MACRO PHOTOMETRY AND ASTROMETRY

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MACHO photometry and astrometry

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Abstract. MACHOs have been discovered by their relativistic amplification of light from distant stars as they crossed very near to the line-of-sight. The very few events were detected from more than a milliard photometric measurements of millions of stars in the LMC. - A mathematical theory of analysis of astrometric and photometric measurements of microlensing events is presented. It is shown that three photometric measurements and three one-dimensional astrometric measurements during an event are, in principle, sufficient to determine the - precisely - six observable physical parameters of the MACHO, including the proper motion and distance of the dark body, provided the position, motion and distance of the undeflected star has been determined from observations outside the event. The practical possibility of such observations is discussed by comparison with the proposed ROEMER satellite, the only known instrument concept with the potential for a sufficient number of photometric and astrometric observations of the required quality. At least 300 photometric amplifications of light from stars brighter than $V = 16.5$ mag could be detected during a ROEMER mission, but the prospects for significant astrometric observations are meager.

Key words: astrometry - gravitational lensing - artificial satellites

\begin{quote}

1. Introduction

The amplification of light from stars, sometimes called 'microlensing', was proposed by Paczynski (1986) as a means to detect Massive Astrophysical Compact Halo Objects (MACHOs). This prediction of photometric lensing events in the light from stars in the large Magellanic Cloud (LMC) was confirmed by Alcock et al. (1993) and Aubourg et al. (1993) through their detection of three events, probably caused by MACHOs of about 0.1 solar masses as they passed within less than one milliarcsecond (mas) of the line-of-sight to a star. A photometric event is fully characterized by the three parameters: the epoch $t_m$ at which the event occurs, the maximum amplification $A_m$, and the characteristic time or duration $t_E$.

The parameter $A_m$ may be replaced by the impact parameter $u_m$ with a more direct physical meaning, as defined below. We show that the three photometric parameters $t_m$, $u_m$, $t_E$ may in principle be derived from just three observed photometric amplifications of an event.

A MACHO is assumed to be a compact, point-like and massive object, fully characterized at a given initial epoch $t_i$ near an event by six physical parameters: mass $m$, distance $D$ or parallax $\pi$, two position coordinates $\alpha, \delta$ and the proper motion $\mu_\alpha, \mu_\delta$. They can under optimal conditions be derived from observations of an encounter event with a star for which the position and proper motion are known. A seventh parameter, and in fact the only remaining physical MACHO parameter, the radial velocity, could be obtained from the two distances to a MACHO observed at two events, but such an occurrence is too improbable to be ever observed and hence does not warrant discussion.

It will be shown that the six physical MACHO parameters (mass, distance, position and proper motion at epoch $t_i$) can in principle be derived from three photometric and three one-dimensional astrometric observations of an event.

The six observable MACHO parameters could, alternatively, be derived from (six) astrometric observations, without use of photometry. But the result would be very much less precise, and it would be a waste of the photometric measurements if they were not used since they are obtained simultaneously with the astrometric observations by any conceivable modern techniques, so this alternative need no further discussion.
\end{quote}
The recommended strategy is to perform milliards of photometric and astrometric observations of stars and to detect any photometric lensing events, alone by analysis of the photometry, deriving the three photometric parameters for each event. The astrometry of the (few) detected events are then used to obtain up to three astrometric parameters of the event: the relative proper motion and parallax, $\mu, \pi_{\text{ad}}$, namely of the star relative to the dark body, see Sect. 2. Finally, the six physical MACHO parameters may be derived by means of the five astrometric parameters of the star: position, proper motion and distance.

For most photometric events no significant astrometric parameters can be derived because the star is too faint and/or the MACHO mass is too small. It is concluded in Sect. 4 that the proposed ROEMER satellite would not give sufficient precision to determine the proper motion of MACHOs. But with 20 times smaller observation errors than ROEMER a significant number of proper motions could be obtained if a large part of MACHOs were of $1M_\odot$, for which no evidence however exists.

Without proper motion data the mass $m$ derived from a photometric event must be based on a tangential velocity from a dynamical model of the dark halo and on a hypothetical distance to the MACHO, mainly inferred from the fact that the MACHO must be closer to us than the star.

It is noted in Sect. 5 that photometric events alone, observed at Galactic stars at closer distances than the LMC will provide statistical values of distances to MACHOs. Also the distribution of masses can be derived by suitable model assumptions. A ROEMER mission could give an interesting contribution to the study of the dark halo by its milliards of photometric observations.

2. Theory of astrometric lensing

The equation of gravitational lensing may be written in the tangential plane on the observer's sky, cf. Fig. 1, as

$$r_\pi^2 - r r_\pi - R_E^2 = 0;$$

where the coordinate system is centered on the lensing point mass $m$, the source is at the position $r$ and the image is at $r_\pi$, along the same vector. The radius of the Einstein ring is given by

$$R_E^2 = \frac{4GM}{c^2} (1/D_d - 1/D_a);$$

where $G$ is the gravitational constant, $m$ is the mass of the MACHO, and $c$ is the velocity of light. The parallax $\pi$ and distance $D$ of the star and the dark body are related as $\pi = R_U/D_d$ and $\pi = R_U/D_d$ where $R_U$ is the astronomical unit. Equations (1) and (2) are adapted from Paczynski (1986) using the tangential plane on the sky as more suited for an astrometric discussion than a plane through the deflector, used by Paczynski. As a consequence, the unit of $r$ and $R_E$ is here radian instead of meter.

Equation (1) has two solutions corresponding to the position of two images:

$$r_{+, -} = [r \pm (r^2 + 4R_E^2)^{1/2}] / 2.$$  

Their amplifications are given by (Vietri & Ostriker 1983)

$$A_{+, -} = \left| \frac{r_{+, -}^2}{r_{+, -}^2 - R_E^2} \right|,$$

and their combined amplification is

$$A = A_+ + A_- = \frac{u^2 + 2}{u(u^2 + 4)^{1/2}}, \quad u \equiv \frac{r}{R_E}.\quad (5a)$$

The inverse formula is accordingly

$$u^2 = 2 \left( \frac{A}{\sqrt{A^2 - 1}} - 1 \right).\quad (5b)$$

The position of the combined image of center-of-light, relative to $m$, is $r + s$ where the astrometric deflection is

$$s = \frac{A_+ r_+ + A_- r_-}{A_+ + A_-} = R_E \frac{u}{(u^2 + 2)},\quad (6)$$

and $r$ and $s$ are the absolute values of the vectors $r$ and $s$, cf. Fig. 1.

The undeflected star is moving with the proper motion $\mu$ relative to $m$, disregarding at first the influence of parallax. We consider a lensing event detected in a set of photometric observations of a given star. Let the amplifications $A_i$ be observed at the times $t_i, i = 1, 2, 3 \ldots$. The corresponding values $u_i$ are derived from Eq. (5b).

A photometric event is completely characterized by the three photometric parameters $t_m, u_m, R_E$ related to the geometry of the event. $t_m$ is the clock time or epoch of minimum distance $r_m$ in Fig. 1;

$$u_m = \frac{r_m}{R_E},\quad (7)$$

is the impact parameter, i.e. the value of $u$ at time $t_m$; and the characteristic time is

$$t_E \equiv R_E / \mu.\quad (8)$$

Therefore, three observed amplifications are generally sufficient to determine the three parameters, but in practice a least squares solution is required if more observations are available.

The observation equation is derived from a rectangular triangle in Fig. 1, assuming a relative motion linear with time, i.e. if we neglect the differential parallax. Thus,

$$\left( t_i - t_m \right)^2 / t_E^2 + u_m^2 = u_i^2, \quad i = 1, 2, 3.\quad (9)$$
Fig. 1. Motion of a star and its images, relative to a dark body at m. \( R_\text{E} \) is the radius of the Einstein circle. The relative proper motion of the undeflected star is \( \mu \). The astrometric deflection is the vector \( s \). The photometric amplifications for the impact parameter \( r_m/R_\text{E} = 0.5 \) are given by the numbers \( A_+, A_-, A \) where \( A \) corresponds to the center-of-light of the two relativistic images.

Having derived the three photometric parameters we can draw a figure similar to Fig. 1 for the particular event. But three parameters are still unknown and can only be determined by astrometry, namely the position angle \( \phi \) (shown in Fig. 1) of the proper motion; the scale defined by \( R_\text{E} \) in radians; and a sign \( q \), defining whether the position angle of \( s \) increases.

\[
q = \text{sign}(d\psi / dt);
\]

(10)

where the angle \( \psi \) is thus defined by

\[
\tan \psi = q(t-t_m)/(u_m t_E) \equiv q(t-t_m)\mu/r_m.
\]

(11)

In principle, a two-dimensional measurement of one vector \( s \) would determine \( \phi \) and \( R_\text{E} \), but not the ambiguity \( q \). ROEMER can only provide one-dimensional measurements, of the projections of \( s \) on the scan direction, therefore three ROEMER observations, e.g. at the above times \( t_i \) would in principle suffice to determine the parameters. The relative motion \( \mu \) is obtained from Eq. (8), and the proper motion vector is

\[
\mu \equiv (\mu_\alpha \cos \delta, \mu_\delta) = \mu (\sin \phi, \cos \phi).
\]

(12)

The basic astrometric parameters are the two components of \( \mu \), or \( \mu \) and \( \phi \), while \( R_\text{E} \) is a parameter derived by Eq. (8).

This geometric solution method would in practice be replaced by a least-squares solution of all available astrometric observations, with proper weighting, resulting in a determination of the astrometric parameters of the star and the deflection event.

So far the discussion has neglected parallax for reasons of simplicity and because it is a good approximation if the difference in parallax between the MACHO and the star is much smaller than \( R_\text{E} \). This is true for cases of astrometric interest. The absolute position of the undeflected star in the tangential plane can be written

\[
r_s = r_{\text{sl}} + \mu_s(t-t_1) + \pi_s P(t);
\]

(13)

where \( r_{\text{sl}} \) is the position at an initial epoch \( t_1 \), \( \mu_s \) and \( \pi_s \) are the proper motion and parallax, and \( P(t) \) contains the annual parallax factors as function of time. The five astrometric parameters for the star are the scalar components of position, proper motion and parallax. They should be determined from all available astrometric observations from space and from ground. By inclusion of ground-based observations a longer interval of time will be covered than possible from space. This would greatly help to decide whether an observed curvature in the star’s path is due to lensing or to an orbiting companion since the latter would persist and lead to large position deviations, proportional to the square of time.

The parallax factors are in tangential coordinates approximately, for right ascension and declination, respectively (van de Kamp 1967)

\[
\begin{align*}
P_\alpha & = (\cos \epsilon \cos \alpha \sin l - \sin \alpha \cos l) R/R_\text{Ul}, \\
P_\delta & = [(\sin \epsilon \cos \delta - \cos \epsilon \sin \alpha \sin \delta) \sin l \\
& - \cos \alpha \sin \delta \cos l] R/R_\text{U};
\end{align*}
\]

where the radius vector of the sun is \( R \approx 1.0 R_\odot \), the obliquity of the ecliptic is \( \epsilon = 23^\circ 27' \), and the ecliptic longitude of the sun is

\[
l = 360^\circ (t + 284)/365,
\]

\( t \) is the clock time in days measured from the beginning of a year.

The absolute position of the combined image is \( r_s \). The position of this image relative to the undeflected star is therefore

\[
s \equiv r_c - r_s = r/(u^2 + 2),
\]

(14)

\( s \) and therefore \( r \) are astrometrically observable quantities because \( r_s \) is available from (13).

The absolute position of the dark body is written similar to (13) as

\[
r_d = r_{\text{sl}} + \mu_d(t-t_1) + \pi_d P(t).
\]

(15)

Since \( r \equiv r_s - r_d \) we have

\[
r = r_{\text{sl}} - r_{\text{dl}} + (\mu_s - \mu_d)(t-t_1) + (\pi_s - \pi_d) P(t),
\]

(16)

and therefore by Eq. (14)

\[
s = (r_m + \mu(t-t_m) - \pi_{\text{sl}} P(t))/(u^2 + 2);
\]

(17)
where the relative parallax \( \pi_{rd} \) is defined as a positive quantity

\[
\pi_{rd} \equiv - (\pi_s - \pi_d).
\]  

(18)

The influence of differential parallax may be taken into account in Eq. (9) by replacing each \( u_i \) with a corrected value \( u + \pi_{rd} P(t) \cdot r/(rR_E) \), obtained from the definition \( u = r/R_E \) and Eq. (16). It can be seen from the values of \( D_s, D_d, R_E \) in Tables 1 and 2 that the correction is \( \ll 1 \) for \( m \lesssim 10 M_\odot \) so that it can be neglected in a first approximation. But even for somewhat larger masses, \( m \lesssim 10 M_\odot \), the parallactic shift changes so little during the observable part of the event that a simpler picture of the parallactic effect is adequate. The earth's orbital velocity of 30 km/s modifies the observed proper motion by a nearly constant and relatively small amount since this velocity is much smaller than the assumed typical tangential velocity of 200 km/s for a MACHO relative to the star.

The proper motion of the star relative to the dark body is

\[
\mu \equiv \mu_s - \mu_d, \tag{19}
\]

and similarly, the relative position is

\[
r_m \equiv r_{rd} - r_{dl} + \mu (t_m - t_1). \tag{20}
\]

We derive from Fig. 1 that

\[
r_m = u_m \mu t_E (- \cos \phi, \sin \phi). \tag{21}
\]

Since \( s \) is in principle observable by astrometric methods it follows from Eq. (17) that there are three independent relative astrometric parameters \( \mu, \phi, \pi_{rd} \) to be observed. The relative position \( r_m \) is not an independent parameter but given by Eq. (20). The absolute astrometric parameters \( r_{dl}, \mu_d, \pi_d \) of the MACHO in the astrometric reference system can finally be obtained from Eqs. (20), (19) and (18).

In conclusion, the complete description of a MACHO at epoch \( t_1 \) requires seven physical parameters: \( (m, \alpha, \delta, D_s, \mu_s, \mu_d, dD/dt; t_1) \). The radial velocity \( dD/dt \) is not observable, but the remaining six parameters can in principle be derived from three photometric and three one-dimensional astrometric measurements of a lensing event, provided the position, motion and distance of the undetected star has been determined from observations outside the event.

3. Observation

The ROEMER mission is briefly described in Sect. 3.1. The conditions for MACHO astrometry are derived in Sect. 3.2, resulting in Table 1 with values for the precision that could be obtained with given assumptions on stellar magnitudes and MACHO masses. Section 3.3 describes the realistic assumptions made on the distribution of stars, and Table 2 gives the number of detectable lensing events and MACHOs by a 5 year ROEMER mission.

3.1. The ROEMER mission

The ROEMER mission is (presently) designed to monitor \( 10^8 \) stars on the whole sky down to a limit about \( V = 16.5 \) (Lindgren et al. 1993; Høg & Lindgren 1993; Høg 1994). We shall here assume a mission duration of 5 years. Photometric and astrometric measurements are obtained in a wide spectral band and through six (or eight) narrow band filters. Considering only the wide-band observations because of the higher signal-to-noise ratio (SNR), each star obtains about 500 elementary observations. Many of these 500 milliard observations would be subject to a significant photometric amplification with a SNR larger than 10. An amplified star will typically have visual magnitude \( V = 16 \) and be located at a distance of 5 kpc, as we shall see in Sect. 3.3.

The elementary measurements are obtained by drift-scan on CCDs each lasting about 4 s per star and giving a one-dimensional astrometric component of the star position and a photometric value. About eight elementary measurements are obtained within a few hours and are averaged for the present purpose. This is possible because the scanning during a few hours goes nearly along the same great circle on which the one-dimensional astrometric components are therefore conveniently projected and averaged, giving a so-called 'abscissa'. The elementary measurements are important for check of internal consistency if a deflection event is detected from the averages, but MACHO masses so small as to produce variations on time scales of hours cannot produce measurable astrometric effects, so that these elementary observations need no further mentioning here. The photometric precision of elementary observations is sufficient to detect photometric amplifications, but this is of limited usefulness because each event would not be well covered with observations, as is required for determining the value of \( t_g \).

The 100 elementary measurements per year result in about 12 abscissae per star. These are far from equidistant in time, but it is adequate for the present first overview of the possibilities to assume for simplification that they are in fact separated by \( \Delta t = 30 \) days. We can also assume that the positions angles are changing considerably between consecutive abscissae so that two-dimensional positions can be derived. The astrometric precision of an abscissa due to photon noise is given by

\[
\sigma_a = \sigma_{a0} 10^{0.2(V-11)} \text{mag, } V \geq 11 \text{mag}; \tag{22}
\]

where \( \sigma_{a0} = 0.20 \) mas. This error is quite realistic for the interesting magnitude interval \( V = 14 - 16 \) mag, whereas brighter stars may be affected by an additional asymptotic error due to other sources than photon noise, and fainter stars will be affected by an additional noise due to background.

The precision of an average photometric measurement belonging to an abscissa measurement is given by

\[
\sigma_{ph} = \sigma_{ph0} 10^{0.2(V-11)} \text{mag, } V \geq 11 \text{mag}; \tag{23}
\]
where $\sigma_{\phi 0} = 0.001$ mag.

### 3.2. Astrometric observations

A deflection event with impact parameter $u_m = 1$ may be considered as typical among the detectable events. It gives rise to a maximum amplification $A_{\text{max}} = 1.342$ and a maximum deflection of $s_{\text{max}} = 0.354 R_E$, according to Eqs. (5) and (6). The deflection decreases to half its maximum at $u \simeq 5$. We can therefore for simplicity assume that observations outside a time interval of $10t_E$ centered on the event contribute nothing to the precision of the relative proper motion $\mu$. Since at least three observations are required to separate the parameters it follows that the condition

$$10t_E > 2\Delta t$$

should be satisfied before $\mu$ can be determined. This and the following condition, and the error propagation law used for Table 1 are tentative and have been derived from simple simulations.

The corresponding condition for determination of the relative parallax $\pi_{\text{rel}}$ is always much more stringent because the deflection must persist during most of a year. The adopted condition is:

$$4t_E > 270 \text{ days}.$$  

A significant determination of either quantity requires in addition that the relative standard errors

$$F_\mu = \frac{\sigma_\mu}{\mu}, \quad F_\pi = \frac{\sigma_{\pi_{\text{rel}}}}{\pi_{\text{rel}}}$$

are much smaller than unity. A limit of $F < 0.3$ should be adopted, corresponding to a SNR $\gtrsim 3$.

![Fig. 2. Amplification $A$ as function of time for the impact parameter $= 1.0$. Photometric observations at the dots are spaced at $\Delta t = 30$ days, as expected for ROEMER abscissa measurements. The curves correspond to the specified values of $t_E$, e.g. as expected for MACHOs of $0.1, 1, 10 M_\odot$ when observed at stars of $V = 12$ mag. According to Table 2.

Figure 2 illustrates the photometric amplification for three MACHO masses.

Table 1 shows the possibilities of MACHO astrometry by relevant values for five $V$ magnitudes of the observed star and for four MACHO masses $m = 100, 10, 1, 0.1 M_\odot$. The typical distance to the dark body is $D_\delta$, calculated as the median of the distance to the star by means of Eqs. (27) and (30). The relative proper motion $\mu$ is calculated from an assumed tangential velocity of 200 km/s. The characteristic time and the Einstein radius of the event are given. The relative standard errors are given as expected from observations of ROEMER precision; a minus ($-$) indicates that $t_E$ is too small to satisfy Eqs. (24) and (25). Finally, the last columns contain 20 times smaller standard errors than the two previous columns as would result if each observation had 20 times smaller errors than given by Eq. (22). Such performance may be achieved some time in future.

### 3.3. Photometric amplifications

The number of observed MACHOs depends on the distribution of stars and MACHOs in the Galaxy and on the observing techniques. Simple formulae are given by which approximate estimates of the size and frequency of observable relativistic lensing can be obtained (Paczynski 1986; Griest 1991; Paczynski 1991; Griest et al. 1991; Kerins & Carr 1994).

We consider stars in intervals of about 1 mag apparent visual magnitude, characterized by the median magnitude $V_{\text{med}}$ so that one half of the stars in the interval are brighter than $V_{\text{med}}$ and the other half is fainter.

The median absolute magnitude $M_{\text{med}}$ of stars counted to a given apparent magnitude limit is required for calculation of the median distance $D_\delta$ to these stars. $M_{\text{med}}$ is $\approx 0$ if the limit is 6 mag, according to Allen (1973, p.244). Another extreme estimate is obtained from star counts at high galactic latitudes (Gilmore 1989, p.21) where the colour index at 15 mag and fainter corresponds to the main-sequence turnoff of an old, metal-poor population. Such a population has $M_{\text{med}} = 4.0$. We shall make the intermediate assumption that $M_{\text{med}} = 2.0$ for the observations obtained by ROEMER.

Given the median magnitude we assume that the length of space $D_\delta$ monitored by the stars in the interval is equal to the distance of stars of absolute magnitude $M_{\text{med}}$ seen at galactic latitude $b = 50$ deg. This is a pessimistic assumption since the mean star density of the sky is rather equal to the density at $b = 20$ deg for $V = 15$ mag. This is assumed to balance the too optimistic assumption about freedom of interstellar absorption.

This results in the following formulae for $D_\delta$ and the number of monitored stars $N_\delta$ in an interval of 1 mag

$$D_\delta = 10 \cdot 10^2 (V_{\text{med}} - M_{\text{med}}) \text{ [pc],}$$

and

$$N_\delta = 41253 \cdot 10^2 3.5 + 0.3 (V_{\text{med}} - 15).$$

This formula for $N_\delta$ corresponds to an average density of stars on the whole sky as the density at $b = 50$ deg for the magnitude $V = 15$ mag, which is representative for the relevant ROEMER observations.
Table 1. Characteristic values for MACHO astrometry as function of median visual magnitude of the stars and for four hypothetical assumptions of MACHO masses.

<table>
<thead>
<tr>
<th>Star V mag</th>
<th>MACHO $m$</th>
<th>$D_d$ pc</th>
<th>$\mu$ mas/yr</th>
<th>Event $t_E$ day</th>
<th>ROEMER $R_e$ mas</th>
<th>$\sigma_{\mu}/\mu$</th>
<th>$\sigma_{\sigma}/\sigma$</th>
<th>20 $\times$ smaller $\sigma$</th>
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<td>790</td>
<td>53</td>
<td>100</td>
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The dark matter is here characterized by $R_H$, the radius of the galactic halo of MACHOs, $\rho_H$, the spatial density of this halo, $v_T$, the (constant) tangential velocity of MACHOs (or, more precisely, of the MACHOs relative to the line of sight to the stars), and $m$, the mass of a MACHO. We assume the values $R_H = 15000$ pc, $\rho_H = 0.5 \times 10^{-24}$ g cm$^{-3}$, corresponding to a total mass of the Halo $= 10^{11}M_\odot$. Furthermore, $v_T = 200$ km/s and $m$ is a free parameter. Thus, $\rho_H, v_T, m$ are assumed to be constant.

The optical depth $\tau$ of the lensing matter is obtained from Paczynski (1986, Eq. (9)). In our discussion the distance to stars is only a few kpc so that we can simply assume that $\rho_H$ is constant along the line of sight and accordingly write

$$\tau = \frac{4\pi GD_e,med}{c^2} \rho_H D_e;$$  \hspace{1cm} (29)

where $\tau = 3.141 \cdots$. The effective distance at the median distance of a MACHO $D_e,med$, is obtained as follows. The effective distance of a MACHO is defined by

$$D_e \equiv \frac{D_d D_{\delta}}{D_{\delta}} = D_d \left(\frac{1}{D_d} - 1/D_{\delta}\right).$$  \hspace{1cm} (30a)

The median distance of a MACHO observed at a star at the distance $D_d$ is

$$D_{d,med} = 2^{-1/3} D_d$$  \hspace{1cm} (30b)

since the space density of MACHOs is assumed to be constant. The effective distance of a MACHO at this distance is then obtained from (30a) as $D_{e,med} = 0.1637D_d$.

The probability that a given star at a given epoch has an amplification larger than a threshold $A_T = 1.342$ is $\tau$. The number of observed photometric amplifications $N_{amp}$ where a star is inside one Einstein radius of a MACHO, with a given observing program is accordingly

$$N_{amp} = \tau n_{obs} N_s$$  \hspace{1cm} (31)

where $n_{obs}$ is the number of observations per star and $N_s$ is the number of stars in the program. $n_{obs} = 60$ absissa observations per star are obtained during 5 years.

We note that $N_{amp}$ is independent of $m$ if the integration time of an observation is short compared with the characteristic time $t_E$, and this is the case for absissa observations with ROEMER if $m > 0.001M_\odot$, cf. Table 1.

If $\Delta t$ is the typical interval between observations of the same star the number of events, i.e., the number of
Table 2. Characteristic values as function of median visual magnitude of the stars. \(N_i\) is the number of observed stars per magnitude interval, and \(D_i\) their median distance. \(N_{\text{amp}}\) is the number of amplifications larger than 1.342, detected by a ROEMER mission of 5 years. \(N_{d1}\) is the number of detected MACHOs if the Halo would consist entirely of MACHOs of \(m = 1M_\odot\). For other masses \(N_d\) is obtained by Eq. (32).

<table>
<thead>
<tr>
<th>(V_{\text{med}})</th>
<th>(D_i)</th>
<th>(N_i)</th>
<th>(N_{\text{amp}})</th>
<th>SNR(T)</th>
<th>(t_0)</th>
<th>(N_{d1})</th>
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<td></td>
<td>day</td>
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<td>32</td>
<td>25.1</td>
<td>18.90</td>
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</table>

\[
N_d = N_{\text{amp}} \frac{\Delta t}{2t_E} = N_{d1}(m/M_\odot)^{-0.5} \tag{32}
\]

since the number of observations per MACHO with amplification larger than \(A_T\) is \(2t_E/\Delta t\). The number \(N_{d1}\) is calculated in Table 2.

This table gives characteristic values as function of magnitude. The signal-to-noise ratio SNR\(T\) of a photometric amplification at the threshold \(A_T\) is calculated by means of Eq. (23) and appears to be large enough for detection at stars much fainter than \(V = 16\). Thus, a large number of photometric amplifications would be detected by a ROEMER mission, and about 40 of these occur for stars of \(V < 16.5\). The first five columns do not depend on \(m\), and the last two columns are calculated for \(m = 1M_\odot\). The amplification at a distance of \(u = 2.0\) is \(A = 1.061\) and can therefore be well measured, given the high values of SNR\(T\) in Table 2. This is required for the determination of \(u\) from Eq. (5b), but it is not counted in our estimates of \(N_{\text{amp}}\).

It appears from the last column that about 24 MACHOs of this mass could be detected down to \(V = 16.5\). The probability is 0.08 for a detection at \(V = 12\).

The photometric amplifications discussed here are obtained at the abscissa level, i.e. as average of 8 elementary observations. The SNR\(T\) \(\simeq 32\) at \(V = 16\) means that each elementary observation has a SNR\(\simeq 10\) and would be easily recognized. Therefore, at least 300 amplifications would be measured in a ROEMER mission.

4. Discussion of MACHO astrometry

The parallax of a MACHO can only be determined if the deflection has sufficient duration, cf. Eq. (25) and Table 1. It appears that \(m \gtrsim 10M_\odot\) is required.

A MACHO will remain very close to the star where it was detected, but its image might still be visible in some other wavelength than the visible where the brightness difference of MACHO and star may be less extreme. Therefore the position of a MACHO could be of interest and is obtained from Eq. (20) if the proper motion is known.

Direct determination of the proper motion of a MACHO would be of astrophysical interest. A statistically reasonable distance \(D_{d,\text{med}}\) could be obtained from Eq. (30b). An observed tangential velocity \(v_T\) in [km/s] would then be available, a quantity otherwise derived from a model of the halo. With the observed \(R_E\) from Eq. (8) the mass \(m\) could be derived from Eq. (2).

Such direct determination of velocity and mass of a MACHO requires the detection of a lensing event and at least three photometric and three one-dimensional astrometric measurements of the event. Less than three observations of each kind could still be of astrophysical value in connection with a statistical analysis of many lensing events, cf. Sect. 5. Detection of an event from photometric observations is by far simpler than the, practically impossible, detection alone from astrometric observations.

This is illustrated in Sect. 3.3 by the ROEMER mission by which it would be possible in 5 years to detect about 40 photometric amplifications at the abscissa level at stars brighter than \(V = 16.5\) from MACHO’s of any mass between 0.001 and 100 solar masses. The lower mass limit given corresponds to \(t_E \approx 1\) day. Astrometric analysis can be limited to those stars, and according to Table 1 a significant proper motion would for instance be expected if the star is brighter than \(V = 14\) and if all MACHOs would have about 10 solar masses or more, for which however no positive evidence exists.

Considering masses of \(m = 1M_\odot\) Table 1 shows that proper motions could not be determined by ROEMER. A satellite giving observational errors 20 times smaller would give a significant relative error of \(\sigma_r / \mu \lesssim 0.4\) at \(V = 16\) mag. Table 2 shows that a total of 24 MACHOs could be observed in a 5 year mission if all MACHOs would have about \(m = 1M_\odot\), for which no positive evidence exists.

In conclusion, the prospects for MACHO astrometry have been quantitatively estimated and found to be meager if all MACHOs have masses of only 0.1\(M_\odot\) as those first discovered.

5. Local space density of MACHOs

The ROEMER observations of microlensing are related to MACHOs much closer to the sun than observations of stars in the LMC and can therefore provide information on the local space density of MACHOs. A set of observations with known degree of completeness is required as basis for a statistical analysis.

In such analysis use is made of Eq. (2), noting that the bracket is statistically well enough known. \(D_0\) is obtained as a photometric distance to the star while a statistical
value of $D_{\text{d,med}}$ is obtained from the assumption that the space density of MACHOs is constant in all directions.

A set of photometric amplifications, without usable astrometry, could provide an estimate of $mr^{-2}$ for each event. Equation (5b) gives $u$ and elimination of $R_E$ with (2) gives the quantity mentioned. A statistical analysis of a set of such values can give an estimate of the column mass density of the dark Halo between the sun and the observed stars.

A set of simultaneous photometric and astrometric ROEMER observations of lensing events can be used to derive statistical values of the mass and space density of MACHOs, even if only one one observation of each event is available with significant precision. From the observed amplification the value of $u$ is obtained from Eq. (5). A projection of the astrometric deflection $\alpha$ may sometimes be derived in which case a statistical value of $R_E$ is obtained from Eq. (6). A statistical value of $m$ is then derived from Eq. (2).

Given the number and the degree of completeness of these observations it is then possible to derive the local number density and the space density of MACHOs. This estimate of the local space density is not dependent on assumptions of tangential velocity of MACHOs and the total mass and radius of the dark Halo, as required if the local space density is obtained from photometric observations of stars far outside the Halo as e.g. in the LMC.

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References

Aubourg, E. et al. 1993, Nature 365, 623


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