Fermion Scattering off a CP-Violating Electroweak Bubble Wall 

By treating CP-violating interaction as a perturbative term, we solved in a previous paper the Dirac equation in the background of electroweak bubble wall, and obtained the transmission and reflection coefficients for a chiral fermion incident from the symmetric-phase region. We give the transmission and reflection coefficients under the other boundary condition, that is, the case of the fermion incident from the broken-phase region. There hold the respective sets of unitarity relations and also reciprocity relations among them. These relations enable us to obtain a simple form of quantum-number flux through the bubble wall, which is the first order quantity of the CP violation. A factor in the integrand of the flux, which originates from the CP violation, is very sensitive to the functional form of the CP-violating phase. The absolute value of this factor is found to decrease as $m_0/a$ increases for $m_0/a > 1$, where $1/a$ is the wall thickness and $m_0$ the fermion mass near the critical temperature.

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1. Introduction

Electroweak baryogenesis is one of the most challenging problems in particle physics and cosmology. The standard model meets the three requirements to generate baryon number \([1]\), provided that the phase transition is of first order \([2]\). One of them, the baryon-number violating process is supplied by the chiral anomaly\([3]\). At high temperature, the rate of the anomalous baryon- and lepton-number violating processes induced by the sphaleron may be rapid enough to be in thermal equilibrium, while the rate becomes negligible at low temperature. The second requirement, out-of-thermal-equilibrium condition, could be realized by the first-order phase transition. The bubble nucleating in the symmetric-phase region expands to convert the whole of the universe into the broken phase. In the minimal standard model or its extension, the phase transition can be of first order by choice of parameters in the Higgs sector, as discussed by many authors \([4]\ [5]\ [6]\ [7]\). The last requirement of CP violation is given by the Kobayashi-Maskawa scheme \([8]\) in the minimal standard model. The effect of CP violation in this scheme, however, may arise in higher orders \([9]\ [10]\), so that its influence on baryogenesis would inevitably be too small to explain the baryon asymmetry observed today \([11]\). Then, one may need a new source of CP violation which would be realized in multi-Higgs models. Thus, a moving bubble wall with CP violation formed by development of the VEV of the Higgs fields would be crucial for electroweak baryogenesis.

In a previous paper\([12]\) (referred to as I hereafter) we presented a general prescription to study the fermion propagation in CP-violating bubble-wall background. By regarding the CP-violating term as a perturbation, we solved the full equation to the first order of the CP violation by DWBA (the distorted-wave Born approximation) method. We derived transmission and reflection coefficients of fermions from the asymptotic form of the wave function with the boundary condition that an incident fermion comes from the symmetric phase.

In this paper we study the other case where an incident fermion comes from the broken phase. We derive the transmission and reflection coefficients and establish relations between quantities derived in this paper and those in I. We show that reciprocity relations between the two cases hold, at least, up to the first order of CP violation. The reciprocity relations enable us to simplify quantum-number flux through the bubble wall. We find that the quantum-number flux is proportional to \((Q_L - Q_R)\Delta R\). Here \(Q_L(Q_R)\) is a quantum number carried by the left (right)-handed fermion, \(\Delta R \equiv R^s_{R \to L} - R^s_{R \to L}\) and

\[Q = \begin{cases} 1 & \text{for left-handed fermions} \\ 0 & \text{for right-handed fermions} \end{cases}\]
$R_{R-L}^s (\bar{R}_{R-L}^s)$ is the reflection coefficient for right-handed chiral fermion (antifermion) incident from the symmetric phase. We analyze $\Delta R$ numerically [13] by taking various forms of CP-violating profile of the bubble wall. The quantity $\Delta R$ is very sensitive to the functional form of the CP-violating phase. Our numerical analyses show that $|\Delta R|$ decreases as $m_0/a$ increases when $1/m_0 \lesssim 1/a$, where $1/a$ is the thickness of the bubble wall and $m_0$ is the fermion mass [14]. This means that the heavier fermion gives the smaller contribution to $|\Delta R|$ for fixed $a$, if the Compton wave length is comparable with or less than the wall thickness. One should note that a larger Yukawa coupling does not necessarily mean a larger effect of CP violation in electroweak baryogenesis.

This paper is organized as follows. We give a summary of I in section 2. In section 3, we present the transmission and reflection coefficients for a fermion incident from the broken phase. We also prove the reciprocity relations to the first order of the CP violation. In section 4 we obtain a simple form of quantum-number flux into the symmetric phase. In the final section we discuss the physical CP-violating phase in the Higgs sector and give concluding remarks.

2. DWBA to CP-Violating Dirac Equation—The Case of Fermion Incident from Symmetric-Phase Region—

In this section we briefly describe how to treat fermion propagation in the bubble-wall background by regarding CP-violating term as a perturbation. The transmission and reflection coefficients for a fermion incident from the symmetric phase are also given.

2.1. Dirac equation and ansatz

We consider one-flavor model described by the lagrangian,

$$\mathcal{L} = \bar{\psi}_L i \partial \psi_L + \bar{\psi}_R i \partial \psi_R + (f \bar{\psi}_L \psi_R \phi + \text{h.c.}). \quad (2.1)$$

In the vacuum, near the first-order phase transition, $\langle \phi \rangle$ may be $x$-dependent field, so that we put

$$m(x) = -f(\langle \phi \rangle(x), \quad (2.2)$$

where $m(x)$ is complex-valued and we neglect the time dependence. If the phase of $m(x)$ has no spatial dependence, it is removed by a constant bi-unitary transformation, which is
outside of our interest. The Dirac equation describing fermion propagation in the bubble-wall background with CP violation is

\[ i\partial_t \psi(t, \mathbf{x}) - m(\mathbf{x}) P_R \psi(t, \mathbf{x}) - m^*(\mathbf{x}) P_L \psi(t, \mathbf{x}) = 0. \] (2.3)

For the bubble wall with large enough radius, \( m(\mathbf{x}) \) could be regarded as a function of only one spatial coordinate, so that we put \( m(\mathbf{x}) = m(z) \).

To solve (2.3), we take the following ansatz:

\[
\psi(t, \mathbf{x}) = (i \partial + m(z) P_R + m(z) P_L) e^{i \sigma(-E t + p_T x)} \psi_E (\mathbf{p}_T, z) = e^{i \sigma(-E t + p_T x)} \\
\times [\sigma(\gamma^0 E - \gamma_T p_T) + i \gamma^3 \partial_z + m(z) P_R + m(z) P_L] \psi_E (\mathbf{p}_T, z),
\]

where \( \sigma = +(-) \) for positive (negative)-energy solution, \( \mathbf{p}_T = (p^1, p^2) \), \( \mathbf{x}_T = (x^1, x^2) \), \( p_T = |p_T| \) and \( \gamma_T p_T = \gamma^1 p^1 + \gamma^2 p^2 \). By putting \( E = E^* \cosh \eta \) and \( p_T = E^* \sinh \eta \) with \( E^* = \sqrt{E^2 - p_T^2} \), the Lorentz transformation eliminates \( \mathbf{p}_T \). After this Lorentz rotation for a fixed \( \mathbf{p}_T \), the Dirac equation is rewritten as

\[ [E^*^2 + \partial_z^2 - |m(z)|^2 + i m_R'(z) \gamma^3 - m_L'(z) \gamma^3] \psi_E(z) = 0, \] (2.5)

where \( m(z) = m_R(z) + i m_I(z) \). Now let us introduce a set of dimensionless variables using a parameter \( a \), whose inverse characterizes the thickness of the wall: \( m_R(z) = m_0 f(az) = m_0 f(x), \) \( m_I(z) = m_0 g(az) = m_0 g(x), \) \( x \equiv az, \tau \equiv at, \epsilon \equiv E^*/a \) and \( \xi \equiv m_0/a \), where \( m_0 \) is the fermion mass in the broken phase. Eq.(2.5) is expressed as

\[ [x^2 + \frac{d^2}{dx^2} - \xi^2 (f(x)^2 + g(x)^2) + i \xi f'(x) \gamma^3 - \xi g'(x) \gamma^3] \psi(x) = 0. \] (2.6)

As for \( f(x) \) and \( g(x) \), we do not specify their functional forms but only assume that

\[ f(x) \rightarrow \begin{cases} 1, & \text{as } x \rightarrow +\infty, \\ 0, & \text{as } x \rightarrow -\infty, \end{cases} \] (2.7)

and that \( |g(x)| << 1 \), i.e., small CP violation. Eq.(2.7) means that the system is in the broken (symmetric) phase at \( x \sim +\infty \) (\( x \sim -\infty \)), the wall height being \( m_0 \).
2.2. DWBA to the Dirac equation

We regard the small \(|g(x)|\) as a perturbation, and keep quantities up to \(O(g^1)\). Put

\[
\psi_e(x) = \psi_e^{(0)}(x) + \psi_e^{(1)}(x),
\]

(2.8)

where \(\psi_e^{(0)}(x)\) is a solution to the unperturbed equation

\[
[e^2 + \frac{d^2}{dx^2} - \xi^2 f(x)^2 + i\xi f'(x)\gamma^3] \psi_e^{(0)}(x) = 0
\]

with an appropriate boundary condition. Then \(\psi_e^{(1)}(x)\) of \(O(g^1)\) is solved as

\[
\psi_e^{(1)}(x) = \int dx' G(x, x') V(x') \psi_e^{(0)}(x') \quad \text{with} \quad V(x) = -\xi f'(x)\gamma_5 \gamma^3.
\]

(2.10)

\(G(x, x')\) is the Green’s function for the operator in (2.9) satisfying the same boundary condition as \(\psi_e^{(0)}(x)\). To this order, the solution to the Dirac equation is given by

\[
\psi(x) \simeq e^{-i\sigma x} \left\{ [\sigma e\gamma^0 + i\gamma^3 \frac{d}{dx} + \xi f(x)] [\psi_e^{(0)}(x) + \psi_e^{(1)}(x)] - i\xi g(x)\gamma_5 \psi_e^{(0)}(x) \right\}.
\]

(2.11)

If we expand \(\psi_e^{(0)}(x)\) in terms of the eigenspinors of \(\gamma^3\) as \(\psi_e^{(0)}(x) \sim \phi_{\pm}(x) u_{\pm}^s\) with \(\gamma^3 u_{\pm}^s = \pm i u_{\pm}^s (s = 1, 2)\), \(\phi_{\pm}(x)\) satisfies

\[
[e^2 + \frac{d^2}{dx^2} - \xi^2 f(x)^2 \mp \xi f'(x)] \phi_{\pm}(x) = 0.
\]

(2.12)

Because of (2.7), the asymptotic forms of \(\phi_{\pm}(x)\) should be \(\phi_{\pm}(x) \to e^{\alpha x}, e^{-\alpha x} (x \to +\infty)\) and \(e^{\beta x}, e^{-\beta x} (x \to -\infty)\), where \(\alpha = i\sqrt{e^2 - \xi^2}\) and \(\beta = i\varepsilon\). Putting all these together, we can obtain the asymptotic forms of the wave function (2.11) at \(x \to \pm \infty\).

2.3. Fermion incident from symmetric-phase region

We consider a state in which the incident wave coming from \(x = -\infty\) is reflected in part at the bubble wall, while at \(x = +\infty\) only the transmitted wave exists. We denote two independent solutions to (2.12) as \(\phi_{\pm, (+\alpha)}(x)\) and \(\phi_{\pm, (-\alpha)}(x) = (\phi_{\pm, (+\alpha)}(x))^*\) which behave as

\[
\phi_{\pm, (+\alpha)}(x) \to e^{\alpha x}, \quad \phi_{\pm, (-\alpha)}(x) \to e^{-\alpha x}
\]

(2.13)

at \(x \to +\infty\). Their asymptotic forms at \(x \to -\infty\) are

\[
\phi_{\pm, (+\alpha)}(x) \sim \gamma_{\pm}(\alpha, \beta) e^{\beta x} + \gamma_{\pm}(\alpha, -\beta) e^{-\beta x},
\]

\[
\phi_{\pm, (-\alpha)}(x) \sim \gamma_{\pm}(-\alpha, \beta) e^{\beta x} + \gamma_{\pm}(-\alpha, -\beta) e^{-\beta x}.
\]

(2.14)
From these, the general solution to (2.9) is eventually given as [12]

\[ \psi^{(0)}(x) = \sum_{s} [A_{s}^{(\pm \alpha)}(x) + A_{s}^{(+ \alpha)}(x)] u_{s}^{\pm}. \tag{2.15} \]

The required boundary condition is achieved by setting \( A_{s}^{(- \alpha)} = 0 \) for \( \sigma = + \) and \( A_{s}^{(+ \alpha)} = 0 \) for \( \sigma = - \). The Green’s function which matches this boundary condition can be constructed from \( \phi_{(\pm \alpha)}(x) \)[12].

From the asymptotic forms of \( (\psi(x)_{\sigma})^{inc} \), \( (\psi(x)_{\sigma})^{trans} \) and \( (\psi(x)_{\sigma})^{refl} \) for \( \sigma = \pm \) in (2.11) [12], we obtain those of the vector and axial-vector currents, \( j_{V}^{\mu} = \bar{\psi} \gamma^{\mu} \psi \) and \( j_{A}^{\mu} = \bar{\psi} \gamma^{\mu} \gamma_{5} \psi \). In terms of the chiral currents, \( j_{V}^{\mu} = (1/2)(j_{V}^{L} - j_{V}^{R}) \) and \( j_{A}^{\mu} = (1/2)(j_{A}^{L} + j_{A}^{R}) \), the transmission and reflection coefficients for the chiral fermion are defined as

\[
T^{(\sigma)}_{L \rightarrow L(R)} = \left( j_{L(R),\sigma}^{3} \right)^{trans} \big|_{A_{s}^{\pm} = \varnothing / (j_{L,R,\sigma}^{3})^{inc}},
\]

\[
T^{(\sigma)}_{R \rightarrow L(R)} = \left( j_{L(R),\sigma}^{3} \right)^{trans} \big|_{A_{s}^{\pm} = \varnothing / (j_{R,L,\sigma}^{3})^{inc}},
\]

\[
R^{(\sigma)}_{L(R) \rightarrow R(L)} = - \left( j_{R(L),\sigma}^{3} \right)^{refl} / (j_{L(R),\sigma}^{3})^{inc}.
\]

If we denote \( R^{s}(T^{s}) = R^{(\pm)}(T^{(\pm)}) \) and \( R^{s}(T^{s}) = R^{(-)}(T^{(-)}) \), where the superscript \( s \) denotes the fermion incident from the symmetric phase, we have[15]

\[
T^{s}_{L \rightarrow L} = \bar{T}^{s}_{R \rightarrow R} = T^{(0)} \left[ \frac{1}{2} + \frac{\epsilon}{2 \sqrt{\epsilon^{2} - \xi^{2}}} \right] (1 - \delta^{CP}),
\]

\[
T^{s}_{L \rightarrow R} = \bar{T}^{s}_{R \rightarrow L} = T^{(0)} \left[ \frac{1}{2} - \frac{\epsilon}{2 \sqrt{\epsilon^{2} - \xi^{2}}} \right] (1 - \delta^{CP}),
\]

\[
T^{s}_{R \rightarrow L} = \bar{T}^{s}_{L \rightarrow R} = T^{(0)} \left[ \frac{1}{2} - \frac{\epsilon}{2 \sqrt{\epsilon^{2} - \xi^{2}}} \right] (1 + \delta^{CP}),
\]

\[
T^{s}_{R \rightarrow R} = \bar{T}^{s}_{L \rightarrow L} = T^{(0)} \left[ \frac{1}{2} + \frac{\epsilon}{2 \sqrt{\epsilon^{2} - \xi^{2}}} \right] (1 + \delta^{CP}),
\]

\[
R^{s}_{L \rightarrow R} = \bar{R}^{s}_{R \rightarrow L} = R^{(0)} + T^{(0)} \delta^{CP},
\]

\[
R^{s}_{R \rightarrow L} = \bar{R}^{s}_{L \rightarrow R} = R^{(0)} - T^{(0)} \delta^{CP}.
\]

Here

\[
T^{(0)} = \frac{\alpha}{\beta} \left| \gamma_{+(\alpha, \beta)} \right|^{2}, \quad R^{(0)} = \left| \frac{\gamma_{-(\alpha, \beta)}}{\gamma_{+(\alpha, \beta)}} \right|^{2} \tag{2.18}
\]

with \( T^{(0)} + R^{(0)} = 1 \) are respectively the transmission and reflection coefficients in the absence of CP violation. The correction by CP violation is

\[
\delta^{CP} = \frac{\xi}{2 \sqrt{\epsilon^{2} - \xi^{2}}} \left( \frac{\gamma_{-(\alpha, \beta)}}{\gamma_{+(\alpha, \beta)}} \right) I_{2} + c.c., \tag{2.19}
\]

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where
\[ I_2 \equiv \int_{-\infty}^{\infty} dx g'(x) \phi_+^{(+\alpha)}(x) \phi_-^{(+\alpha)}(x). \] (2.20)

Among these, the following unitarity relations hold:
\[ T_{L\to L}^* + T_{L\to R}^* + R_{L\to R}^* = 1, \quad T_{R\to L}^* + T_{R\to R}^* + R_{R\to L}^* = 1. \] (2.21)

3. Fermion Incident from Broken-Phase Region and Reciprocity

3.1. Asymptotic forms of the wave function

In place of \( \phi_+^{(+\alpha)}(x) \), we start with \( \phi_+^{(-\beta)}(x) \) that behave as
\[ \phi_+^{(-\beta)}(x) \to e^{-\beta x}, \quad \phi_+^{(+\beta)}(x) = (\phi_+^{(-\beta)}(x))^* \to e^{+\beta x} \] (3.1)
at \( x \to -\infty \), while at \( x \to +\infty \)
\[ \phi_+^{(-\beta)}(x) \sim \tilde{\gamma}_\beta(-\beta, \alpha)e^{\alpha x} + \tilde{\gamma}_\beta(-\beta, -\alpha)e^{-\alpha x}, \]
\[ \phi_+^{(+\beta)}(x) \sim \tilde{\gamma}_\beta(\beta, \alpha)e^{\alpha x} + \tilde{\gamma}_\beta(\beta, -\alpha)e^{-\alpha x}. \] (3.2)

From these, the general solution to (2.9) is given as
\[ \psi^{(0)}(x) = \sum_s [B_{s}^{(-)} \phi_+^{(-\beta)}(x) + B_{s}^{(+)\phi_+^{(+\beta)}(x)}] u_s^+. \] (3.3)

The boundary condition is achieved by setting \( B_{s}^{(+)} = 0 \) for \( \sigma = + \) and \( B_{s}^{(-)} = 0 \) for \( \sigma = - \).

Most of the following formulae are derived completely in parallel to \( I \), so we skip the detailed algebra. The asymptotic forms for the positive-energy wave are
\[
\begin{align*}
(\psi(x)_{\sigma=+})^{\text{trans}} &= e^{-i\epsilon x} \sum_s B_{s}^{(+)} \{ \beta [1 - (-)^s \frac{\xi}{\epsilon} (\hat{I}_1 - g(+\infty))] u_s^+ \\
&\quad + \epsilon [1 - (-)^s \frac{1}{\epsilon} \frac{\xi}{2} \hat{I}_1 - \frac{1}{2} g(+\infty) + g(-\infty)] u_s^\pm \}, \\
(\psi(x)_{\sigma=+})^{\text{inc}} &= e^{-i\epsilon x} \sum_s B_{s}^{(+)} \tilde{\gamma}_\beta(-\beta, -\alpha) u_s^+, \\
&\quad \times \{ \xi + (\xi + \alpha) [1 + (-)^s \frac{\xi}{\epsilon} (\frac{\xi - \alpha}{2\beta} \frac{\tilde{\gamma}_\beta(-\beta, -\alpha)}{\tilde{\gamma}_\beta(-\beta, -\alpha)} \hat{I}_2 + \frac{1}{2} g(+\infty)))] u_s^+ \\
&\quad + \epsilon [1 + (-)^s \frac{\xi}{\epsilon} (\frac{\xi - \alpha}{2\beta} \frac{\tilde{\gamma}_\beta(-\beta, -\alpha)}{\tilde{\gamma}_\beta(-\beta, -\alpha)} \hat{I}_2 - \frac{1}{2} g(+\infty)))] u_s^\pm \}, \\
(\psi(x)_{\sigma=+})^{\text{refl}} &= e^{-i\epsilon x} \sum_s B_{s}^{(+)} \tilde{\gamma}_\beta(-\beta, \alpha) u_s^+, \\
&\quad \times \{ \xi + (\xi - \alpha) [1 + (-)^s \frac{\xi}{\epsilon} (\frac{\xi + \alpha}{2\beta} \frac{\tilde{\gamma}_\beta(-\beta, \alpha)}{\tilde{\gamma}_\beta(-\beta, \alpha)} \hat{I}_2 + \frac{1}{2} g(+\infty)))] u_s^+ \\
&\quad + \epsilon [1 + (-)^s \frac{\xi}{\epsilon} (\frac{\xi + \alpha}{2\beta} \frac{\tilde{\gamma}_\beta(-\beta, \alpha)}{\tilde{\gamma}_\beta(-\beta, \alpha)} \hat{I}_2 - \frac{1}{2} g(+\infty)))] u_s^\pm \},
\end{align*}
\] (3.4)
and
\[ (\psi(x)_{\sigma=-})^{asym} = (\psi(x)_{\sigma=+})^{asym} |_{(\alpha,\beta,\epsilon,\tilde{\epsilon},\tilde{I}_1,\tilde{I}_2) \to (-\alpha, -\beta, -\epsilon, \tilde{\epsilon}, \tilde{I}_1, \tilde{I}_2)} \]  \hspace{1cm} (3.5)

for negative-energy wave. \( \tilde{I}_1 \) and \( \tilde{I}_2 \) are defined by
\[ \tilde{I}_1 = \int_{-\infty}^{+\infty} dx g'(x) \phi_-(^{+\beta}) (x) \phi_+^{(-\beta)} (x), \]
\[ \tilde{I}_2 = \int_{-\infty}^{+\infty} dx g'(x) \phi_+^{(-\beta)} (x) \phi_+^{(-\beta)} (x). \]  \hspace{1cm} (3.6)

The first one \( \tilde{I}_1 \) is found to be irrelevant to the final results similarly to \( I_1 \equiv \int_{-\infty}^{+\infty} dx g'(x) \phi_-(^{(-\alpha)}) \phi_+^{(+\alpha)} (x) \) in \( I \).

3.2. Transmission and reflection coefficients

These are defined in the way similar to (2.16), and are expressed as follows:
\[ T^b_{L\to L} = \tilde{T}^b_{R\to R} = \tilde{T}^{(0)} \left( \frac{1}{2} + \frac{\sqrt{\epsilon^2 - \xi^2}}{2\epsilon} \right) (1 + \tilde{\delta}_{CP}), \]
\[ T^b_{L\to R} = \tilde{T}^b_{R\to L} = \tilde{T}^{(0)} \left( \frac{1}{2} - \frac{\sqrt{\epsilon^2 - \xi^2}}{2\epsilon} \right) (1 - \tilde{\delta}_{CP}), \]
\[ T^b_{R\to L} = \tilde{T}^b_{R\to L} = \tilde{T}^{(0)} \left( \frac{1}{2} - \frac{\sqrt{\epsilon^2 - \xi^2}}{2\epsilon} \right) (1 + \tilde{\delta}_{CP}), \]
\[ T^b_{R\to R} = \tilde{T}^b_{L\to L} = \tilde{T}^{(0)} \left( \frac{1}{2} + \frac{\sqrt{\epsilon^2 - \xi^2}}{2\epsilon} \right) (1 - \tilde{\delta}_{CP}), \]  \hspace{1cm} (3.7)

and
\[ \tilde{R}^b_{L\to L} = \tilde{R}^b_{R\to R} = -\tilde{R}^b_{R\to R} = -\tilde{R}^b_{L\to L} = \tilde{T}^{(0)} \frac{\xi^2}{2\epsilon \sqrt{\epsilon^2 - \xi^2}} \tilde{\delta}_{CP}, \]
\[ \tilde{R}^b_{L\to R} = \tilde{R}^b_{R\to L} = \tilde{T}^{(0)} \frac{\sqrt{\epsilon^2 - \xi^2}}{\epsilon} + \frac{\xi^2}{2\epsilon \sqrt{\epsilon^2 - \xi^2}} \tilde{\delta}_{CP}, \]
\[ \tilde{R}^b_{R\to L} = \tilde{R}^b_{L\to R} = \tilde{T}^{(0)} \frac{\sqrt{\epsilon^2 - \xi^2}}{\epsilon} + \frac{\xi^2}{2\epsilon \sqrt{\epsilon^2 - \xi^2}} \tilde{\delta}_{CP}, \]  \hspace{1cm} (3.8)

where superscript \( b \) denotes the fermion incident from the broken phase. Note that, in contrast to the previous case, we have nonzero \( \tilde{R}^b_{L(R)\to L(R)} \) because the fermion is massive in the broken phase. \( \tilde{T}^{(0)} \) and \( \tilde{R}^{(0)} \), the transmission and reflection coefficients in the absence of CP violation respectively, are given by
\[ \tilde{T}^{(0)} = \frac{\beta}{\alpha} \frac{1}{|\tilde{\gamma}_+(\beta, \alpha)|^2}, \quad \tilde{R}^{(0)} = \left| \frac{\gamma_+^{(\beta, -\alpha)}}{\tilde{\gamma}_+^{(\beta, \alpha)}} \right|^2, \]  \hspace{1cm} (3.9)
with $\tilde{T}^{(0)} + \tilde{R}^{(0)} = 1$. The correction by CP violation is given by

$$\tilde{\xi}_{CP} = \frac{\xi}{e} \left[ \frac{\xi - \alpha}{2\beta} \tilde{\gamma}(\beta, -\alpha) \bigg| T_{2} + c.c. \right]. \quad (3.10)$$

Among these the following unitarity relations hold as they should:

$$T_{L-L}^{h} + T_{L-R}^{h} + R_{L-L}^{h} + R_{L-R}^{h} = 1,$$
$$T_{R-L}^{h} + T_{R-R}^{h} + R_{R-L}^{h} + R_{R-R}^{h} = 1. \quad (3.11)$$

3.3. Reciprocity relations

$\phi_{\pm}(x)$ are linearly dependent on $\phi_{\pm}(x)$ and vice versa. By comparing their asymptotic forms, we have

$$\phi_{\pm}^{(+\beta)}(x) = \tilde{\gamma}_{\pm}(\beta, \alpha) \phi_{\pm}^{(+\alpha)}(x) + \tilde{\gamma}_{\pm}(\beta, -\alpha) \phi_{\pm}^{(-\alpha)}(x),$$
$$\phi_{\pm}^{(-\beta)}(x) = \tilde{\gamma}_{\pm}(-\beta, \alpha) \phi_{\pm}^{(+\alpha)}(x) + \tilde{\gamma}_{\pm}(-\beta, -\alpha) \phi_{\pm}^{(-\alpha)}(x), \quad (3.12)$$

and

$$\phi_{\pm}^{(+\alpha)}(x) = \gamma_{\pm}(\alpha, \beta) \phi_{\pm}^{(+\beta)}(x) + \gamma_{\pm}(-\alpha, \beta) \phi_{\pm}^{(-\beta)}(x),$$
$$\phi_{\pm}^{(-\alpha)}(x) = \gamma_{\pm}(-\alpha, \beta) \phi_{\pm}^{(+\beta)}(x) + \gamma_{\pm}(-\alpha, -\beta) \phi_{\pm}^{(-\beta)}(x), \quad (3.13)$$

which yield relations between $\tilde{\gamma}$ and $\gamma$:

$$\tilde{\gamma}_{\pm}(\beta, \alpha) = \frac{\beta}{\alpha} \gamma_{\pm}(-\alpha, -\beta), \quad \tilde{\gamma}_{\pm}(\beta, -\alpha) = \frac{-\beta}{\alpha} \gamma_{\pm}(\alpha, -\beta),$$
$$\tilde{\gamma}_{\pm}(-\beta, \alpha) = \frac{-\beta}{\alpha} \gamma_{\pm}(-\alpha, \beta), \quad \tilde{\gamma}_{\pm}(-\beta, -\alpha) = \frac{\beta}{\alpha} \gamma_{\pm}(\alpha, \beta). \quad (3.14)$$

Eq.(3.14) immediately leads us to

$$\tilde{T}^{(0)} = T^{(0)}, \quad \tilde{R}^{(0)} = R^{(0)}, \quad (3.15)$$

that is, the reciprocity relations in the absence of CP violation, which are well known in the nonrelativistic scattering problem.

Furthermore, the following reciprocity relations are proved to hold, at least, up to the first order of CP violation:

$$T_{L-L}^{h} + T_{R-L}^{h} = T_{R-L}^{h} + T_{R-R}^{h} (= 1 - R_{R-L}^{h}),$$
$$T_{L-R}^{h} + T_{R-R}^{h} = T_{L-L}^{h} + T_{L-R}^{h} (= 1 - R_{L-R}^{h}). \quad (3.16)$$
The proof goes as follows. By inserting (3.12) into $\hat{I}_2$ and using (3.14), we have

$$
\hat{I}_2 = \frac{\beta^2}{\alpha^2} [\gamma_+(-\alpha, \beta)\gamma_+(-\alpha, \beta)I_2 - \gamma_-(-\alpha, \beta)\gamma_+(\alpha, \beta)I_1^* - \gamma_+(-\alpha, \beta)\gamma_+(\alpha, \beta)I_1 + \gamma_-(-\alpha, \beta)\gamma_+(\alpha, \beta)I_2^*].
$$

(3.17)

Remembering that $\gamma_-$'s are written in terms of $\gamma_+$'s as shown in I, we arrive at

$$
\tilde{\delta}^{CP} \equiv -\frac{\xi}{\beta} \frac{\alpha}{2\epsilon} \left[ \frac{\alpha - \xi \gamma_+(\alpha, \beta)}{\alpha} \left( |\gamma_+(\alpha, \beta)|^2 - |\gamma_+(\alpha, \beta)|^2 I_2 + c.c. \right) \right]
$$

$$
= -\frac{\xi}{\beta} \frac{\alpha}{2\epsilon} \left[ \frac{\alpha - \xi \gamma_+(\alpha, \beta)}{\alpha} \frac{\alpha}{\beta} I_2 + c.c. \right]
$$

$$
= +\frac{\xi}{\beta} \frac{\alpha}{2\epsilon} \left[ \gamma_-(\alpha, \beta) I_2 + c.c. \right] = \delta^{CP}.
$$

(3.18)

Namely, the amount of CP violation is independent of the direction from which the incident fermion comes. Eq. (3.18) combined with (3.15) leads to (3.16).

We emphasize the fact that various coefficients obtained here are expressed in terms of only two quantities, $T^{(0)}$ and $\delta^{CP}$.

4. Quantum-Number Flux through Bubble Wall

4.1. The flux into the symmetric phase

Suppose that a left (right)-handed fermion has a quantum number $Q_{L(R)}$, which is conserved in the symmetric phase. Let us estimate the expectation value of the changes of the quantum number in the symmetric phase brought by a fermion incident from the symmetric phase. It is given by

$$
\Delta Q^\alpha = (Q_R - Q_L)R_{L\rightarrow R} + (Q_L - Q_R)R_{R\rightarrow L} + (-Q_R + Q_L)R_{L\rightarrow R}^s + (-Q_L + Q_R)R_{R\rightarrow L}^s + (-Q_L)(T_{L\rightarrow L} + T_{L\rightarrow R}^s) + (-Q_R)(T_{R\rightarrow L} + T_{R\rightarrow R}^s)
$$

$$
- (-Q_L)(\tilde{T}_{L\rightarrow L}^s + \tilde{T}_{L\rightarrow R}^s) - (-Q_R)(\tilde{T}_{R\rightarrow L}^s + \tilde{T}_{R\rightarrow R}^s)
$$

$$
= (Q_L - Q_R)(R_{R\rightarrow L}^s - R_{R\rightarrow R}^s),
$$

(4.1)

where we have used the unitarity relations (2.21). On the other hand, that brought by a fermion from the broken phase is

$$
\Delta Q^b = Q_L(T_{L\rightarrow L}^b + T_{R\rightarrow L}^b) + Q_R(T_{L\rightarrow R}^b + T_{R\rightarrow R}^b)
$$

$$
+ (-Q_L)(T_{L\rightarrow L}^b + T_{L\rightarrow R}^b) + (-Q_R)(T_{L\rightarrow R}^b + T_{R\rightarrow R}^b)
$$

$$
= (Q_L - Q_R)(T_{L\rightarrow L}^b + T_{R\rightarrow R}^b - T_{L\rightarrow R}^b - T_{R\rightarrow L}^b).
$$

(4.2)
From (4.1) and (4.2), the quantum-number flux into the symmetric phase just in front of the bubble wall moving with velocity $u$ is given by

$$F_Q = \frac{1}{\gamma} \int_{m_0}^{\infty} d p_L \int_{0}^{\infty} \frac{d p_T p_T}{4 \pi^2} (Q_L - Q_R)$$

$$\times \left[ (R_{R \rightarrow L}^a - \tilde{R}_{R \rightarrow L}^a) f^a(p_L, p_T)ight.$$  

$$\left. - (T_{L \rightarrow R}^b + T_{R \rightarrow R}^b - T_{L \rightarrow R}^b - T_{R \rightarrow L}^b) f^b(-p_L, p_T) \right],$$

(4.3)

where $p_{L(R)}$ is the longitudinal (transverse) momentum of the fermion to the bubble wall, $\gamma = \sqrt{1 - u^2}$ and the fermion-flux density in the symmetric (broken) phase $f^a(f^b)$ is given by [16]

$$f^a(p_L, p_T) = (p_L/E)(\exp[\gamma(E - u p_L)/T] + 1)^{-1},$$

$$f^b(-p_L, p_T) = (p_L/E)(\exp[\gamma(E + u \sqrt{p_L^2 - m_0^2})/T] + 1)^{-1},$$

(4.4)

with the chemical potential being omitted for simplicity. Thanks to the reciprocity relations (3.16), (4.3) is reduced to a simple expression:

$$F_Q = \frac{1}{\gamma} \int_{m_0}^{\infty} d p_L \int_{0}^{\infty} \frac{d p_T p_T}{4 \pi^2} (Q_L - Q_R) \left[ f^a(p_L, p_T) - f^b(-p_L, p_T) \right] \Delta R.$$

(4.5)

Here $\Delta R$ is the difference between the chiral fermion and its anti-fermion in the reflection coefficients incident from the symmetric phase:

$$\Delta R \equiv R_{R \rightarrow L}^a - \tilde{R}_{R \rightarrow L}^a = -2 T^{(0)} \delta^{CP},$$

(4.6)

where $T^{(0)}$ and $\delta^{CP}$ are given in (2.18) and (2.19) respectively.

We now recognize that $\Delta R$ is the dynamical quantity of primary importance, since, if its absolute value is very small, any quantum-number flow is almost forbidden. Note that the vector-like quantum numbers such as baryon and lepton numbers do not flow through the CP-violating bubble wall. This statement would hold, at least, up to $O(g^2)$ [17]. The candidates for $Q_L(R)$ in the electroweak theory with usual quantum-number assignment for the matter fields are the hypercharge or the third component of the weak isospin $T_3$. In the charge-transport scenario in [16], the hypercharge flux $F_Q$ carried by the top quark is subsequently converted into net baryon asymmetry by the sphaleron transition.
4.2. **Numerical analyses of \( \Delta R \): the kink-type bubble wall**

We have carried out numerical analyses of \( \Delta R \) by taking several forms of \( g(x) \) [13] assuming that the bubble-wall profile without CP violation is of the kink type [18]:

\[
\begin{align*}
&f(x) = (1 + \tanh x)/2. \\
&f'(x) = 2f(x) - 2f(x)^2.
\end{align*}
\]

(4.7)

The unperturbed solutions \( \phi_{\pm}^{(a\alpha)}(x) \) in this background are expressed in terms of the hypergeometric functions. The transmission and reflection coefficients without CP violation are

\[
\begin{align*}
T^{(a)} &= \frac{\sin(\pi\alpha)\sin(\pi\beta)}{\sin[\frac{\pi}{2}(\alpha + \beta + \xi)] \sin[\frac{\pi}{2}(\alpha + \beta - \xi)]}, \\
R^{(a)} &= \frac{\sin[\frac{\pi}{2}(\alpha - \beta + \xi)] \sin[\frac{\pi}{2}(\alpha - \beta - \xi)]}{\sin[\frac{\pi}{2}(\alpha + \beta + \xi)] \sin[\frac{\pi}{2}(\alpha + \beta - \xi)]},
\end{align*}
\]

(4.8)

respectively.

The effects of CP violation can be evaluated, once the functional form of \( g(x) \) is given. Note that the CP-angle can be removed by a constant bi-unitary transformation for \( g(x) = \Delta \theta f(x) \), where the parameter \( \Delta \theta \) characterizes the magnitude of CP violation. As a check, we have confirmed \( \Delta R/\Delta \theta = 0 \) numerically in this case. We have studied the following examples of \( g(x) \) with various ranges of \( dg(x)/dx \):

\[
\begin{align*}
g(x) &= \Delta \theta f(x)^2, \quad g(x) = \Delta \theta f(x)^3, \quad g(x) = \Delta \theta f'(x), \\
g'(x) &= \Delta \theta \sech x, \quad g'(x) = \Delta \theta \sech(2x), \\
g'(x) &= \Delta \theta \sech(3x), \quad g'(x) = \Delta \theta \sech x \tanh x, \\
g'(x) &= \Delta \theta \sech(2x) \tanh x, \quad g'(x) = \Delta \theta \sech(3x) \tanh x.
\end{align*}
\]

Among these, \( \Delta R/\Delta \theta \) for \( g(x) = \Delta \theta f'(x) \) is just \(-2\) times that for \( g(x) = \Delta \theta f(x)^2 \), since \( f'(x) = 2f(x) - 2f(x)^2 \).

The shape, magnitude and sign of \( \Delta R/\Delta \theta \) are very sensitive to the functional form of \( g(x) \). However the dependence of \( |\Delta R/\Delta \theta| \) on \( m_0/a \) shows an interesting general trend. Namely, \( |\Delta R/\Delta \theta| \) decreases as \( m_0/a \) increases when \( m_0/a \gtrsim 1 \), as far as our numerical analyses are concerned. In other words, \( |\Delta R/\Delta \theta| \) becomes smaller as the relevant fermion is heavier for fixed \( a \) when \( m_0 \gtrsim a \). In Fig. 1 and Fig. 2, we have plotted the \( E^* \) dependence of \( \Delta R/\Delta \theta \) for several inverse thickness of the bubble wall, \( a \), in the cases \( g(x) = \Delta \theta f(x)^2 \) and \( g(x) = \Delta \theta f(x)^3 \), respectively. \( |\Delta R/\Delta \theta| \) in these figures is extremely small for \( m_0/a \gtrsim 2 \). We could say that a heavier fermion with the larger Yukawa coupling does not always take part in baryogenesis game.
5. Concluding Remarks

We have proved that the reciprocity relations among the transmission and reflection coefficients hold, at least, up to the first order of CP violation. All the coefficients are written in terms of only two quantities, $T^{(0)}$ and $a^{CP}$, irrespective of whether an incident fermion comes from the symmetric phase or the broken phase. We have obtained the simple form of the quantum-number flux through the CP-violating bubble wall with the help of the unitarity and reciprocity. If there is difference in the quantum number between left- and right-handed fermions, it can be left in the symmetric phase.

The numerical analyses of $\Delta R/\Delta \theta$ indicate an interesting feature that the heavier fermion plays a minor role to generate the baryon asymmetry when the Compton wave length is comparable with or less than the wall thickness. Specifically in the cases $g(x) = \Delta \theta f(x)^2$ and $g(x) = \Delta \theta f(x)^3$, $|\Delta R|$ is negligibly small in the region $2/m_0 < 1/a$, which means that twice the Compton wave length of the relevant fermion must be larger than the wall thickness in order for $\Delta R$ to have a sizable value. The above feature seems to be reasonable, since the heavier fermion would experience thicker bubble wall due to its short Compton wave length. One cannot say that the top quark is the most relevant one contributing to the quantum-number flux $F_Q$. A larger Yukawa coupling does not necessarily mean a larger effect of CP violation in electroweak baryogenesis.

The chemical-equilibrium relations in particle interactions in the symmetric phase play an important role to convert some quantum number, say the hypercharge, into the net baryon number density by the sphaleron transition [16]. Yukawa interactions for light fermions like $e$, $\mu$ and $u$ are out of chemical equilibrium in the symmetric phase, because their mean free times are comparable with the expansion time of the universe. Thus the candidate fermions contributing to the baryon-number generation would be $\tau$ lepton and $d$, $c$, $s$, $t$ and $b$ quarks. Once the wall thickness and the functional form of $g(x)$ are known, we can select, by using the formulae for $\Delta R$ and $F_Q$, what species of fermions take part in the baryogenesis game.

Here we make a comment on how to determine the CP-violating imaginary part $g(x)$ in two-Higgs-doublet models. One should note that $g(x)$ itself is not physical since it is a gauge-variant quantity. The imaginary part $g(x)$ and the corresponding classical gauge-field configurations would be determined by solving the classical equations of motion of the gauge-Higgs system in some gauge. It is, however, difficult to solve the Dirac equation in the background of classical solutions of gauge and Higgs fields.
Now, let us assume that the gauge fields are pure-gauge type:

\[ ig \frac{r^a}{2} A^a_\mu = \partial_\mu U_2(x) U_2^{-1}(x), \quad ig \frac{1}{2} B_\mu = \partial_\mu U_1(x) U_1^{-1}(x), \]

where \( U_1 \) and \( U_2 \) are elements of \( U(1)_Y \) and \( SU(2)_L \) respectively. We are interested in a bubble-wall solution which may be assumed to be spherically symmetric. In this case we know an example that the pure-gauge assumption is reasonable. As shown by Ratra and Yaffe in [19], the \( SU(2) \) gauge-Higgs system can be reduced to \( 1 + 1 \) dimensional abelian gauge-Higgs system under the spherically symmetric ansatz for the gauge and Higgs fields. In that case the gauge field is written in terms of pure-gauge. Once we admit the assumption, the lagrangian of the gauge-Higgs system is reduced to

\[ \mathcal{L} = |\partial_\mu \varphi_1|^2 + |\partial_\mu \varphi_2|^2 - V_{\text{eff}}(\varphi_1, \varphi_2; T), \]  

where \( \varphi_i(i = 1, 2) \) are the Higgs fields in the pure-gauge background. If we decompose \( \varphi_i \) as \( \frac{1}{\sqrt{2}} \rho_i(x) e^{i \theta_i(x)} \), the phases \( \theta_i(x) \) are unambiguously determined by the equations of motion, which follow from (5.2). Note that \( V_{\text{eff}} \) depends only on \( \theta_1, \theta_2 \) due to the original gauge-invariance so that no \( x \)-dependent phase arises in the Higgs potential in one-Higgs-doublet model. The equations of motion for \( \theta_i \) are

\[ \partial_\mu (\rho_i^2 \partial^\mu \theta_i) + \frac{\partial V_{\text{eff}}}{\partial \theta_i} = 0, \quad (i = 1, 2). \]  

From (5.3), we have

\[ \partial_\mu (\rho_1^2 \partial^\mu \theta_1 + \rho_2^2 \partial^\mu \theta_2) = 0 \]  

by using the fact that \( V_{\text{eff}} \) depends only on \( \theta_1, \theta_2 \). On the other hand, the condition that the gauge fields stay in pure gauge (the sourcelessness condition) is given by

\[ \rho_1^2(x) \partial_\mu \theta_1(x) + \rho_2^2(x) \partial_\mu \theta_2(x) = 0, \]  

which follows from the equation of motion for the gauge fields. It is important that we should generally impose (5.5) rather than (5.4) when we solve classical equations of motion under the pure-gauge assumption for gauge fields. From the phases \( \theta_i \) determined in this way, one can apply the prescription to solve the Dirac equation with CP-violating bubble wall background developed in I and this paper.

Although many complicated factors would affect baryogenesis, a sufficient amount of baryon-number asymmetry could not be produced unless \( |\Delta R/\Delta \theta| \) due to CP violation is
large enough. We expect that our DWBA prescription and the results would serve to build models to generate baryon asymmetry of the universe.

References

[14] Here the fermion mass $m_0$ is determined by the VEV of the Higgs field near the critical temperature. See Eq. (2.2).
[15] In this paper we denote $\delta^{inc}$ in I as $\delta^{CP}$.
[17] This statement may be exact because the reciprocity relations would be expected to hold to all order of the perturbation theory with respect to the CP violation.

Figure Captions

Fig. 1. $\Delta R/\Delta \theta$ as a function of $E^*$ for various $a$, in the case where $g(x) = \Delta \theta f(x)^2$. The numerical values of $E^*$ and $a$ are given in the unit of $m_0$, the height of the bubble wall.
Fig. 2. $\Delta R/\Delta \theta$ as a function of $E^*$ for various $a$, in the case where $g(x) = \Delta \theta f(x)^3$. The numerical values of $E^*$ and $a$ are given in the unit of $m_0$, the height of the bubble wall.