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MACHO PHOTOMETRY AND ASTROMETRY NORDITA - 94/26 A / 865423

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MACHO photometry and astrometry

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Abstract. MACHOs have been discovered by their relativistic amplification of light from distant stars as they crossed very near to the line-of-sight. The very few events were detected from more than a milliard photometric measurements of millions of stars in the LMC. – A mathematical theory of analysis of astrometric and photometric measurements of microlensing events is presented. It is shown that three photometric measurements and three one-dimensional astrometric measurements during an event are, in principle, sufficient to determine the – precisely – six observable physical parameters of the MACHO, including the proper motion and distance of the dark body, provided the position, motion and distance of the undeflected star has been determined from observations outside the event. The practical possibility of such observations is discussed by comparison with the proposed ROEMER satellite, the only known instrument concept with the potential for a sufficient number of photometric and astrometric observations of the required quality. At least 300 photometric amplifications of light from stars brighter than $V = 16.5$ mag could be detected during a ROEMER mission, but the prospects for significant astrometric observations are meager.

Key words: astrometry – gravitational lensing – artificial satellites

1. Introduction

The amplification of light from stars, sometimes called ‘microlensing’, was proposed by Paczynski (1986) as a means to detect Massive Astrophysical Compact Halo Objects (MACHOs). This prediction of photometric lensing events in the light from stars in the large Magellanic Cloud (LMC) was confirmed by Alcock et al. (1993) and

Aubourg et al. (1993) through their detection of three events, probably caused by MACHOs of about 0.1 solar masses as they passed within less than one milliarcsecond (mas) of the line-of-sight to a star. A photometric event is fully characterized by the three parameters: the epoch t_m at which the event occurs, the maximum amplification A_m , and the characteristic time or duration t_E .

The parameter A_m may be replaced by the impact parameter u_m with a more direct physical meaning, as defined below. We show that the three *photometric* parameters t_m, u_m, t_E may in principle be derived from just three observed photometric amplifications of an event.

A MACHO is assumed to be a compact, point-like and massive object, fully characterized at a given initial epoch t_1 near an event by six physical parameters: mass m , distance D or parallax π , two position coordinates α, δ and the proper motion μ_α, μ_δ . They can under optimal conditions be derived from observations of an encounter event with a star for which the position and proper motion are known. A seventh parameter, and in fact the only remaining physical MACHO parameter, the radial velocity, could be obtained from the two distances to a MACHO observed at two events, but such an occurrence is too improbable to be ever observed and hence does not warrant discussion.

It will be shown that the six physical MACHO parameters (mass, distance, position and proper motion at epoch t_1) can in principle be derived from three photometric and three one-dimensional astrometric observations of an event.

The six observable MACHO parameters could, alternatively, be derived from (six) astrometric observations, without use of photometry. But the result would be very much less precise, and it would be a waste of the photometric measurements if they were not used since they are obtained simultaneously with the astrometric observations by any conceivable modern techniques, so this alternative need no further discussion.

The recommended strategy is to perform milliads of photometric and astrometric observations of stars and to detect any photometric lensing events, alone by analysis of the photometry, deriving the three photometric parameters for each event. The astrometry of the (few) detected events are then used to obtain up to three *astrometric* parameters of the event: the *relative* proper motion and parallax, μ , π_{sd} , namely of the star relative to the dark body, see Sect. 2. Finally, the six physical MACHO parameters may be derived by means of the five astrometric parameters of the star: position, proper motion and distance.

For most photometric events no significant astrometric parameters can be derived because the star is too faint and/or the MACHO mass is too small. It is concluded in Sect. 4 that the proposed ROEMER satellite would not give sufficient precision to determine the proper motion of MACHOs. But with 20 times smaller observation errors than ROEMER a significant number of proper motions could be obtained if a large part of MACHOs were of $1M_\odot$, for which no evidence however exists.

Without proper motion data the mass m derived from a photometric event must be based on a tangential velocity from a dynamical model of the dark halo and on a hypothetical distance to the MACHO, mainly inferred from the fact that the MACHO must be closer to us than the star.

It is noted in Sect. 5 that photometric events alone, observed at Galactic stars at closer distances than the LMC will provide statistical values of distances to MACHOs. Also the distribution of masses can be derived by suitable model assumptions. A ROEMER mission could give an interesting contribution to the study of the dark halo by its milliads of photometric observations.

2. Theory of astrometric lensing

The equation of gravitational lensing may be written in the tangential plane on the observer's sky, cf. Fig. 1, as

$$r_x^2 - rr_x - R_E^2 = 0; \quad (1)$$

where the coordinate system is centered on the lensing point mass m , the source is at the position $\mathbf{r}$ and the image is at $\mathbf{r}_x$, along the same vector. The radius of the Einstein ring is given by

$$R_E^2 = \frac{4Gm}{c^2}(1/D_d - 1/D_s); \quad (2)$$

where G is the gravitational constant, m is the mass of the MACHO, and c is the velocity of light. The parallax π and distance D of the star and the dark body are related as $\pi_s = R_U/D_s$ and $\pi_d = R_U/D_d$ where R_U is the astronomical unit. Equations (1) and (2) are adapted from Paczynski (1986) using the tangential plane on the sky as more suited for an astrometric discussion than a

plane through the deflector, used by Paczynski. As a consequence, the unit of r and R_E is here radian instead of meter.

Equation (1) has two solutions corresponding to the position of two images:

$$r_{+,-} = [r \pm (r^2 + 4R_E^2)^{1/2}]/2. \quad (3)$$

Their amplifications are given by (Vietri & Ostriker 1983)

$$A_{+,-} = \text{abs} \left(\frac{r_{+,-}^4}{r_{+,-}^4 - R_E^4} \right), \quad (4)$$

and their combined amplification is

$$A \equiv A_+ + A_- = \frac{u^2 + 2}{u(u^2 + 4)^{1/2}}, \quad u \equiv \frac{r}{R_E}. \quad (5a)$$

The inverse formula is accordingly

$$u^2 = 2 \left(\frac{A}{\sqrt{A^2 - 1}} - 1 \right). \quad (5b)$$

The position of the combined image of center-of-light, relative to m , is $\mathbf{r} + \mathbf{s}$ where the astrometric deflection is

$$\mathbf{s} = \frac{A_+ \mathbf{r}_+ + A_- \mathbf{r}_-}{A_+ + A_-} = R_E \mathbf{u}/(u^2 + 2), \quad (6)$$

and r and s are the absolute values of the vectors $\mathbf{r}$ and $\mathbf{s}$, cf. Fig. 1.

The undeflected star is moving with the proper motion μ relative to m , disregarding at first the influence of parallax. We consider a lensing event detected in a set of photometric observations of a given star. Let the amplifications A_i be observed at the times t_i , $i = 1, 2, 3 \dots$. The corresponding values u_i are derived from Eq. (5b).

A photometric event is completely characterized by the three photometric parameters t_m , u_m , t_E related to the geometry of the event. t_m is the clock time or epoch of minimum distance r_m in Fig. 1;

$$u_m \equiv r_m/R_E \quad (7)$$

is the impact parameter, i.e. the value of u at time t_m ; and the characteristic time is

$$t_E \equiv R_E/\mu. \quad (8)$$

Therefore, three observed amplifications are generally sufficient to determine the three parameters, but in practice a least squares solution is required if more observations are available.

The observation equation is derived from a rectangular triangle in Fig. 1, assuming a relative motion linear with time, i.e. if we neglect the differential parallax. Thus,

$$(t_i - t_m)^2/t_E^2 + u_m^2 = u_i^2, \quad i = 1, 2, 3. \quad (9)$$

Fig. 1. Motion of a star and its images, relative to a dark body at m . R_E is the radius of the Einstein circle. The relative proper motion of the undeflected star is μ . The astrometric deflection is the vector s . The photometric amplifications for the impact parameter $r_m/R_E = 0.5$ are given by the numbers A_+, A_-, A where A corresponds to the center-of-light of the two relativistic images.

Having derived the three photometric parameters we can draw a figure similar to Fig. 1 for the particular event. But three parameters are still unknown and can only be determined by astrometry, namely the position angle ϕ (shown in Fig. 1) of the proper motion; the scale defined by R_E in radians; and a sign q , defining whether the position angle of s increases.

$$q = \text{sign}(d\psi/dt); \quad (10)$$

where the angle ψ is thus defined by

$$\tan \psi = q(t - t_m)/(u_m t_E) \equiv q(t - t_m)\mu/r_m. \quad (11)$$

In principle, a two-dimensional measurement of one vector s would determine ϕ and R_E , but not the ambiguity q . ROEMER can only provide one-dimensional measurements, of the projections of s on the scan direction, therefore three ROEMER observations, e.g. at the above times t_i would in principle suffice to determine the parameters. The relative motion μ is obtained from Eq. (8), and the proper motion vector is

$$\mu \equiv (\mu_\alpha \cos \delta, \mu_\delta) = \mu(\sin \phi, \cos \phi). \quad (12)$$

The basic astrometric parameters are the two components of μ , or μ and ϕ , while R_E is a parameter derived by Eq. (8).

This geometric solution method would in practice be replaced by a least-squares solution of all available astrometric observations; with proper weighting, resulting in a

determination of the astrometric parameters of the star and the deflection event.

So far the discussion has neglected parallax for reasons of simplicity and because it is a good approximation if the difference in parallax between the MACHO and the star is much smaller than R_E . This is true for cases of astrometric interest. The absolute position of the undeflected star in the tangential plane can be written

$$\mathbf{r}_s = \mathbf{r}_{sI} + \mu_s(t - t_I) + \pi_s \mathbf{P}(t); \quad (13)$$

where $\mathbf{r}_{sI}$ is the position at an initial epoch t_I , μ_s and π_s are the proper motion and parallax, and $\mathbf{P}(t)$ contains the annual parallax factors as function of time. The five astrometric parameters for the star are the scalar components of position, proper motion and parallax. They should be determined from all available astrometric observations from space and from ground. By inclusion of ground-based observations a longer interval of time will be covered than possible from space. This would greatly help to decide whether an observed curvature in the star's path is due to lensing or to an orbiting companion since the latter would persist and lead to large position deviations, proportional to the square of time.

The parallax factors are in tangential coordinates approximately, for right ascension and declination, respectively (van de Kamp 1967)

$$\begin{aligned} P_\alpha &= (\cos \epsilon \cos \alpha \sin l - \sin \alpha \cos l)R/R_U, \\ P_\delta &= [(\sin \epsilon \cos \delta - \cos \epsilon \sin \alpha \sin \delta) \sin l \\ &\quad - \cos \alpha \sin \delta \cos l]R/R_U; \end{aligned}$$

where the radius vector of the sun is $R \simeq 1.0R_U$, the obliquity of the ecliptic is $\epsilon = 23^\circ 27'$, and the ecliptic longitude of the sun is

$$l = 360^\circ(t + 284)/365,$$

t is the clock time in days measured from the beginning of a year.

The absolute position of the combined image is $\mathbf{r}_c$. The position of this image relative to the undeflected star is therefore

$$\mathbf{s} \equiv \mathbf{r}_c - \mathbf{r}_s = \mathbf{r}/(u^2 + 2). \quad (14)$$

s and therefore $\mathbf{r}$ are astrometrically observable quantities because $\mathbf{r}_s$ is available from (13).

The absolute position of the dark body is written similar to (13) as

$$\mathbf{r}_d = \mathbf{r}_{dI} + \mu_d(t - t_I) + \pi_d \mathbf{P}(t). \quad (15)$$

Since $\mathbf{r} \equiv \mathbf{r}_s - \mathbf{r}_d$ we have

$$\mathbf{r} = \mathbf{r}_{sI} - \mathbf{r}_{dI} + (\mu_s - \mu_d)(t - t_I) + (\pi_s - \pi_d)\mathbf{P}(t), \quad (16)$$

and therefore by Eq. (14)

$$\mathbf{s} = (\mathbf{r}_m + \mu(t - t_m) - \pi_{sd}\mathbf{P}(t))/(u^2 + 2); \quad (17)$$

where the relative parallax π_{sd} is defined as a *positive* quantity

$$\pi_{sd} \equiv -(\pi_s - \pi_d). \quad (18)$$

The influence of differential parallax may be taken into account in Eq. (9) by replacing each u_i with a corrected value $u + \pi_{sd} \mathbf{P}(t) \cdot \mathbf{r} / (r R_E)$, obtained from the definition $u = r/R_E$ and Eq. (16). It can be seen from the values of D_s, D_d, R_E in Tables 1 and 2 that the correction is $\ll 1$ for $m \gtrsim 1 M_\odot$ so that it can be neglected in a first approximation. But even for somewhat larger masses, $m \lesssim 10 M_\odot$, the parallactic shift changes so little during the observable part of the event that a simpler picture of the parallactic effect is adequate. The earth's orbital velocity of 30 km/s modifies the observed proper motion by a nearly constant and relatively small amount since this velocity is much smaller than the assumed typical tangential velocity of 200 km/s for a MACHO relative to the star.

The proper motion of the star relative to the dark body is

$$\boldsymbol{\mu} \equiv \boldsymbol{\mu}_s - \boldsymbol{\mu}_d, \quad (19)$$

and similarly, the relative position is

$$\mathbf{r}_m \equiv \mathbf{r}_{sl} - \mathbf{r}_{dl} + \boldsymbol{\mu}(t_m - t_1). \quad (20)$$

We derive from Fig. 1 that

$$\mathbf{r}_m = u_m \mu t_E (-\cos \phi, \sin \phi). \quad (21)$$

Since $\mathbf{s}$ is in principle observable by astrometric methods it follows from Eq. (17) that there are three independent relative astrometric parameters μ, ϕ, π_{sd} to be observed. The relative position $\mathbf{r}_m$ is not an independent parameter but given by Eq. (20). The absolute astrometric parameters $\mathbf{r}_{dl}, \boldsymbol{\mu}_d, \pi_d$ of the MACHO in the astrometric reference system can finally be obtained from Eqs. (20), (19) and (18).

In conclusion, the complete description of a MACHO at epoch t_1 requires seven physical parameters: $(m, \alpha, \delta, D, \mu_\alpha, \mu_\delta, dD/dt; t_1)$. The radial velocity dD/dt is not observable, but the remaining six parameters can in principle be derived from three photometric and three one-dimensional astrometric measurements of a lensing event, provided the position, motion and distance of the undeflected star has been determined from observations outside the event.

3. Observation

The ROEMER mission is briefly described in Sect. 3.1. The conditions for MACHO astrometry are derived in Sect. 3.2, resulting in Table 1 with values for the precision that could be obtained with given assumptions on stellar magnitudes and MACHO masses. Section 3.3 describes the realistic assumptions made on the distribution of stars, and Table 2 gives the number of detectable lensing events and MACHOs by a 5 year ROEMER mission.

3.1. The ROEMER mission

The ROEMER mission is (presently) designed to monitor 10^8 stars on the whole sky down to a limit about $V = 16.5$ (Lindgren et al. 1993; Høg & Lindgren 1993; Høg 1994). We shall here assume a mission duration of 5 years. Photometric and astrometric measurements are obtained in a wide spectral band and through six (or eight) narrow band filters. Considering only the wide-band observations because of the higher signal-to-noise ratio (SNR), each star obtains about 500 elementary observations. Many of these 50 milliard observations would be subject to a significant photometric amplification with a SNR larger than 10. An amplified star will typically have visual magnitude $V=16$ and be located at a distance of 5 kpc, as we shall see in Sect. 3.3.

The elementary measurements are obtained by drift-scan on CCDs each lasting about 4 s per star and giving a one-dimensional astrometric component of the star position and a photometric value. About eight elementary measurements are obtained within a few hours and are averaged for the present purpose. This is possible because the scanning during a few hours goes nearly along the same great circle on which the one-dimensional astrometric components are therefore conveniently projected and averaged, giving a so-called 'abscissa'. The elementary measurements are important for check of internal consistency if a deflection event is detected from the averages, but MACHO masses so small as to produce variations on time scales of hours cannot produce measurable astrometric effects, so that these elementary observations need no further mentioning here. The photometric precision of elementary observations is sufficient to detect photometric amplifications, but this is of limited usefulness because each event would not be well covered with observations, as is required for determining the value of t_E .

The 100 elementary measurements per year result in about 12 abscissae per star. These are far from equidistant in time, but it is adequate for the present first overview of the possibilities to assume for simplification that they are in fact separated by $\Delta t = 30$ days. We can also assume that the positions angles are changing considerably between consecutive abscissae so that two-dimensional positions can be derived. The astrometric precision of an abscissa due to photon noise is given by

$$\sigma_a = \sigma_{a0} 10^{0.2(V-11)}, \quad V \geq 11 \text{ mag}; \quad (22)$$

where $\sigma_{a0} = 0.20$ mas. This error is quite realistic for the interesting magnitude interval $V = 14 - 16$ mag, whereas brighter stars may be affected by an additional asymptotic error due to other sources than photon noise, and fainter stars will be affected by an additional noise due to background.

The precision of an average photometric measurement belonging to an abscissa measurement is given by

$$\sigma_{ph} = \sigma_{ph0} 10^{0.2(V-11)}, \quad V \geq 11 \text{ mag}; \quad (23)$$

where $\sigma_{\text{ph0}} = 0.001 \text{ mag}$.

3.2. Astrometric observations

A deflection event with impact parameter $u_m = 1$ may be considered as typical among the detectable events. It gives rise to a maximum amplification $A_{\text{max}} = 1.342$ and a maximum deflection of $s_{\text{max}} = 0.354 R_E$, according to Eqs. (5) and (6). The deflection decreases to half its maximum at $u \simeq 5$. We can therefore for simplicity assume that observations outside a time interval of $10t_E$ centered on the event contribute nothing to the precision of the relative proper motion μ . Since at least three observations are required to separate the parameters it follows that the condition

$$10t_E > 2\Delta t \quad (24)$$

should be satisfied before μ can be determined. This and the following condition, and the error propagation law used for Table 1 are tentative and have been derived from simple simulations.

The corresponding condition for determination of the relative parallax π_{sd} is always much more stringent because the deflection must persist during most of a year. The adopted condition is:

$$4t_E > 270 \text{ days.} \quad (25)$$

A significant determination of either quantity requires in addition that the relative standard errors

$$F_\mu \equiv \sigma_\mu / \mu, \quad F_\pi \equiv \sigma_{\pi_{\text{sd}}} / \pi_{\text{sd}} \quad (26)$$

are much smaller than unity. A limit of $F < 0.3$ should be adopted, corresponding to a $\text{SNR} \gtrsim 3$.

Fig. 2. Amplification A as function of time for the impact parameter = 1.0. Photometric observations at the dots are spaced at $\Delta t = 30$ days, as expected for ROEMER abscissa measurements. The curves correspond to the specified values of t_E , e.g. as expected for MACHOs of 0.1, 1, $10M_\odot$ when observed at stars of $V = 12 \text{ mag}$, cf. Table 2.

Figure 2 illustrates the photometric amplification for three MACHO masses.

Table 1 shows the possibilities of MACHO astrometry by relevant values for five V magnitudes of the observed star and for four MACHO masses $m = 100, 10, 1, 0.1 M_\odot$. The typical distance to the dark body is D_d , calculated

as the median of the distance to the star by means of Eqs. (27) and (30). The relative proper motion μ is calculated from an assumed tangential velocity of 200 km/s. The characteristic time and the Einstein radius of the event are given. The relative standard errors are given as expected from observations of ROEMER precision; a minus (–) indicates that t_E is too small to satisfy Eqs. (24) and (25). Finally, the last columns contain 20 times smaller standard errors than the two previous columns as would result if each observation had 20 times smaller errors than given by Eq. (22). Such performance may be achieved some time in future.

3.3. Photometric amplifications

The number of observed MACHOs depends on the distribution of stars and MACHOs in the Galaxy and on the observing techniques. Simple formulae are given by which approximate estimates of the size and frequency of observable relativistic lensing can be obtained (Paczynski 1986; Griest 1991; Paczynski 1991; Griest et al. 1991; Kerins & Carr 1994).

We consider stars in intervals of about 1 mag apparent visual magnitude, characterized by the median magnitude V_{med} so that one half of the stars in the interval are brighter than V_{med} and the other half is fainter.

The median absolute magnitude M_{med} of stars counted to a given apparent magnitude limit is required for calculation of the median distance D_s to these stars. M_{med} is $\simeq 0$ if the limit is 6 mag, according to Allen (1973, p.244). Another extreme estimate is obtained from star counts at high galactic latitudes (Gilmore 1989, p.21) where the colour index at 15 mag and fainter corresponds to the main-sequence turnoff of an old, metal-poor population. Such a population has $M_{\text{med}} = 4.0$. We shall make the intermediate assumption that $M_{\text{med}} = 2.0$ for the observations obtained by ROEMER.

Given the median magnitude we assume that the length of space D_s monitored by the stars in the interval is equal to the distance of stars of absolute magnitude M_{med} seen at galactic latitude $b = 50 \text{ deg}$. This is a pessimistic assumption since the mean star density of the sky is rather equal to the density at $b = 20 \text{ deg}$ for $V = 15 \text{ mag}$. This is assumed to balance the too optimistic assumption about freedom for interstellar absorption.

This results in the following formulae for D_s and the number of monitored stars N_s in an interval of 1 mag

$$D_s = 10 \cdot 10^{0.2(V_{\text{med}} - M_{\text{med}})} [\text{pc}], \quad (27)$$

and

$$N_s = 41253 \cdot 10^{2.35+0.3(V_{\text{med}}-15)}. \quad (28)$$

This formula for N_s corresponds to an average density of stars on the whole sky as the density at $b = 50 \text{ deg}$ for the magnitude $V = 15 \text{ mag}$, which is representative for the relevant ROEMER observations.

Table 1. Characteristic values for MACHO astrometry as function of median visual magnitude of the stars and for four hypothetical assumptions of MACHO masses.

Star <i>V</i> mag	MACHO			Event		ROEMER		20 × smaller σ	
	<i>m</i> $M_\odot$	D_d pc	μ mas/yr	t_E day	R_E mas	σ_μ/μ 1	σ_π/π 1	σ_μ/μ 1	σ_π/π 1
12	100.0	790	53	100	14.5	0.023	> 1	0.001	0.14
13	100.0	1260	34	126	11.5	0.041	> 1	0.002	0.30
14	100.0	1990	21	158	9.2	0.073	> 1	0.004	0.68
15	100.0	3160	13	199	7.3	0.130	> 1	0.007	> 1
16	100.0	5010	8	251	5.8	0.232	> 1	0.012	> 1
12	10.0	790	53	32	4.6	0.130	—	0.007	—
13	10.0	1260	34	40	3.6	0.232	—	0.012	—
14	10.0	1990	21	50	2.9	0.412	—	0.021	—
15	10.0	3160	13	63	2.3	0.732	—	0.037	—
16	10.0	5010	8	79	1.8	> 1	> 1	0.065	> 1
12	1.0	790	53	10	1.5	0.732	—	0.037	—
13	1.0	1260	34	13	1.2	> 1	—	0.065	—
14	1.0	1990	21	16	0.9	> 1	—	0.116	—
15	1.0	3160	13	20	0.7	> 1	—	0.206	—
16	1.0	5010	8	25	0.6	> 1	—	0.366	—
12	0.1	790	53	3	0.5	—	—	—	—
13	0.1	1260	34	4	0.4	—	—	—	—
14	0.1	1990	21	5	0.3	—	—	—	—
15	0.1	3160	13	6	0.2	> 1	—	> 1	—
16	0.1	5010	8	8	0.2	> 1	—	> 1	—

The dark matter is here characterized by R_H , the radius of the galactic halo of MACHOs, ρ_H , the spatial density of this halo, v_T , the (constant) tangential velocity of MACHOs (or, more precisely, of the MACHOs relative to the line of sight to the stars), and m , the mass of a MACHO. We assume the values $R_H = 15000$ pc, $\rho_H = 0.5 \cdot 10^{-24}$ g cm $^{-3}$, corresponding to a total mass of the Halo = $10^{11} M_\odot$. Furthermore, $v_T = 200$ km/s and m is a free parameter. Thus, ρ_H, v_T, m are assumed to be constant.

The optical depth τ of the lensing matter is obtained from Paczynski (1986, Eq. (9)). In our discussion the distance to stars is only a few kpc so that we can simply assume that ρ_H is constant along the line of sight and accordingly write

$$\tau = \frac{4\pi G D_{e,med}}{c^2} \rho_H D_s; \quad (29)$$

where $\pi = 3.141 \dots$. The effective distance at the median distance of a MACHO $D_{e,med}$, is obtained as follows. The effective distance of a MACHO is defined by

$$D_e \equiv \frac{D_d D_{ds}}{D_s} = D_d^2 (1/D_d - 1/D_s). \quad (30a)$$

The median distance of a MACHO observed at a star at the distance D_s is

$$D_{d,med} = 2^{-1/3} D_s \quad (30b)$$

since the space density of MACHOs is assumed to be constant. The effective distance of a MACHO at this distance is then obtained from (30a) as $D_{e,med} = 0.1637 D_s$.

The probability that a given star at a given epoch has an amplification larger than a threshold $A_T = 1.342$ is τ . The number of observed photometric amplifications N_{amp} where a star is inside one Einstein radius of a MACHO, with a given observing program is accordingly

$$N_{amp} = \tau n_{obs} N_s \quad (31)$$

where n_{obs} is the number of observations per star and N_s is the number of stars in the program. $n_{obs} = 60$ abscissa observations per star are obtained during 5 years.

We note that N_{amp} is independent of m if the integration time of an observation is short compared with the characteristic time t_E , and this is the case for abscissa observations with ROEMER if $m > 0.001 M_\odot$, cf. Table 1.

If Δt is the typical interval between observations of the same star the number of events, i.e., the number of

Table 2. Characteristic values as function of median visual magnitude of the stars. N_s is the number of observed stars per magnitude interval, and D_s their median distance. N_{amp} is the number of amplifications larger than 1.342, detected by a ROEMER mission of 5 years. N_{d1} is the number of detected MACHOs if the Halo would consist entirely of MACHOs of $m = 1M_\odot$. For other masses N_d is obtained by Eq. (32).

V_{med} mag	D_s pc	N_s 1	N_{amp} 1	SNR_T 1	t_E day	N_{d1} 1
12	1000	1162668	0.05	202	10.0	0.08
13	1585	2319827	0.25	127	12.6	0.30
14	2512	4628663	1.26	80	15.8	1.19
15	3981	9235396	6.31	51	19.9	4.75
16	6310	18427038	31.60	32	25.1	18.90

observed dark bodies N_d , is approximately

$$N_d = N_{\text{amp}} \frac{\Delta t}{2t_E} = N_{\text{d1}} (m/M_\odot)^{-0.5} \quad (32)$$

since the number of observations per MACHO with amplification larger than A_T is $2t_E/\Delta t$. The number N_{d1} is calculated in Table 2.

This table gives characteristic values as function of magnitude. The signal-to-noise ratio SNR_T of a photometric amplification at the threshold A_T is calculated by means of Eq. (23) and appears to be large enough for detection at stars much fainter than $V = 16$. Thus, a large number of photometric amplifications would be detected by a ROEMER mission, and about 40 of these occur for stars of $V < 16.5$. The first five columns do not depend on m , and the last two columns are calculated for $m = 1M_\odot$. The amplification at a distance of $u = 2.0$ is $A = 1.061$ and can therefore be well measured, given the high values of SNR_T in Table 2. This is required for the determination of u from Eq. (5b), but it is not counted in our estimates of N_{amp} .

It appears from the last column that about 24 MACHOs of this mass could be detected down to $V = 16.5$. The probability is 0.08 for a detection at $V = 12$.

The photometric amplifications discussed here are obtained at the abscissa level, i.e. as average of 8 elementary observations. The $\text{SNR}_T \simeq 32$ at $V = 16$ means that each elementary observation has a $\text{SNR} \simeq 10$ and would be easily recognized. Therefore, at least 300 amplifications would be measured in a ROEMER mission.

4. Discussion of MACHO astrometry

The parallax of a MACHO can only be determined if the deflection has sufficient duration, cf. Eq. (25) and Table 1. It appears that $m \gtrsim 10M_\odot$ is required.

A MACHO will remain very close to the star where it was detected, but its image might still be visible in some other wavelength than the visible where the brightness difference of MACHO and star may be less extreme. Therefore the position of a MACHO could be of interest and is obtained from Eq. (20) if the proper motion is known.

Direct determination of the proper motion of a MACHO would be of astrophysical interest. A statistically reasonable distance $D_{\text{d,med}}$ could be obtained from Eq. (30b). An observed tangential velocity v_T in [km/s] would then be available, a quantity otherwise derived from a model of the halo. With the observed R_E from Eq. (8) the mass m could be derived from Eq. (2).

Such direct determination of velocity and mass of a MACHO requires the detection of a lensing event and at least three photometric and three one-dimensional astrometric measurements of the event. Less than three observations of each kind could still be of astrophysical value in connection with a statistical analysis of many lensing events, cf. Sect. 5. Detection of an event from photometric observations is by far simpler than the, practically impossible, detection alone from astrometric observations. This is illustrated in Sect. 3.3 by the ROEMER mission by which it would be possible in 5 years to detect about 40 photometric amplifications at the abscissa level at stars brighter than $V = 16.5$ from MACHO's of any mass between 0.001 and 100 solar masses. The lower mass limit given corresponds to $t_E \simeq 1$ day. Astrometric analysis can be limited to those stars, and according to Table 1 a significant proper motion would for instance be expected if the star is brighter than $V = 14$ and if all MACHOs would have about 10 solar masses or more, for which however no positive evidence exists.

Considering masses of $m = 1M_\odot$ Table 1 shows that proper motions could not be determined by ROEMER. A satellite giving observational errors 20 times smaller would give a significant relative error of $\sigma_\mu/\mu \lesssim 0.4$ at $V = 16$ mag. Table 2 shows that a total of 24 MACHOs could be observed in a 5 year mission if all MACHOs would have about $m = 1M_\odot$, for which no positive evidence exists.

In conclusion, the prospects for MACHO astrometry have been quantitatively estimated and found to be meager if all MACHOs have masses of only $0.1M_\odot$ as those first discovered.

5. Local space density of MACHOs

The ROEMER observations of microlensing are related to MACHOs much closer to the sun than observations of stars in the LMC and can therefore provide information on the local space density of MACHOs. A set of observations with known degree of completeness is required as basis for a statistical analysis.

In such analysis use is made of Eq. (2), noting that the bracket is statistically well enough known. D_s is obtained as a photometric distance to the star while a statistical

value of $D_{d,med}$ is obtained from the assumption that the space density of MACHOs is constant in all directions.

A set of photometric amplifications, without usable astrometry, could provide an estimate of mr^{-2} for each event. Equation (5b) gives u and elimination of R_E with (2) gives the quantity mentioned. A statistical analysis of a set of such values can give an estimate of the column mass density of the dark Halo between the sun and the observed stars.

A set of simultaneous photometric and astrometric ROEMER observations of lensing events can be used to derive statistical values of the mass and space density of MACHOs, even if only one observation of each event is available with significant precision. From the observed amplification the value of u is obtained from Eq. (5). A projection of the astrometric deflection s may sometimes be derived in which case a statistical value of R_E is obtained from Eq. (6). A statistical value of m is then derived from Eq. (2).

Given the number and the degree of completeness of these observations it is then possible to derive the local number density and the space density of MACHOs. This estimate of the local space density is not dependent on assumptions of tangential velocity of MACHOs and the total mass and radius of the dark Halo, as required if the local space density is obtained from photometric observations of stars far outside the Halo as e.g. in the LMC.

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