the chiral symmetry breaking condensates, $< \bar{T}_L t_R + \bar{t}_R T_L >\neq 0$. We shall assume the latter condensates form yielding a triplet of technipions $\pi_{TC}^{\pm 0} \sim \bar{t} \sigma^{\pm \gamma_5} t$, associated with a much smaller pion
decay constant \( f_{TC} \sim 10 - 50 \text{GeV} \). These pions do not get eaten, and remain in the physical spectrum.

The physical technipions \( \pi_{TC}^{\pm 0} \) will receive a mass from the mixed operator with coefficient \( G_{iT} \). The “explicit” masses of the low scale technifermions resulting from this operator are obviously just \( m_{F,m} = G_{iT} < (\bar{T}_L T_R) > \). These masses break the chiral symmetry of the second technidoublet resulting in a physical technipion mass analogous to the way in which the physical pion mass results from explicit quark masses. The technipion mass \( m_{\pi_{TC}} \) may be estimated by scaling up the usual result for the ordinary pion mass \( m_{\pi} \),

\[
m_{\pi_{TC}} = m_{\pi} \sqrt{\left( \frac{m_u + m_d}{m_u + m_d} \right) \frac{f_{TC}}{f_{\pi}}}.
\]  

(111)

Using these equations we find rather heavy technipion masses of order tens of GeV, or perhaps hundreds of GeV, depending on \( f_{TC} \).

The charged technipions decay via ordinary \( W \) exchange into the heaviest fermion channels available such as \( c \bar{b} \). The physical neutral technipion \( \pi_{TC}^{0} \) will decay into two photons via a chiral symmetry suppressed anomalous \( \pi_{TC}^{0} \gamma \gamma \) coupling. The partial width is given by (King 1993),

\[
\Gamma(\pi_{TC}^{0} \rightarrow \gamma \gamma) = A_{\pi_{TC}^{0} \gamma \gamma}^2 \left( \frac{g^2}{16 \pi^3 f^2_{TC}} \right) m_{\pi_{TC}^{0}}^3
\]

(112)

where in the present model \( A_{\pi_{TC}^{0} \gamma \gamma} = -\frac{1}{8} \left( \frac{m_{\pi_{TC}^{0}}}{f_{TC}} \right)^2 \). Despite its small partial width, this is expected to be an important decay mode of the \( \pi_{TC}^{0} \) leading to a striking signature. For example, in 1993 the L3 collaboration at LEP reported four events, one \( e^+ e^- \gamma \gamma \) and three \( \mu^+ \mu^- \gamma \gamma \), each with two energetic photons with an invariant mass \( M_{\gamma \gamma} \approx 60 \text{GeV} \). This would be a typical signature for a neutral technipion of mass 60 GeV. Although in conventional models the production rate would be very small (Lubicz 1993), in the present model the technipion may be produced in association with a virtual techniomega which serves to enhance its production rate (King 1993).

Apart from the low-scale technipions, the technidoublet \( t \) will give rise to technivector mesons \( V \) analogous to the QCD vector resonances. However here the masses of such technivectors will be an order of magnitude smaller than table 2. For example we may expect a \( J^{PC} = 1^{--} \) technirho triplet \( \rho_{TC}^{\pm 0} \) and techniomega singlet \( \omega_{TC} \) with masses in the LEPII range 100-200 GeV. Later on we shall consider extending this mass range to 200-300 GeV, which is relevant for the Tevatron. The vector masses
may be estimated by scaling up the ordinary $\rho$ and $\omega$ mass $m_{\rho TC,\omega TC} \approx m_{\rho,\omega} \frac{f_{TC}}{f_{\pi}}$. The $\rho^0_{TC}$ may be detected at LEPI via its couplings to the photon and Z,

$$m_{\rho TC}^2 g_{\rho TC} \rho_{TC}^{0\mu} \left[ eA_\mu + \frac{e}{\tan 2\theta W} Z_\mu \right]$$ (113)

leading to resonances in R. The techniomega $\omega^0_{TC}$ being associated with the isosinglet current $\bar{t} \gamma^\mu t$ does not couple to the photon or Z, but may have a direct coupling to leptons.

There are other models in the literature which also have a low TC scale, for example Eichten and Lane (1989). These authors considered the Tevatron signatures of a light technirho. The technirhos are produced via their electroweak couplings to the $W$ and $Z, \gamma$. The dominant decay of the technirho $\rho_{TC}$ is into $\pi_{TC} \pi_{TC}$, but this channel is not kinematically accessible if the technipions are heavier than half the technirho mass. In this case the technirho will decay dominantly into $\pi_{TC} W_L$, or $\pi_{TC} Z_L$, at a rate suppressed by $(f_{TC}/F_{TC})^2$. For example we may have,

$$\bar{q} q \rightarrow Z^* \rightarrow \rho^0_{TC} \rightarrow \pi^+_{TC} W^-$$ (114)

with the technipion decaying into $tb$ or $cb$, leading to a Tevatron signal consisting of a $W$ recoiling against a di-jet, with the whole system having an invariant mass equal to that of the technirho.

Finally note that since in low-scale TC the dynamical mass of the sub-dominant technifermions may not be much heavier than the $Z$ mass the $S, T, U$ analysis must be extended to include $V, W, X$ as discussed in the previous section, leading to a relaxation of the radiative correction constraints as in equation 101. Since the top mass arises from a top quark condensate rather than ETC, the $Zb\bar{b}$ vertex radiative corrections should not be a problem either for our low-scale TC scenario.

5 Top Quark Condensates

5.1 Four-fermion theory

We have seen that obtaining a large top quark mass in TC theories is non-trivial. One is led to consider strong ETC which as discussed in the section 3.3 involves the consideration of four-technifermion operators which contribute to the gap equation. Such theories are characterised by fine-tuning and light scalar bound states. It is a small step from these types of theory to top quark condensate theories, first postulated

The starting point of top quark condensate models is to postulate a four-fermion operator,

\[ G(Q_L t_R)(t_R Q_L) \]  

where \( Q_L = (t_L, b_L) \) and we may write \( G = (8\pi^2/3)\lambda/\Lambda^2 \) where \( \Lambda \) is called the ultraviolet cut-off, and is a dimensionful parameter of the theory. From our experience with strong ETC we may regard the contact operator as arising from some heavy gauge boson of mass \( \lambda \) being exchanged between top quarks. Such a heavy boson must couple strongly to top quarks, as the coefficient \( 8\pi^2/3 \) suggests. In section 5.5 we shall discuss a model in which such a heavy boson can emerge but for now we shall live with the non-renormalisable operator above.

The gap equation arising from the operator in equation 115 is represented by figure 18. This yields a self-consistent equation for \( m_t \),

\[ m_t = 6Gm_t \int_0^\Lambda \frac{d^4k}{(k^2 - m_t^2)} = \Lambda^2(1 - \frac{1}{\lambda}) \]  

Assuming that \( m_t \neq 0 \) the top mass can be cancelled from both sides of the equation, and the integration can be performed to yield,

\[ m_t^2 \ln \frac{\Lambda^2}{m_t^2} = \Lambda^2 - \frac{8\pi^2}{3G} \]  

For \( \lambda \gg 1 \) it is clear that \( m_t = 0(\Lambda) \). For \( \lambda < 1 \) the solution \( m_t = 0 \) is preferred, and the gap equation is trivial. If we fine-tune \( \lambda = 1 + \epsilon \) then we can obtain \( m_t \ll \Lambda \).

We expect fine-tuning to be associated with the appearance of light scalar bound states, similar to our discussion in the section 3.3. The light bound states may be revealed by looking for poles in the \( \bar{t}_L t_R \rightarrow \bar{t}_L t_R \) scattering amplitude. A standard bubble summation can be performed, which like the gap equation is valid in the large \( N_c \) limit, where \( N_c = 3 \) is the number of colours. The \( 1/N_c \) corrections have been studied by Hands et al (1991). The result of the bubble summation is to exhibit a pole in the scalar \( \bar{t} t \) channel (once the gap equation has been implemented to cancel the leading divergence) at \( p = 2m_t \). This is the Higgs boson pole. There are also poles
in the pseudoscalar channels $\bar{t}\gamma_5 t$, $\bar{t}\gamma_5 b$ and $\bar{b}\gamma_5 t$ at $p = 0$ corresponding to the three massless Goldstone bosons originating from the Higgs doublet. Note that the mass of the Higgs boson and the top quark are related, and are kept light by a common fine tuning. The simple relation above indicates that the Higgs boson is a bound state of $\bar{t}t$ with zero binding energy. However, this result is only valid in the pure fermion approximation, and it neglects any gauge corrections such as QCD gluon exchange. It also neglects Higgs exchange corrections. The point is that the Higgs boson must be regarded as a light propagating degree of freedom, which can help to bind itself by a bootstrap mechanism as originally envisaged by Nambu (1989).

### 5.2 Renormalisation Group Approach

There are two ways in which the gauge and Higgs corrections can be calculated. The first way is to simply add a QCD gluon exchange term to the right-hand side of the gap equation (King and Mannan 1990). QCD gluon exchange corrections can also be added to the bubble sums in a similar way (Chesterman et al 1991). These are difficult calculations, and do not include the important Higgs exchange corrections. However there is a much easier method in which all corrections (at leading order) may be conveniently considered, namely the equivalent Lagrangian approach of Bardeen et al (1990).

The starting point of the equivalent Lagrangian approach is to observe that the four-fermion operator in equation 115 may be expressed in an alternative way in terms of an auxiliary complex scalar doublet $H$ as follows,

$$ (\bar{Q}_L t_R)(\bar{t}_R Q_L) = \bar{Q}_L t_R H + h.c. - \frac{1}{G} H \bar{H} H $$

where $H = G(\bar{t}_R Q_L)$ will become the composite Higgs doublet. The equivalent Lagrangian in equation 118 is valid at the energy scale $\mu = \Lambda$. The effective low-energy Lagrangian at an energy scale $\mu \ll \Lambda$ is obtained by integrating out the high-energy fermion degrees of freedom (Bardeen et al 1990). It is straightforward to show that the low energy effective theory at $\mu \ll \Lambda$ is just the minimal standard model, a few terms of which are shown below,

$$ L^{eff} = g_t \bar{Q}_L t_R H + h.c + |D_\mu H|^2 - M^2 H \bar{H} - \frac{\lambda}{2} (H \bar{H} H)^2 + 0\left(\frac{1}{\Lambda^2}\right) $$

but with the compositeness boundary conditions,

$$ g_t(\mu), \lambda(\mu) \rightarrow \infty \mid_{\mu \rightarrow \Lambda} $$
These compositeness boundary conditions arise from the simple requirement that at $\mu = \Lambda$ is satisfied.

Equations 119, 120 correspond to a quasi-fixed point of the standard model (Pendleton and Ross 1981, Hill 1981, 1990), and lead to predictions for the Higgs boson mass and top quark mass as a function of the ultraviolet cut-off $\Lambda$. One simply integrates the standard model renormalisation group (RG) equations using the boundary conditions above. This is the great power of this method, because the resulting predictions have all the gauge corrections and Higgs exchange corrections taken account of by the standard model RG equations. The words quasi-fixed point mean that the RG trajectory of the Yukawa couplings $g_t(\mu)$ and $\lambda(\mu)$ tend to converge on a certain (fixed) low energy value almost independently of their boundary value at $\mu = \Lambda$ providing that this boundary value exceeds a certain minimum value. It is this feature which is very important in giving reliable predictions, since the precise boundary conditions in equation 120 cannot be taken literally since they are infinitely far outside the perturbative region. The resulting predictions are shown in table 3 for three values of cut-off $\Lambda$ and for both the top quark condensate model and a degenerate four family model.

It must be said that the results are disappointing. The top quark mass is always much heavier than the tentative current experimental measurement of about 170 GeV. One way to improve the situation is to make the top quark condensate model supersymmetric (Clark et al 1990, Carena et al 1992). This approach exploits the fact that in the minimal supersymmetric standard model, the triviality limit of the top quark mass is less than in the standard model. There appears to be nothing wrong with this approach, apart from the observation that the supersymmetric gap equation leads to a critical coupling constant which is very much larger than in the non-supersymmetric model. However it is difficult to get excited about such models because the scale of new physics is of order the Planck scale, or grand unified scale. As discussed in section 5.4 it is quite likely that such models with high scales are isomorphic to the standard model (or supersymmetric standard model). What is much more interesting is if the scale of new physics is reduced to the TeV region. This can be achieved in several ways. One way is immediately apparent from table 3: introduce a fourth family of fermions, so that the top quark mass becomes unconstrained and the scale of new physics may be reduced to the TeV region. Note that the four-family case with $\Lambda = 2 \text{ TeV}$ involves no fine-tuning. With such a low scale of new physics, one would expect experimental signatures from longitudinal $W$
Table 3: Table 2. Predictions of top mass ($t$) and Higgs ($h$) mass as a function of cut-off $\Lambda$. Similar predictions are also shown for a degenerate fourth family ($m_t = m_h$).

<table>
<thead>
<tr>
<th>$\Lambda$</th>
<th>$m_t$ (GeV)</th>
<th>$m_h$ (GeV)</th>
<th>$m_t = m_h$ (GeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^3 GeV$</td>
<td>220</td>
<td>250</td>
<td>300</td>
</tr>
<tr>
<td>$10^7 GeV$</td>
<td>450</td>
<td>600</td>
<td>550</td>
</tr>
<tr>
<td>$2 TeV$</td>
<td></td>
<td></td>
<td>$\approx 1 TeV$</td>
</tr>
</tbody>
</table>

and Z scattering experiments at LHC. This was discussed by Chestermann and King (1992).

5.3 A fourth family after all?

One may wonder whether a fourth family is still a viable possibility after the LEP measurement of the $Z$ width which indicates three and only three light neutrinos? If there is a fourth family then the fourth neutrino must be heavier than half the $Z$ mass in order that the $Z$ does not decay in this channel. But having a fourth neutrino heavier than 45 GeV, seems a little odd given that the first three neutrino species are so light. However it turns out that it is possible to write down models which quite naturally lead to the desired neutrino hierarchy. For example a singlet Majoron model has been proposed by Hill et al (1990, 1991) in which the four right-handed neutrinos gain Majorana masses of order 1 TeV. This scheme then predicts three species of Majorana neutrino near their laboratory mass limits plus a heavy fourth neutrino. The model also predicts massless Majoron which is necessary in order that the heavier neutrinos can decay into the lighter ones (via Majoron emission) at a fast enough rate to avoid over-closing the Universe. However the Majorons also couple to the $Z$ via neutrino loops, and since the $Z$ couples to all fermions, the Majoron also may couple to any fermion. As shown by Mohapatra and Zhang (1993) the see-saw four-family model is in trouble because the Majorons associated with it lead to conflicts with astrophysics. Essentially stars can emit Majorons as a cooling mechanism, which constrains the effective Majoron coupling to electrons, and this constraint turns out to be violated in the case where the symmetry breaking scale associated with the Majoron is of order 1 TeV, as assumed in the four-family see-saw model.

However there are other natural models of the fourth family which are not subject to the above constraints. The minimal four family model (King 1992c, 1992d) introduces only one right-handed neutrino (with zero Majorana mass), rather then
three. This single right-handed neutrino has to be shared amongst three left-handed neutrinos, and results in a mass spectrum consisting of three massless neutrinos plus a heavy Dirac neutrino. Because the single right-handed neutrino is assumed to be precisely massless total lepton number $L$ is exactly conserved in this model. However the separate lepton numbers $L_e, L_\mu, L_\tau$ are not conserved. It is $L$ conservation which forbids all Majorana neutrino masses and enforces the masslessness of the first three neutrinos. A non-minimal four-family model with a single right-handed neutrino was in fact proposed by Babu et al. (1988, 1989) before the LEP result of three light neutrinos. In the non-minimal four family model the single right-handed neutrino is given a Majorana mass, thereby breaking $L$ number and allowing the first three neutrinos to acquire small masses from a two $W$ exchange mechanism. The idea of the earlier non-minimal four-family model was not to provide an explanation for why the fourth neutrino was heavier than $M_Z/2$ but rather to discuss the neutrino masses and mixings in such a model. In the minimal four-family model of King (1992c), which was invented independently, the fact that lepton number is conserved leads to a particularly simple parametrisation of lepton mixing as discussed by King (1992d). In summary, the minimal four-family model provides an elegant mechanism for having three massless neutrinos plus one heavy Dirac neutrino, and therefore the idea of a fourth family should certainly not be excluded on the basis of the LEP measurement of the $Z$ width. The above models are of course independent of the idea of DEWSB.

### 5.4 Irrelevant Operators

We wanted to account for a heavy top quark, but the results in table 2 look like overkill. The top quark mass of 220 GeV is just too heavy. The superheavy scale $\Lambda = 10^{17} GeV$ implies fine-tuning, and no prospect of direct experimental verification of the theory. However the predictions in table 2 have been criticised by Suzuki (1990b) and later by King and Mannan (1991) due to the neglect of higher dimensional or “irrelevant” operators, which if they have sufficiently large dimensionless coefficients, may alter the predictions significantly. A generalised NJL model has been proposed by Hasenfratz et al. (1991),

$$L_{NJL}^{ren} = G(\bar{t}t + \frac{X}{\Lambda^2}\partial_\mu F^\mu t)(1 + \frac{f_1}{\Lambda^2}\partial^2 + \ldots)(\bar{t}t + \frac{X}{\Lambda^2}\partial_\mu F^\mu t) + \ldots$$  \hspace{1cm} (121)

where the term with coefficient $f_1$ represents the first term in an infinite tower of terms, and the ellipsis at the end of the equation represents a similar pseudoscalar term plus kinetic terms. In the large $N_c$ limit for $\mu \ll \Lambda$ it was argued that the
generalised NJL model in equation 121 is equivalent to the minimal standard model in the sense that the two models occupy the same parameter space. In other words $m_t$ and $m_H$ are arbitrary.

The above analysis seems to pour cold water over the whole top quark condensate approach. If such models are merely re-statements of the standard model (or supersymmetric standard model) then why bother? However all is not lost since if one can reduce the scale of new physics down to the TeV region then it will be possible to distinguish the new physics which is responsible for the top quark condensate from standard physics. In other words if the dimensional scale associated with the non-renormalisable operators is a TeV or so, then the whole operator approach will begin to break down, just as Fermi theory of weak interactions breaks down at energies of order the $W$ mass. In this case the Hasenfratz et al operator approach is inadequate, and one must confront the new physics responsible for the non-renormalisable operators directly. Of course new physics at a TeV is dangerous because we know that it can potentially lead to FCNC’s. However in the next section we shall discuss a model based on heavy gauge boson exchange (of mass $\sim 1\text{ TeV}$) which can lead to a top quark condensate without leading to excessive FCNC’s.

### 5.5 TeV scale models of the top quark condensate

There are many examples of top quark condensate models in the literature, for example Hill (1991), King (1991), Clague and Ross (1991), Lindner and Lust (1991), Lindner and Ross (1992). The model we shall discuss below (King 1992a, 1992b) has the virtue that it has a built-in GIM mechanism, which allows the scale of new physics $\Lambda$ to be reduced down to a TeV without incurring excessive FCNC’s. If electroweak symmetry were broken solely by a top quark condensate then we would expect the top quark mass associated with operators at the TeV scale to be too heavy, as in table 3. Since we shall assume that $\Lambda \sim \text{ TeV}$ it is necessary that electroweak symmetry is broken by some other sector in addition to the top quark condensate. As we shall see later the lepton sector of this model is associated with a low-scale TC theory which will also break electroweak symmetry along the lines of the discussion in section 4.4. Therefore we are free to assume that the scale of new physics is reduced down to a TeV or so in the following model. In addition it is also possible to extend the discussion to a fourth family, although for simplicity we shall not do so here.

The following discussion of a simplified version of the model is taken from Elliott
and King (1992). The basic idea of the model is to introduce a gauged horizontal symmetry broken by non–quark condensates driven by a technicolour–like interaction. These condensates in turn allow the formation of quark condensates resulting in both dynamically generated quark masses and electroweak symmetry breaking. For simplicity we consider only the u,c,t sector and regard the up and charm quarks as having only the usual QCD dynamical mass. This is simply for pedagogical purposes, and those readers interested in the complete model should consult the literature. In this idealised world, the non–abelian global symmetries of the standard model would be $SU(2)_L \otimes SU(2)_R$, corresponding to massless up and charm quarks, but with a massive top quark.

The technicolour–like interaction acts on new left–handed Weyl fermions. The relevant symmetry groups, where ‘ga’ means gauged and ‘gl’ means global, are (ignoring $SU(2)_L \otimes U(1)_Y$ in this toy model)

$$SU(3)_C \otimes SU(3)^{ga}_{L,R} \otimes SU(3)^{gl}_{L,R} \otimes SU(2)^{gl}_{L,R}$$

and the quarks $U_L, U_R$ and new fermions $a, b, c, d$ transform as

$$
\begin{align*}
U_L & \sim (3, 1, 3, 1, 1, 1) \\
U_R & \sim (\bar{3}, 1, 1, 1, 1) \\
a & \sim (1, 3, 1, 1, 1) \\
b & \sim (1, \bar{3}, 1, 1, 1) \\
c & \sim (1, 3, 1, 1, 1) \\
d & \sim (1, \bar{3}, 1, 1, 1)
\end{align*}
$$

where $SU(3)_C$ is the QCD gauge group, $SU(3)^{ga}_{L,R}$ are gauged horizontal flavour symmetries acting on the left–handed quarks $U^T_L \equiv (u_L, c_L, t_L)$ and left–handed charge conjugate quarks $U^T_R \equiv (u_R^c, c_R^c, t_R^c)$, respectively. The superscript $c$ denotes charge conjugation. We take the mass matrix connecting the $c$ and $d$ fermions to be identically zero, which will lead to massless $u, c$ quarks at low energies.

We assume that $SU(3)^{ga}_f$ becomes strongly coupling at some scale $\Lambda_f$ and drives the formation of the following condensates (for a detailed discussion of this assumption see Evans et al 1993a and Luty 1992)

$$< a_i d_i > \neq 0, < b_i c_i > \neq 0, i = 1, 2 \text{ and } < a_3 b_3 > \neq 0$$

These condensates break the groups $SU(3)^{ga}_L \otimes SU(3)^{ga}_R$ and $SU(2)^{gl}_L \otimes SU(2)^{gl}_R$ down to the diagonal subgroup $SU(2)_L \otimes SU(2)_R$ – the global symmetry of the standard model. The structure of the model in equations (122), (123) thus ensures that the
global symmetry of the standard model is reproduced at low energies. This in turn ensures that we have a GIM mechanism which is essential if \( \Lambda_f \) is to be lowered below about \( 10^3 \) TeV, so that flavour-changing neutral-currents are suppressed.

A simple calculation gives the following 16×16 mass squared matrix for the 8 gauge bosons of \( SU(3)_L^a \) and the 8 gauge bosons of \( SU(3)_R^a \):

\[
\begin{pmatrix}
A^1_{\mu L} & \cdots & \frac{1}{4} F_{1L}^2 g_L^2 & \frac{1}{4} F_{1R}^2 g_R^2 & \cdots & \frac{1}{4} F_{8L}^2 g_L^2 & \frac{1}{4} F_{8R}^2 g_R^2 \\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\
A^8_{\mu L} & \cdots & -\frac{1}{6} F_{1L}^2 g_L^2 g_R & \frac{1}{4} F_{1R}^2 g_R & \cdots & \frac{1}{6} F_{8L}^2 g_L^2 g_R & \frac{1}{4} F_{8R}^2 g_R \\
A^1_{\mu R} & \cdots & \frac{1}{4} F_{1L}^2 g_L^2 & -\frac{1}{4} F_{1R}^2 g_R^2 & \cdots & \frac{1}{4} F_{8L}^2 g_L^2 & -\frac{1}{4} F_{8R}^2 g_R^2 \\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\
A^8_{\mu R} & \cdots & -\frac{1}{4} F_{1L}^2 g_L^2 g_R & \frac{1}{4} F_{1R}^2 g_R & \cdots & \frac{1}{4} F_{8L}^2 g_L^2 g_R & \frac{1}{4} F_{8R}^2 g_R.
\end{pmatrix}
\]  

(125)

where \( F_{1L} \) is the characteristic pion decay constant of the \( SU(3)_L^a \) group, and \( g_L \) and \( g_R \) are the coupling constants of \( SU(3)_L^a \) and \( SU(3)_R^a \), respectively. In order to calculate the mass eigenstates we need to diagonalise the mass matrix. Since there is mixing between \( A^8_{\mu L} \) and \( A^8_{\mu R} \) only, we may restrict ourselves to a 2×2 mass matrix:

\[
\begin{pmatrix}
\frac{1}{4} F_{1L}^2 g_L^2 & -\frac{1}{6} F_{1L}^2 g_L^2 g_R \\
-\frac{1}{6} F_{1L}^2 g_L^2 g_R & \frac{1}{4} F_{1R}^2 g_R^2
\end{pmatrix}
\]  

(126)

Defining mass eigenstates \( A_\mu \) and \( B_\mu \) and a mixing angle \( \theta \), where

\[
A^8_{\mu L} = A_\mu \cos \theta + B_\mu \sin \theta
\]  

(127)

\[
A^8_{\mu R} = -A_\mu \sin \theta + B_\mu \cos \theta
\]  

(128)

a trivial calculation reveals that

\[
M^2_A = \frac{1}{8} F_{1L}^2 \left[ g_L^2 + g_R^2 + \sqrt{g_L^4 + 2 g_L^2 g_R} \right]
\]  

(129)

\[
M^2_B = \frac{1}{8} F_{1L}^2 \left[ g_L^2 + g_R^2 - \sqrt{g_L^4 + 2 g_L^2 g_R} \right]
\]  

(130)

\[
\sin 2\theta = \frac{4 g_L g_R}{\sqrt{g_L^4 + 2 g_L^2 g_R}}
\]  

(131)

The massive gauge bosons \( A_\mu \) and \( B_\mu \) couple the L– and the R–sectors of the top quark and we obtain the interaction

\[
- \frac{1}{\sqrt{3}} A_\mu [g_L \cos \theta \bar{t}_L \gamma^\mu t_L - g_R \sin \theta \bar{t}_R \gamma^\mu t_R]
\]

\[
- \frac{1}{\sqrt{3}} B_\mu [g_L \sin \theta \bar{t}_L \gamma^\mu t_L + g_R \cos \theta \bar{t}_R \gamma^\mu t_R]
\]  

(132)
where the $-\frac{1}{\sqrt{3}}$ comes from $(T_8)^3$, and $T_a = \frac{1}{2} \lambda_a$ are the generators of $SU(3)$. Similar interactions involving the up and charm quarks will have the $-\frac{1}{\sqrt{3}}$ replaced by $\frac{1}{\sqrt{3}}$ which comes from $(T_8)^1$ or $(T_8)^2$. Because the $c - d$ mass matrix is identically zero, terms involving the other diagonal generator of $SU(3)$, $T_3$, do not give rise to additional up and charm quark interactions.

In the low energy limit, where we may approximate the massive gauge propagator by $\frac{g}{M^2}$, we obtain, after a Fierz rearrangement

$$i \frac{g_{L} g_{R} \sin 2\theta}{3} \left( \frac{1}{M_{B}} - \frac{1}{M_{A}} \right) (\bar{t}_L t_R)(\bar{t}_R t_L)$$

for the amplitude for the process $t_L + \bar{t}_L \rightarrow t_L + \bar{t}_R$ by $A$ and $B$ exchange. We clearly see that the exchange of a $B$ represents an attractive channel, and an $A$ a repulsive channel. Since $M_{B} < M_{A}$, the condition for an overall attractive interaction is $\sin 2\theta > 0$. This condition is always satisfied.

Thus we have induced a contact term in equation of the same form as equation 115. When the interactions become sufficiently strong such a term results in the formation of a top condensate and electroweak symmetry breaking. There will be similar contact terms involving the up and charm quarks, with a coefficient $\frac{1}{4}$ times that in equation (133). Such contact terms will be too weak for up and charm quark condensates to form. Since $g_L$ and $g_R$ will be large in order to force the top to condense, it is natural to worry about the possibility that $SU(3)_L^2$ and $SU(3)_R^2$ cannot themselves drive other, potentially disastrous, non-zero condensates. The condensates $< U_{L}^{i} A^{i} >$ and $< U_{R}^{i} b^{i} >$, $(i = 1, 2, 3)$ would, of course, break the technicolour group $SU(3)^2$ and the ordinary $SU(3)$ colour group. We therefore assume that this does not occur.

The above model, and a fourth family variant of the model, were subsequently studied, using gap equation techniques by Elliott and King (1992). In the full version of the model with both top and bottom quark condensates (or their fourth family equivalents) there is an axion due to a global $U(1)$ symmetry. As shown by Elliott and King (1993) this axion can become very heavy, and may even provide a resolution to the strong CP problem.

The inspiration for the above model comes from Georgi’s ETC models of quarks which also have a built-in GIM mechanism (Chivukula et al 1987, Georgi 1992). In effect we have removed the TC from the quark sector of these models, and shown that what is left has the correct properties to yield strong operators capable of inducing top and bottom quark condensates (or fourth family quark condensates). In this type of model leptons are a big problem because, unlike the quarks, they do not carry
colour. The lepton problem occurs because leptons do not carry colour, and so it proves to be very difficult to construct an anomaly-free model as in equation 123.

A natural solution to these difficulties for the lepton sector was suggested by King (1992b). The basic idea is to endow the leptons with three colours (red, white and blue) and then write down a model of leptons which is identical to the quark sector in equation 123. We thus end up with a model which is symmetric in the quarks and leptons as in the model by Foot and Lew (1990). Unlike the quark sector, lepton colour (a gauge group) is broken down from $SU(3)$ to $SU(2)$, with the decomposition $3 \rightarrow 2 \oplus 1$, where the $SU(2)$ singlet (white) is identified with the physical leptons, and the $SU(2)$ doublet (red, blue) represents some exotic states. The $SU(2)$ gauge group may be interpreted as a low-scale TC group $SU(2)_{TC}$ (King 1993) as discussed in section 4.4, leading to the possibility of experimental verification at LEP or the Tevatron. Although this scenario is attractive, the model cannot be complete since the quark and lepton sectors of the model each separately break electroweak symmetry, resulting in three massless or light GB's in the physical spectrum. The solution to this problem is to introduce four-fermion operators which connect the quark and lepton sectors together. An alternative approach which does not treat the quarks and leptons separately is discussed by Randall (1993).

Finally we note that in the above model the bottom and tau receive mass from bottom and tau condensates, which requires fine-tuning. However if there were a fourth family of fermions all fine-tuning could be completely eliminated.

6 Conclusion

In this review we have traced the development of the subject of dynamical symmetry breaking from the invention of technicolour in 1979 through the idea of top quark condensates a decade later and up to the present day. We have discussed the simple idea of technicolour, which breaks electroweak symmetry by analogy with QCD, but which does not provide fermion masses. This led us to consider extended technicolour and its phenomenological problems. However since the time that the review by Farhi and Susskind (1981) was written there has been real progress in tackling the problems of extended technicolour, and it has been one of the purposes of the present review to discuss this progress. Much of the progress has been concerned with the idea of condensate enhancement, which enables fermion and pseudo-Goldstone boson masses to be enhanced, thereby allowing the extended technicolour breaking scale to be raised.
and flavour-changing neutral currents to be suppressed. Condensate enhancement may be achieved in either walking technicolour theories, or strong extended technicolour theories, and both these ideas have been reviewed in as non-technical a way as possible. The basic message of this approach is that it is possible to enhance the condensate sufficiently to avoid problems with flavour-changing neutral currents, but the price that must be paid appears to be fine-tuning of one sort or another. Walking technicolour is not a generic property of technicolour models, and so some tuning in the space of models is required. In strong extended technicolour theories, more direct fine-tuning is required and such fine-tuning is accompanied by light scalar bound states.

We have also discussed the experimental prospects for technicolour theories. We have seen that the sure-fire way of testing technicolour theories is to perform longitudinal gauge boson scattering experiments at TeV energies. This is analogous to doing pion-pion scattering experiments and as in that case, we would expect a resonance structure of some sort which will hold the key to the strong dynamics responsible for electroweak symmetry breaking. A more indirect indication of technicolour would be the discovery of pseudo-Goldstone bosons, which we have also discussed. A third way to look for evidence of technicolour is in precision electroweak measurements which are sensitive to new physics. We have seen that current data disfavour models with too many new heavy fermions, such as the single techni-family, but it is impossible to rule out the technifamily this way because of calculational uncertainties, and unconventional dynamics which can lead to effects which partly cancel the large corrections. For example, if the technicolour scale is low the constraints from radiative corrections are greatly relaxed. Low-scale technicolour is a fairly recent idea which we have chosen to highlight because it can lead to spectacular technicolour signatures at LEP and the Tevatron.

We have also discussed the recent idea of top quark condensates. This idea is initially quite an attractive one since it promises to account for the large top quark mass without relying on extended technicolour. However it turns out that the top quark mass always comes out too large in the simplest models, even for a scale of new physics equal to the Planck scale, and in addition there appears to be a severe fine-tuning problem associated with keeping the top quark mass much lower than the natural scale of the operators which must be introduced. It was in fact claimed that such models are in fact isomorphic to the standard model, a claim which seems to pull the rug from under this approach. One solution to all these problems is to bring
down the scale of the new physics responsible for the top quark condensate down to a TeV or so. Of course this means that the top quark condensate cannot break electroweak symmetry by itself otherwise the top quark would come out to be much too heavy; electroweak symmetry must be broken by some other physics in addition to top quark condensates. One example is the idea of fourth family condensates.

Given all these ideas it is important to try and put them into practice and construct explicit models of dynamical electroweak symmetry breaking. There is no universally agreed upon model, but there are very good candidates which look quite promising. We have considered two examples of models in this review – a simple (unrealistic) ETC model, which we discussed in order to get a feeling for the issues and problems that models face; and a more complicated model of the top quark condensate based on TeV-scale gauge boson exchange, with a built-in GIM mechanism in order to avoid excessive FCNC’s. There are so many models in the literature that it would have been impossible to review them all, and since there is no leading candidate dynamical model we have focussed on the above two models simply because the author is familiar with them.

At the time of writing in 1994 the standard model looks in remarkably good shape. The theory agrees with experiment at the 1% level, and the evidence for the discovery of the top quark has just been announced. However the Higgs boson of the standard model has not been discovered, and the mechanism of electroweak symmetry breaking remains untested experimentally. This lack of experimental information about the mechanism of electroweak symmetry breaking is accompanied by a general feeling of theoretical dissatisfaction with the single Higgs doublet of the minimal standard model. The hope is that there is some new physics beyond the standard model which plays an important part in electroweak symmetry breaking. The question is what is the nature of this new physics?

Although in this article we have only discussed the possibility that the effect of the Higgs doublet is replaced by some kind of fermion-antifermion condensate, it is worth emphasising that there are other alternatives. We have already mentioned that the naturalness problems accompanying elementary Higgs scalars may be eliminated by postulating an $N = 1$ supersymmetry which is broken at the TeV scale. It is also perhaps worth mentioning the ideas of composite quarks and leptons, and composite gauge bosons. Alternatively the new physics may involve some combination of the above ideas, or possibly some new ingredient which we have not yet imagined.

The problem of electroweak symmetry breaking, or the problem of the origin of
mass, is the number one problem of late twentieth century physics. From our present viewpoint in 1994 the answer to this question remains as open today as it always was, but it is a question whose answer lies at the TeV energy scale. It will therefore take a supercollider capable of probing the TeV energy scale to answer this question.

The great American dream, the superconducting supercollider (SSC), has just been cancelled due to budgetary considerations. The European hope, the large hadron collider (LHC), which would occupy the LEP tunnel at CERN, is facing a critical period ahead. These are uncertain times in particle physics, yet the fundamental questions such as the origin of mass have never been more urgent or more pressing, and the answers to these questions have never been more tantalisingly within our grasp, than now.

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Figure Captions

Figure 1: Energy diagram of a superconductor.

Figure 2: BCS energy gap function.

Figure 3: Corrections to the $W^\pm$ propagator from Goldstone boson exchange.

Figure 4: Mixing between the $W^0$ and the $B$ gauge bosons.

Figure 5: Extended technicolour gauge boson vertices which can connect fermions (f) and technifermions (T) in all possible ways.

Figure 6: Radiative mechanism for extended technicolour generation of fermion mass. The exchanged gauge boson is a heavy ETC boson of mass $M_{ETC}$.

Figure 7: Dangerous neutral flavour-changing diagram which can contribute to $K^0 - \bar{K}^0$ mixing. The exchanged gauge boson is a heavy ETC boson of mass $M_{ETC}$.

Figure 8: Diagrams responsible for generating the quark and charged lepton masses in this simple extended technicolour model.

Figure 9: The diagrammatic representation of the gap equation.

Figure 10: The numerical solutions of the gap equation for (a) a normal QCD-like theory, (b) a Walking TC theory, (c) a Fixed Point theory. Both $\Sigma(p^2)$ and $\alpha_{TC}(p^2)$ are plotted. The solutions can be simply described. For $p < \Lambda_{TC}$, $\Sigma(p) = \Sigma(0)$ (flat). For $p > \mu$, where perturbation theory is more trustworthy, the behaviour of $\Sigma(p)$ depends upon the value of $\alpha_{TC}(p)$. In the region where $\alpha_{TC}(p) \approx \alpha_{TC}^\star$, $\Sigma(p) \sim 1/p$ (see for example Peskin 1982). However for $\alpha_{TC}(p) \ll \alpha_{TC}^\star$, $\Sigma(p) \sim 1/p^2$ (up to logarithms) as shown by Lane (1974) and Politzer (1976). In the intermediate region denoted as a fuzzy band the fall-off of $\Sigma$ is intermediate between these two cases. In the case of normally running theories (figure 10a) the logarithmic fall-off of $\alpha_{TC}$ is
sufficiently fast to ensure that the asymptotic solution sets in almost immediately, leading to a value of the condensate corresponding to \( \gamma \approx 0 \). In walking TC theories (figure 10b) the coupling falls off more slowly than a logarithm, and hence the solution \( \Sigma(p) \sim 1/p \) persists over a larger range of \( p \), resulting in an enhanced condensate with \( 0 < \gamma < 1 \) (Appelquist et al 1986, 1987). In the case of a fixed point (figure 10c) \( \alpha_{TC}(p) = \alpha_{TC}^* \) over the whole range of \( p > \mu \), and maximum condensate enhancement is achieved with \( \gamma \approx 1 \).

**Figure 11:** Additional contributions to the right-hand side of the gap equation in figure 9 coming from (a) higher order corrections, (b) QCD gluon exchange, (c) heavy ETC gauge boson exchange, (d) a feedback mechanism involving an internal top quark propagator.

**Figure 12:** Criticality curve for quenched QED with a four-fermion operator.

**Figure 13:** Signal and background events expected at the LHC from the decay of a 1.5 TeV technirho into WZ in (a) the BESS model, and (b) the DHT model. In both cases the signal plus background (upper histogram) is clearly visible above the background.

**Figure 14:** Spectrum of pseudo-Goldsone bosons (sometimes called technipions) in the one-family \( SU(N)_{TC} \) model.

**Figure 15:** Spectrum of pseudo-Goldsone bosons (sometimes called technipions) in the one-family \( SO(N)_{TC} \) model.

**Figure 16:** Experimental constraints on the \( S, T \) radiative correction parameters. The smaller ellipses represent the constrained fit in which it is assumed that \( V, W, X = 0 \) (at 68\% and 90\% c.l.). The larger ellipses represent an unconstrained fit in which \( V, W, X \neq 0 \) (at 68\% and 90\% c.l.).

**Figure 17:** A contribution to the \( Z b\bar{b} \) vertex from neutral ETC gauge boson exchange.
Figure 18: The gap equation in the top quark condensate model.