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WITH SPONTANEOUS LOCALIZATION
AND EXPERIMENTS**

**INTERNATIONAL
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**QUANTUM MECHANICS
WITH SPONTANEOUS LOCALIZATION
AND EXPERIMENTS ¹**

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ABSTRACT

We examine from an experimental point of view the recently proposed models of spontaneous reduction. We compare their implications about decoherence with those of environmental effects. We discuss the treatment, within the considered models, of the so called quantum telegraph phenomenon and we show that, contrary to what has been recently stated, no problems are met. Finally, we review recent interesting work investigating the implications of dynamical reduction for the proton decay.

1. INTRODUCTION

One of the most intriguing features of Quantum Mechanics (QM) is that it provides too good a description of microphenomena to be considered a mere recipe to predict probabilities of prospective measurement outcomes from given initial conditions and, yet, as a fundamental theory of reality, it gives rise to serious conceptual problems, among which the impossibility, in its orthodox interpretation, to think of all physical observables of the system under consideration as possessing in all instances definite values.

This fact can be (and actually is) accepted as far as microsystems are involved, far less so if we consider that individual macrosystems, as aggregates of microsystems, should also be thoroughly described by QM, whence they too ought to be associated with vectors $|\psi\rangle$ in some Hilbert space.

In fact, if Q is a macroobservable with two eigenstates $|\pm\rangle$ such that:

$$Q|\pm\rangle = \pm|\pm\rangle, \quad Q = P_+ - P_-, \quad P_{\pm} = |\pm\rangle\langle\pm|, \quad (1)$$

then, according to QM, Q does not possess a definite value unless the macrosystem is in one of the eigenstates $|\pm\rangle$, with the consequence that linear superpositions of macrostates like

$$|\psi_{\pm}\rangle = \frac{1}{\sqrt{2}}\{ |+\rangle \pm |-\rangle \}, \quad (2)$$

do conflict with macrorealism, namely with the sensible prejudice about reality that macroobservables possess definite values in all macrostates independently of any observation that can be carried out on the corresponding macrosystems. This motivates the search for a theory allowing a macroobjective description of the world.

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be changed so that microsystems keep on behaving according to (11) on any reasonable interval of time, whereas a reduction mechanism like in (3), on a time-scale suited to a macrorealist position, is consistently supplied by the unique dynamics itself.

A simple example: let the physical system be described in a two-dimensional Hilbert space $\mathbb{H} = \mathbb{C}^2$, $\sigma_x, \sigma_y, \sigma_z$ be the Pauli matrices and choose a Hamiltonian H

$$H = \hbar \frac{\omega}{2} \sigma_y = \hbar \frac{\omega}{2} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}. \quad (12)$$

If the initial state is P_+ , then at time t , $P_+(t)$ will in general have nonvanishing off-diagonal terms, since

$$|+\rangle \rightarrow \exp(-it \frac{\omega}{2} \sigma_y) |+\rangle = \cos \frac{\omega}{2} t |+\rangle + \sin \frac{\omega}{2} t |-\rangle. \quad (13)$$

Adding a term $\lambda T[\rho_t]$ to (11) (compare (3)), where λ^{-1} is some characteristic time, and a counterterm $-\lambda \rho_t$ to assure that $\text{Tr} \rho_t = 1$ for all $t \geq 0$, the new evolution equation

$$\partial_t \rho_t = -\frac{i}{\hbar} [H, \rho_t] - \lambda \rho_t + \lambda T[\rho_t], \quad (14)$$

is such that, for times much larger than λ^{-1} , the off-diagonal elements of ρ_t with respect to the basis $|\pm\rangle$ will be negligible. Indeed, if we can neglect the commutator (generator of the Schrödinger evolution) in (14), the solution ρ_t with initial condition $\rho(0) = \rho$ is:

$$\rho_t = e^{-\lambda t} \rho + [1 - e^{-\lambda t}] T[\rho] \xrightarrow{t \rightarrow +\infty} \rho_\infty = T[\rho]. \quad (15)$$

A modified QM incorporating a dynamical reduction (DR) mechanism can be called a Collapse Theory to be contrasted with a Non-Collapse Theory like the one sketched in point 4 above.

In the following, after a sketchy presentation of dynamical reduction models^[2,3,4,5], we shall examine explicit models of this type i.e. Quantum Mechanics with Spontaneous Localization (QMSL) and Continuous Spontaneous Localization (CSL), respectively, in connection with some experimental situations:

Dynamical Reduction and Non-Collapse Theories

Dynamical Reduction and the Quantum Telegraph

Dynamical Reduction and Proton Decay.

2. THE DYNAMICAL REDUCTION PROGRAM

2.1 Quantum Mechanics with Spontaneous Localization

We start considering one-particle systems described in $\mathbb{H} = L^2(\mathbb{R}^3)$.

QMSL1. At random times, with mean frequency λ , any system is affected by localization processes in position around points $\mathbf{x} \in \mathbb{R}^3$ occurring with probability $P_{loc}(\mathbf{x})$ and transforming their states $\Psi(\mathbf{q}) \in \mathbb{H}$ according to

$$\Psi(\mathbf{q}) \rightarrow \frac{\Phi_{\mathbf{x}}(\mathbf{q})}{\|\Phi_{\mathbf{x}}\|} \quad (16)$$

$$\Phi_{\mathbf{x}}(\mathbf{q}) = \left(\frac{\alpha}{\pi}\right)^{\frac{3}{2}} \exp\left(-\frac{\alpha}{2}(\mathbf{q} - \mathbf{x})^2\right) \Psi(\mathbf{q}) \quad (17)$$

$$P_{loc}(\mathbf{x}) = \|\Phi_{\mathbf{x}}\|^2, \quad (18)$$

the characteristic length $\alpha^{-\frac{1}{2}}$ and the frequency of localization being chosen to be

$$\frac{1}{\sqrt{\alpha}} = 10^{-5} \text{ cm}, \quad \lambda = 10^{-16} \text{ sec}^{-1}. \quad (19)$$

The projection operator $P^\Psi = |\Psi\rangle\langle\Psi|$ undergoes a process similar to the one in (5):

$$P^\Psi \rightarrow T_{GRW}[P^\Psi] = \left(\frac{\alpha}{\pi}\right)^{\frac{3}{2}} \int_{\mathbb{R}^3} d\mathbf{x} e^{-\frac{\alpha}{2}(\mathbf{q}-\mathbf{x})^2} P^\Psi e^{-\frac{\alpha}{2}(\mathbf{q}-\mathbf{x})^2}, \quad (20)$$

$\mathbf{q}$ being the position operator of the system.

In spite of the fact that the conceptual relevance of QMSL, embodied in (16)-(18) resides in its yielding reduction at the individual level¹, its main physical consequences can also be studied by considering what the dynamics looks like at the ensemble level. The extension of (20) to the density matrices ρ corresponding to the considered physical states is, in fact, easily obtained by linearity with the result that the localization processes depress on the scale of $\frac{1}{\sqrt{\alpha}}$ the off-diagonal matrix elements in position representation. In fact, $|\mathbf{x} - \mathbf{y}|^2 \alpha \gg 1$ implies:

$$|\langle \mathbf{x} | T_{GRW}[\rho] | \mathbf{y} \rangle| = \exp\left(-\frac{\alpha}{4}(\mathbf{x} - \mathbf{y})^2\right) |\langle \mathbf{x} | \rho | \mathbf{y} \rangle| \ll 1. \quad (21)$$

With the above assumptions one gets the evolution equation for the statistical operator

$$\partial_t \rho_t = -\frac{i}{\hbar} [H, \rho_t] - \lambda \rho_t + \lambda T_{GRW}[\rho_t]. \quad (22)$$

With the choice (19) the dynamics of microsystems determined by (22) is almost never distinguishable from (11), the usual one.

¹ There is an unavoidable ambiguity with density matrices. A state like in (6) can be read as an incoherent mixture of the states $|\pm\rangle\langle\pm|$ as well as of $|\psi_\pm\rangle\langle\psi_\pm|$ with equal weights. On the contrary, the Hilbert space processes (16-18) allow to conclude that states which are linear (coherent) superpositions of states widely separated on the scale of $\alpha^{-1/2}$ lose coherence in positions (see also the detailed discussion of CSL in Subsection 2.2).

those i 's that label particles within a volume $\alpha^{-\frac{3}{2}}$ around the j -th one and there are about 10^8 such particles in normal density conditions. Accordingly the values of τ_N of table 1 decrease by a factor 10^8 .

3. ENVIRONMENT INDUCED REDUCTION PROCESSES

We begin by recalling that points of view 4 and 5 concerning the macroobjectification problem in QM are conceptually quite different.

One of the most interesting attempts to justify an environment-induced macroobjectification has been made by Joos and Zeh^[6] who considered reasonable interactions between a macroobject like a pointer and a background of photons and/or neutrinos, air molecules and so on. The specific modification of the standard dynamics has not a universal character, but depends on the details of the process, in particular on which kind of environment exhibits relevant interactions with the system.

A collapse theory points, instead, to a modified QM whereby it is a fundamental new dynamics that eliminates the interferences on a macroscopic scale and is not merely devised to account for some unavoidable ignorance of all the details intervening in the Schrödinger evolution for S+E.

In particular, contrary to QMSL which, as the true dynamics, introduces two new fundamental constants of nature, α and λ , experimentally testable in principle, the reduction mechanisms of non-collapse theories depend crucially on the specific environmental situation. It might then be useful to compare the "reduction rates" and the physical consequences of specific examples of decoherence mechanisms of non-collapse theories with those of QMSL and CSL. Such a comparison has been presented in an interesting recent paper^[7] we are going to discuss.

In it the environment is felt by the physical system of interest as a background noise due to the (instantaneous) scattering of photons, neutrinos or air molecules (various environments) off a system, the effect on the compound initial state ρ_{S+E}^i being determined by a transition matrix T

$$\rho_{S+E}^i \longrightarrow \rho_{S+E}^f = T \rho_{S+E}^i T^\dagger. \quad (37)$$

Let $\mathbf{p}$, $\mathbf{k}$ be the momenta of the system and of a background incident particle, respectively, and $a_{\mathbf{pk}}(\mathbf{q})$ the probability amplitude that the momentum transferred to the system is $\mathbf{q}$. Reasonable assumptions on T and approximations about the nature of the scattering processes are the following.

EIRM1. Conservation of energy and momentum:

$$\langle \mathbf{p}', \mathbf{k}' | T | \mathbf{p}, \mathbf{k} \rangle = \delta(\mathbf{p}' + \mathbf{k}' - \mathbf{p} - \mathbf{k}) a_{\mathbf{pk}}(\mathbf{p}' - \mathbf{p}). \quad (38)$$

EIRM2. Independence of $a_{\mathbf{pk}}(\mathbf{q})$ from the motion of the system due to the higher velocity of the incident particle:

$$a_{\mathbf{pk}}(\mathbf{q}) = a_{\mathbf{k}}(\mathbf{q}). \quad (39)$$

EIRM3. The system is supposed to be exposed to a constant particle flux Φ per unit area and unit time scattered off with a total scattering cross section σ and a temporal

distribution modelled by a Poisson process with intensity $\Lambda = \sigma\Phi$. Furthermore, the background incident particles are supposed to be in momentum eigenstates or incoherent mixtures of them with probability momentum distribution $\mu(\mathbf{k})$.

If $\rho_S^{i,f}$ are the initial (before) and final (after a scattering process) states of the system obtained as in (7), and

$$P_{\mathbf{k}}(\mathbf{q}) = |a_{\mathbf{k}}(\mathbf{q})|^2 \quad (40)$$

$$\hat{P}_{\mathbf{k}}(\mathbf{x}) = \frac{1}{(2\pi)^3} \int_{\mathbb{R}^3} d\mathbf{q} e^{-i\mathbf{q}\mathbf{x}} P_{\mathbf{k}}(\mathbf{q}) \quad (41)$$

are the probability distribution of momentum transfers, respectively its Fourier transform, it follows that, in coordinate representation,

$$\langle \mathbf{x} | \rho_S^f | \mathbf{y} \rangle = \hat{P}(\mathbf{x} - \mathbf{y}) \langle \mathbf{x} | \rho_S^i | \mathbf{y} \rangle \quad (42)$$

$$\hat{P}(\mathbf{x} - \mathbf{y}) = \int_{\mathbb{R}^3} d\mathbf{k} \mu(\mathbf{k}) \hat{P}_{\mathbf{k}}(\mathbf{x} - \mathbf{y}). \quad (43)$$

Taking into account the third assumption above and denoting by $T_{Sca}[\rho]$ the ρ_S^f in (42), in the time interval $[t, t + dt]$ a scattering induced process occurs with probability Λdt . Consequently the unitary Schrödinger evolution turns into a master equation very much similar to (22):

$$\rho_{t+\delta t} = \left[H, \rho_t \right] \delta t + (1 - \Lambda \delta t) \rho_t + \Lambda \delta t T_{Sca}[\rho_t]. \quad (44)$$

A comparison of $T_{Sca}[\rho]$ and $T_{GRW}[\rho]$ is possible by looking at the expansions of the Gaussian damping factor that appears in (21) and the factor $\hat{P}(\mathbf{x})$, which plays an analogous role in (42) ($|\hat{P}(\mathbf{x})| \leq 1$), in powers of $\mathbf{x} = (x_1, x_2, x_3)$:

$$\langle \mathbf{x} | T_{GRW}[\rho] | \mathbf{y} \rangle \simeq \left(1 - \frac{\alpha}{4} |\mathbf{x} - \mathbf{y}|^2 + \dots \right) \langle \mathbf{x} | \rho | \mathbf{y} \rangle \quad (45)$$

$$\begin{aligned} \langle \mathbf{x} | T_{Sca}[\rho] | \mathbf{y} \rangle &\simeq \left(1 - i \sum_{j=1}^3 (x_j - y_j) \int_{\mathbb{R}^3} d\mathbf{q} q_j P(\mathbf{q}) - \right. \\ &\quad \left. - \frac{1}{2} \sum_{j=1}^3 \sum_{k=1}^3 (x_j - y_j)(x_k - y_k) \int_{\mathbb{R}^3} d\mathbf{q} q_j q_k P(\mathbf{q}) + \dots \right) \langle \mathbf{x} | \rho | \mathbf{y} \rangle. \end{aligned} \quad (46)$$

EIRM4. The incident particles are considered isotropically distributed with

$$\mu(\mathbf{k}) = \frac{1}{4\pi k^2} \lambda_0 \nu(\lambda_0 |\mathbf{k}|), \quad (47)$$

where λ_0 is a typical wavelength of the hitting particle and $\nu(x)$ is a probability distribution on the positive real axis.

Consequently, the linear term in (46) vanishes and the quadratic one (covariance matrix) is completely determined by

$$l_{eff}^{-2} = \int_{\mathbb{R}^3} d\mathbf{q} q_i^2 P(\mathbf{q}) = \frac{\partial^2}{\partial x_i^2} \hat{P}(\mathbf{x})|_{\mathbf{x}=0}, \quad i = 1, 2, 3, \quad (48)$$

4.1 The Phenomenology of the Quantum Telegraph

The physical system consists of two laser beams of intensities I_1, I_2 scattered off a single trapped atomic system which can be dealt with as a three-level system with a ground state $|0\rangle$ and two excitable states: a higher level $|1\rangle$ and a metastable lower level $|2\rangle$, with mean lives (see Fig. 1)

$$\beta_1^{-1} \simeq 10^{-8} \text{ sec} \ll \beta_2^{-1} \simeq 1 \text{ sec}. \quad (52)$$

The laser beams are tuned so that the one of intensity I_1 excites the atom from the ground state $|0\rangle$ to $|1\rangle$, that of intensity I_2 provokes the transition $|0\rangle \rightarrow |2\rangle$, followed by emissions of blue, respectively red photons and return to $|0\rangle$.

The emission pattern of an experiment conducted with $I_1 \gg I_2$, nearly 10^8 vs. 10 photons per second, reveals an intermittent blue fluorescence randomly interrupted by periods of darkness (see Fig. 2).

4.2 Quantum Mechanical Interpretations

At a first glance, the experimental evidence seems to be explainable by a naive argument based on the concepts of photons and of transitions among energy levels.

A. Because of the higher intensity of the beam I_1 , the atom is most of the time excited to the short-lived level $|1\rangle$ from which it jumps down to the ground state in approximately 10^{-8} seconds with the emission a blue photon. But, every now and then, a red photon from the beam I_2 sneaks in and the atom is excited to the metastable state $|2\rangle$, where it gets shelved for approximately 1 second before emitting a red photon and starting again a period of blue fluorescence.

However, these conclusions are far too classical (à la Bohr) and underestimate a relevant quantum effect, namely the interference between blue and red photons in the laser beams which is propagated to the atom by the linearity of the quantum evolution and results in the emergence of linear superpositions of the atomic levels.

B. After interacting with the laser beams, the atom, initially in its ground state $|0\rangle$, evolves in t seconds into a new state $|\phi(t)\rangle$ which is the coherent superposition of the three levels:

$$|0\rangle \rightarrow |\phi(t)\rangle = c_0(t)|0\rangle + c_1(t)|1\rangle + c_2(t)|2\rangle, \quad (53)$$

the probability P_i of a spontaneous emission corresponding to the jump $|i\rangle \rightarrow |0\rangle$ being $P_i = \beta_i |c_i(t)|^2$.

Being the amplitude $|c_2(t)|^2 \ll |c_1(t)|^2$ for almost all t , one expects an emission pattern consisting in continuous fluorescence and, sometimes, the emission of a red photon, periods of darkness resulting extremely unlikely (Fig. 3).

The above attempt to embody linearity does not explain the occurrence of intermittency in the emission pattern and in ref.[10] (compare also corresponding references in [9]) it is suggested that an explanation is only possible if a reduction mechanism corresponding to null measurements (seeing no photons) is introduced into the game.

In view of these facts A. Shimony concludes^[9]:

"Two propositions seem to me to suggest themselves quite strongly. The first is that a stochastic modification of quantum dynamics is a natural way to accommodate the jumps from a period of darkness to a period of fluorescence. The second is that the natural locus of the jumps is the interaction of a physical system with the electromagnetic vacuum. Whether stochasticity is exhibited when the system in question is simple and microscopic like a single atom, or only when it is macroscopic and complex like the phosphor of a photo-detector, is not suggested preferentially by the quantum telegraph, for the simple reason that the single trapped atom and the photo-detector are both essential ingredients in the phenomenon..."

But, whichever choice is made points to a stochastic modification of quantum dynamics which has little to do with spontaneous localization."

4.3 The Correct Quantum Argument

Concerning the alleged impossibility of explaining the intermittent fluorescence of the quantum telegraph by resorting to a dynamical reduction model with localization, it must be stressed that

C. The presence of periods of darkness in the emission pattern can be deduced within a purely unitary quantum mechanical scheme^[11], by taking into account the whole system

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without any need of invoking reduction processes induced by detecting no photons.

The analysis to be correct must consider states $|\psi(t)\rangle$ of the form

$$|\psi(t)\rangle = \sum_{i=0}^2 \sum_{\{n\}} c_{i,\{n\}}(t) |i\rangle \otimes |\{n\}\rangle, \quad (54)$$

where $|\{n\}\rangle$ is a state with n scattered photons. Then, with

$$P = \left(\sum_{i=0}^2 |i\rangle\langle i| \right) \otimes |\{0\}\rangle\langle\{0\}| \quad (55)$$

the orthogonal projection onto the Fock sector with no scattered photons, the probability $P(t)$ of periods of darkness extending in the interval of time $[0, t]$ when, initially, $|\psi(0)\rangle = |0\rangle \otimes |\{0\}\rangle$, is:

$$P(t) = \|P|\psi(t)\rangle\|^2 = \sum_{i=0}^2 |c_{i,\{0\}}(t)|^2. \quad (56)$$

The study of $P(t)$ leads to the correct prediction of periods of darkness in a purely quantum dynamical context. Moreover, during a period of darkness the state of the system ATOM+RADIATION FIELD is

$$(c_0(t)|0\rangle + c_1(t)|1\rangle + c_2(t)|2\rangle) \otimes |\{0\}\rangle, \quad (57)$$

so that periods of darkness can end with the emission of both red and blue photons with an emission pattern like in Fig. 4.

However, such a change of the value of Δ would lead us to the limit of acceptability for small objects: the dynamics will not reduce within the perception time an object containing 10^{15} particles like a carbon particle of radius 10^{-3} cm. Similarly, the argument of^[15] about the perception mechanism would no longer be correct.

5.2 The Pearle and Squires Argument

Considering a macroscopic body of total mass M made up of different types of identical particles of mass m_j , one may think of relating the reduction process to the mass density operator

$$M(\mathbf{x}) = \sum_k m_k \left(\frac{\alpha}{2\pi}\right)^{\frac{3}{2}} \int_{\mathbb{R}^3} d\mathbf{y} e^{-\frac{\alpha}{2}(\mathbf{y}-\mathbf{x})^2} a_k^\dagger(\mathbf{y}) a_k(\mathbf{y}), \quad (64)$$

rather than considering independent stochastic processes for the various classes of particles².

Specifically, one should replace in equation (30) $\gamma T_{CSL}[\rho_t]$ with $\gamma T_M[\rho_t]$

$$T_M[\rho_t] = \frac{1}{m_0^2} \int_{\mathbb{R}^3} d\mathbf{x} \left\{ M(\mathbf{x}) \rho_t M(\mathbf{x}) - \frac{1}{2} (M^2(\mathbf{x}) \rho_t + \rho_t M^2(\mathbf{x})) \right\}, \quad (65)$$

m_0 being some typical reference mass.

The first order analysis that has led to (63) can be similarly carried out in this case, only, one has to consider the total centre of mass $\mathbf{Q}$ of the system and not only the various species centres of mass $\mathbf{Q}_j$

$$\mathbf{Q} = \frac{1}{M} \sum_j m_j \sum_{i=1}^{N_j} \mathbf{q}_{ji},$$

whence the rate of internal excitation and/or dissociation appears as a second order effect. Indeed, the total centre of mass $\mathbf{Q}$ cannot excite any internal degree of freedom:

$$\frac{d}{dt} P(t)|_{t=0} \simeq \frac{M^2 \alpha \lambda}{2m_0^2} | \langle \Psi_E | \mathbf{Q} | \Psi_H \rangle |^2 = 0. \quad (66)$$

If one takes the QMSL value of λ for nucleons, that is the reference mass m_0 is identified with the nucleon mass, one has a remarkable decrease of the QMSL rate Q_{D+E} in (61) for atoms: from one atom per mole being either excited or dissociated every second to one of these possible events per mole every 10^{12} seconds (10^5 years). Nevertheless, all the important features of QMSL are preserved:

The decoupling of macroscopically distinguishable states is taken care of by the nucleons of the macroscopic bodies.

The energy increase is almost identical to the acceptable figure which is a consequence of the QMSL localization.

The collapse induced decay probability of a proton is depressed by a factor 10^{-16} making the CSL predicted lifetime well compatible with the experimental data.

² The advantages of taking such a position have been discussed also by A. Rimini in these proceedings.

Some concluding comments are in order at this point: the above analysis has appropriately pointed out the nice features deriving from relating reduction to the mass of the elementary particles. However,

i. The DR program has made clear that one can try to follow a new line to solve the conceptual problems that QM meets with macroobjects and measurement processes, namely, modifying the dynamics so that the physics of microsystems remains unaltered, while macrosystems exhibit an acceptable behaviour. For the time being, the program still requires many improvements, in particular the crucial problem to be faced is getting reasonable relativistic generalizations. Being the quark dynamics fundamentally relativistic, and due to the great difficulties haunting the so called “nonrelativistic quark models”, simply applying the specific models of QMSL and CSL to nucleons seems a little bit too hazardous.

ii. Other collapse theories are still under consideration. A model presented in^[16] tries to relate the decoherence mechanism to gravity and to reduce the number of new fundamental constants characterizing CSL. In particular, the reduction mechanism is linked to the mass of the systems involved, already meeting the basic request of^[11].

iii. The difficulties connected with nucleon decay might also be avoided by slight modifications of the standard CSL, for instance, by using a higher power $N^{\frac{2}{3}}(\mathbf{x})$ of the smeared number operator appearing in (32).

Fig. 1: atomic system in the quantum telegraph phenomenon.

Fig. 2: emission pattern in the quantum telegraph phenomenon.

Fig. 3: emission pattern according to argument B, thick lines correspond to red photons.

Fig. 4: emission pattern according to argument C.

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As a further remark, it must be noted that a complete account of the quantum telegraph experiment ought to include the macroscopic detectors that are involved in the measurement of the emission pattern. Consequently, the physical system to be dealt with is

ATOM+RADIATION FIELD+MACROSCOPIC DETECTORS

Within the DR-program the actualization of the different macrostates of the detecting apparatuses is accounted by the new dynamics and the corresponding objectification of macroproperties is thus obtained. One could raise the question whether this can have any appreciable influence on the quantum telegraph phenomenon.

It is sufficient to observe that, according to the analysis of the previous Section, the effects of the reduction mechanisms are comparable with those of environment-induced reductions that occur at the detectors level.

Indeed, on the basis of the agreement of the correct quantum mechanical computations of the probability of occurrence of periods of darkness and of their duration with the experimental results, one can safely conclude that

i. QMSTL and CSL dynamics play, for the process under consideration, exactly the same role as for any macroscopic detection process, namely they objectively macro-properties.

ii. There is no need to require that new nonlinear and stochastic modifications of standard QM become effective at the microscopic level to account for the quantum telegraph phenomenon.

iii. In particular, nothing in the considered experiments suggests that reductions take place with respect to an "energy" rather than to a "position" preferred basis.

5. DYNAMICAL REDUCTION AND NUCLEON DECAY

The presence in nature of a mechanism that localizes particles would be accompanied by a corresponding spreading in their momenta. It is thus interesting to study its effect on the stability of atoms and nuclei. It is possible to get a rough idea of the consequences of QMSTL by modelling atoms and nuclei as one dimensional systems moving in a harmonic potential so that their ground states can be approximated by Gaussian wave functions $\Psi_G(q)$ of appropriate width γ^{-1} :

$$\Psi_G(q) = \left(\frac{\pi}{2\gamma^2}\right)^{\frac{1}{2}} e^{-\frac{1}{2}\gamma^2 q^2}, \quad \begin{array}{l} \gamma \simeq 10^8 \text{ cm}^{-1} \quad \text{for an atom} \\ \gamma \simeq 10^{12} \text{ cm}^{-1} \quad \text{for a nucleus} \end{array} \quad (58)$$

If the particle undergoes a localization around x as in (16), $\Psi_G(q)$ changes into:

$$\|\Phi_x(q)\| = \left(\frac{\pi}{\alpha}\right)^{\frac{1}{2}} e^{-\frac{\alpha}{2}(b-x)^2} \Psi_G(q). \quad \Phi_x(q) = \left(\frac{\pi}{\alpha}\right)^{\frac{1}{2}} e^{-\frac{\alpha}{2}(b-x)^2} \Psi_G(q). \quad (59)$$

From (18), the probability density that a localization takes place at x is given by $\|\Phi_x\|^2$. Accordingly, the probability that, if a hitting process occurs, the state of the system is

still $\Psi_G(q)$ is given by:

$$P^{ND} = \int dx |\langle \Psi_G | \Phi_x \rangle|^2 = \frac{1}{1 + \frac{\alpha}{4\gamma^2}} \approx \begin{cases} 1 - 10^{-7} & \text{for an atom} \\ 1 - 10^{-15} & \text{for a nucleus} \end{cases} \quad (60)$$

Since microsystems are supposed to undergo one localization every 10^{16} seconds, the transition rate Q_{ND} to an excited or dissociated state is

$$Q_{ND} = \lambda(1 - P^{ND}) \approx \frac{\lambda\alpha}{8\gamma^2} = \begin{cases} 10^{-23} & \text{for an atom} \\ 10^{-31} & \text{for a nucleus} \end{cases} \quad (61)$$

[m] Pearle examines the case of the hydrogen atom and compares the flux of Lyman- α ultraviolet photons emitted by intergalactic hydrogen with the one expected if a GRW mechanism were at work^[3,12,13], the latter turning out to be much weaker than the one observed.

Applying the above argument to the quark model of proton we would get a decaying time of the same order of magnitude of the one of a nucleus (10^{31} sec), whereas the proton lifetime is estimated longer than 10^{31} years, that is 10^{38} sec. This fact seems to indicate that the decoherence rate Δ should be decreased by a factor 10^7 .

However, the consequences of $\Delta \simeq 10^{-13} \text{ cm}^{-2} \text{ sec}^{-1}$ would be unacceptable. In fact, being $\alpha^{-\frac{1}{2}} = 10^{-5} \text{ cm}$ a reasonable value for the localization length (l_{lf} in Table 2.), it would yield the value $N \simeq N 10^{-23} \text{ sec}$ for the macroscopic coherence time in (28). Thus, linear superpositions of spatially separated states of any reasonable macroscopic "pointer" ($N \sim 10^{23}$) would typically take times of the order of the second to be suppressed.

5.1 Reconsidering the Argument within the CSL Approach

In [1] the authors let an initial bound state $\rho_H = |\Psi_H\rangle\langle\Psi_H|$ evolve into ρ according to the dynamics (30) of CSL and study the transition probability $P(t)$ to an excited orthogonal state $|\Psi_H\rangle$:

$$\frac{d}{dt} P(t)|_{t=0} = \partial_t \langle \Psi_H | \rho | \Psi_H \rangle > |_{t=0}. \quad (62)$$

The only contribution is that from the reducing term $\gamma T_{CSL}[\rho]$ in (31). By developing up to the first order in α one gets:

$$\frac{d}{dt} P(t)|_{t=0} \approx \sum^f \frac{\alpha \lambda N_j^2}{2} |\langle \Psi_H | Q_j | \Psi_H \rangle|^2, \quad Q_j = \frac{1}{N_j} \sum_{i=1}^f \mathbf{q}_j, \quad (63)$$

f numbering the species of identical particles making up the system, $\mathbf{q}_j$ being the position operators of the j -th type of particles and Q_j their centre of mass. For just one nucleon the result agrees with that of QMSTL, while, for macrosystems, due to the more efficient decoupling rate, it appears that a correction of $\Delta = \alpha\lambda/2$ which would lead to no conflict with the proton lifetime is possible.

which has dimension cm^{-2} and introduces a characteristic length l_{ff} of the reduction processes to be compared with $(\frac{\alpha}{2})^{\frac{1}{2}}$ of QMSL. Moreover, a natural time scale is given by $\tau = \Lambda^{-1}$. Solving (44) for short times $\Lambda \delta t \gg 1$, yields as in (23)

$$< x|p_{x+\hbar}^2|y > \simeq \exp(-\Lambda \delta t(1 - P(x - y))) < x|p|y > . \quad (49)$$

Off the diagonal ($|x - y|l_{ff}^{-1} \gg 1$) the damping is dominated by $\exp(-\Lambda \delta t)$, whereas near the diagonal ($|x - y|l_{ff}^{-1} \ll 1$) the damping goes as $\exp(-\frac{1}{\Lambda}l_{ff}^2\delta t|x - y|^2)$. We can then introduce:

$$\begin{aligned} \text{Coherence Time: } \tau &= \Lambda^{-1} \\ \text{Decoherence Rate: } \Delta &= \frac{l_{ff}^2}{\Lambda} \end{aligned} \quad (50)$$

$$\quad \quad \quad (51)$$

The decoherence time τ is fixed by the total scattering cross section σ and the flux of incident particles per unit area and unit time, while the decoherence rate Δ by the differential cross section that enters the expression of $P(\mathbf{q})$ in (40). The author calculates the values of l_{ff} and τ for a microsystem (electron) in different physical backgrounds.

Cause of collapse	$l_{ff}[cm]$	$\Phi[cm^{-2}sec^{-1}]$	$\tau_{electron}[sec]$
300K air at 1 atm	10^{-9}	10^{21}	10^{-13}
300K air in lab vacuum	10^{-9}	10^{11}	1
Sunlight on earth	9×10^{-5}	10^{17}	10^7
300K photons	2×10^{-3}	10^{19}	10^5
Background radioactivity	10^{-12}	10^{-1}	10^{18}
Quantum gravity	$10^5 - 10^{12}$	10^{109}	30
GRW effect	10^{-5}	10^{16}	10^{16}
Cosmic microwave background	2×10^{-1}	10^{13}	10^{11}
Solar neutrinos	10^{-9}	10^{11}	10^{31}
Cosmic background neutrinos	3×10^{-1}	10^{13}	10^{51}

Table 2.: l_{ff} for various scattering processes and $\tau_{electron}$.

[The absence of a second figure in the GRW effect row emphasizes the deep conceptual difference between collapse and noncollapse theories: there is no particle flux scattered off the system inducing the localization mechanism on the length scale $l_{ff} = (\frac{\alpha}{2})^{\frac{1}{2}}$, the effect being due to a completely new dynamics and not to the environment. Analogously the author gives estimates of Δ for various objects in different backgrounds.

Cause of apparent	Free	Dust	Bowling ball
wave function collapse	electron	particle	ball
300K air at 1 atm	10^{31}	10^{37}	10^{46}
300K air in lab vacuum	10^{18}	10^{23}	10^{31}
Sunlight on earth	10	10^{20}	10^{28}
300K photons	1	10^{19}	10^{27}
Background radioactivity	10^{-4}	10^{15}	10^{23}
Quantum gravity	10^{-25}	10^{10}	10^{22}
GRW effect	10^{-6}	10^9	10^{21}
Cosmic microwave background	10^{-10}	10^6	10^{17}
Solar neutrinos	10^{-15}	10	10^{13}

Table 3.: Δ in $cm^{-2}sec^{-1}$ for various scattering processes.

As it is evident from Table 3., the GRW effect, e.g. for a free electron, is weaker than those of air molecules in lab vacuum or photons on earth by a factor in between 10^{26} and 10^6 , respectively, hence masked by them. Therefore, to put into evidence effects due to spontaneous decoherence mechanisms one should isolate the physical system of interest from the environment to a presently hardly attainable degree of accuracy.

However, the figures in Table 3. might lead to erroneous conclusions, if not correctly understood. In fact, e.g. for bound electrons, QMSL can, as we shall discuss later, induce, as a result of the localization mechanisms, transitions (which are not considered in the preceding analysis of environment induced decoherence) leading to excitations or dissociations of the composed systems to which the electrons belong.

As an example, in a recent paper^[9] it is argued that the Lyman- α ultraviolet radiation emitted by hydrogen atoms (about $1 - 10$ photons per second per mole) as a consequence of the spontaneous localizations suffered by electrons could be detected by an appropriate experimental setup. On the other hand, on the basis of energy balance considerations, it can be shown that an analogous effect due to collisions of an atom with 300 Kelvin air molecules at 1 atmosphere is much smaller in comparison.

4 DYNAMICAL REDUCTION AND THE QUANTUM TELEGRAPH

In this section we examine not a suggested experimental test for the DR-program, but, rather, some recent claims that there exists some already available empirical evidence that spontaneous collapses are necessary, but of a nature different from either QMSL or CSL. We quote directly from A. Shimony^[9]:

“A great weakness of the investigations carried out so far in search of modifications of quantum dynamics is the absence of empirical hypotheses: that body of empirical fact which motivates all the advocates of nonlinear modifications is, the occurrence of definite events, and, in particular, the achievement of definite outcomes of measurement. But this body of fact is singularly unsuggestive of the details of a reasonable modification of Quantum Mechanics. What is needed are phenomena which are suggesting and revelatory...”

No more promising phenomena for this purpose have been found than the intermittent-

If we consider an interval of time $[t, t + \delta t]$ where the effects of the Hamiltonian evolution are negligible compared with the localization processes ($\lambda \delta t \gg 1$), we get:

$$\langle x | \rho_{t+\delta t} | y \rangle \simeq \exp \left(-\lambda \delta t \left(1 - e^{-\frac{\lambda}{2}(\mathbf{x}-\mathbf{y})^2} \right) \right) \langle x | \rho_t | y \rangle$$

$$\simeq \begin{cases} e^{-\lambda \delta t} & |\mathbf{x} - \mathbf{y}|^2 \alpha \gg 1 \\ e^{-\frac{\lambda}{2} \delta t |\mathbf{x} - \mathbf{y}|^2} & |\mathbf{x} - \mathbf{y}|^2 \alpha \ll 1 \end{cases} \quad (23)$$

Then, we are led to single out two basic parameters of QMSL:

$$\begin{aligned} \text{Coherence Time: } \tau = \lambda^{-1} &= 10^{16} \text{ sec} \\ \text{Decoherence Rate: } \Delta &= \frac{\alpha}{2} \simeq 10^{-6} \text{ cm}^{-2} \text{ sec}^{-1}. \end{aligned} \quad (24)$$

In going from microsystems to macrosystems we first consider the case of aggregates of N distinguishable particles described in $\mathbb{H} = L^2(\mathbb{R}^{3N})$.

QMSL2. Each i -th particle is assumed to undergo an independent localization process as in (16)-(18). There follows that the evolution equation for the density matrix of an N -particle system becomes:

$$\partial_t \rho_i = -\frac{\hbar}{i} [H, \rho_i] - N \lambda \rho_i + \lambda T^{QMSL}[\rho_i] \quad (26)$$

$$T^{QMSL}[\rho_i] = \sum_{N=1}^{\infty} \int \mathbb{R}^3 d\mathbf{x} e^{-\frac{\alpha}{2}(\mathbf{q}_i - \mathbf{x})^2} \rho_i e^{-\frac{\alpha}{2}(\mathbf{q}_i - \mathbf{x})^2}. \quad (27)$$

$\mathbf{q}_i$ being the position operator relative to the i -th particle.

As a main consequence, the centre of mass motion is such that the corresponding density matrix, where the internal motion has been traced out, obeys an evolution equation similar to (22) with a frequency λ_N of localization which is N times larger than the microscopic λ . This is reasonable for macroscopic rigid bodies since each localization process, one in λ^{-1} seconds, suffered by any of the N particles is felt as a localization process by the center of their aggregate. Hence, for macroscopic bodies made up of N constituents:

$$\begin{aligned} \text{Coherence Time: } \tau_N &= \frac{N}{\tau} \\ \text{Decoherence Rate: } \Delta_N &= N \Delta. \end{aligned} \quad (28)$$

The effects of the QMSL dynamics (26) are such that the interference between states of almost rigid macroscopic bodies in which the centre of mass is differently located with respect to the length $\alpha^{-\frac{1}{2}}$ is suppressed after times of the order of τ_N or less:

System	Radius (cm)	N	$\Delta_N [\text{cm}^{-2} \text{sec}^{-1}]$	$\tau_N [\text{sec}]$
Dust Particle	10^{-3}	10^{15}	10^9	10
Bowling Ball	10	10^{27}	10^{21}	10^{-11}

Table 1.

2.2 Quantum Mechanics With Continuous Spontaneous Localization

Macroscopic bodies composed by one or more species of indistinguishable particles are dealt with by a generalization of QMSL called CSL. In CSL a random potential generates a continuous, stochastic evolution of Hilbert space vectors. The probability of occurrence of the random potential depends on the corresponding evolved state vector through a "cooking procedure" which makes the dynamics nonlinear. After averaging out the stochasticity, density matrices change in time according to the following evolution equation with localization:

$$\partial_t \rho_i = -\frac{\hbar}{i} [H, \rho_i] + \gamma T^{CSL}[\rho_i] \quad (30)$$

$$T^{CSL}[\rho_i] = \sum_k \int \mathbb{R}^3 d\mathbf{x} \left\{ N_k(\mathbf{x}) \rho_i N_k(\mathbf{x}) - \frac{1}{2} \left(N_k^2(\mathbf{x}) \rho_i + \rho_i N_k^2(\mathbf{x}) \right) \right\} \quad (31)$$

$$N_k(\mathbf{x}) = \left(\frac{\alpha}{2\pi} \right)^{\frac{3}{2}} \int \mathbb{R}^3 d\mathbf{y} e^{\frac{\alpha}{2}(\mathbf{x}-\mathbf{y})^2} a_k^\dagger(\mathbf{y}) a_k(\mathbf{y}), \quad (32)$$

where the index k numbers the types of particles and $a_k^\dagger(\mathbf{y})$, $a_k(\mathbf{y})$ are creation and annihilation operators of type k at points $\mathbf{y} \in \mathbb{R}^3$.

Let us consider, for the moment, a physical system made up of just one species of particles ($k = 1$). With the choice $\gamma = \lambda \left(\frac{4\pi}{\alpha} \right)^{\frac{3}{2}}$, (31) reduces to (27) on the one-particle sector. If $N > 1$, denoting with $|\mathbf{q}\rangle \simeq |\mathbf{q}_1, \dots, \mathbf{q}_N\rangle$ the improper eigenstates of the position operators of the N particles, then

$$\begin{aligned} \gamma \langle \mathbf{q} | T^{CSL}[\rho_i] | \mathbf{q}' \rangle &= -\frac{\lambda}{2} \sum_N \sum_{n=1}^N \left\{ e^{-\frac{\alpha}{2}(\mathbf{q}_n - \mathbf{q}_1)^2} + e^{-\frac{\alpha}{2}(\mathbf{q}'_n - \mathbf{q}_1)^2} \right. \\ &\quad \left. - 2e^{-\frac{\alpha}{2}(\mathbf{q}_n - \mathbf{q}'_n)^2} \right\} \langle \mathbf{q} | \rho_i | \mathbf{q}' \rangle, \end{aligned} \quad (33)$$

whereas QMSL, which does not take into account particle indistinguishability, gives:

$$\langle \mathbf{q} | -N \lambda \rho_i + \lambda T^{QMSL}[\rho_i] | \mathbf{q}' \rangle = -\lambda \sum_{j=1}^N \left\{ 1 - e^{-\frac{\alpha}{2}(\mathbf{q}_j - \mathbf{q}'_j)^2} \right\} \langle \mathbf{q} | \rho_i | \mathbf{q}' \rangle. \quad (34)$$

While (22) (see footnote 1) diagonalizes with respect to a preferred basis identifiable with the improper eigenstates of the position operators, (30) induces decoherence of macrostates with macroscopically different densities over volumes $\alpha^{-\frac{3}{2}}$. For normal densities the decoupling rate for a superposition of two macrostates completely spatially separated is much faster than for QMSL; compare

$$\gamma \langle \mathbf{q} | T^{CSL}[\rho_i] | \mathbf{q}' \rangle \simeq -\lambda N 10^8 \langle \mathbf{q} | \rho_i | \mathbf{q}' \rangle, \quad (35)$$

against

$$\langle \mathbf{q} | -N \lambda \rho_i + \lambda T^{QMSL}[\rho_i] | \mathbf{q}' \rangle \simeq -\lambda N \langle \mathbf{q} | \rho_i | \mathbf{q}' \rangle. \quad (36)$$

In fact, of the three terms in (33), the third one is negligible ($|\mathbf{q}_i - \mathbf{q}'_j|^2 \alpha \gg 1$) for all $i, j = 1, \dots, N$, while the other two, for given $j = 1, \dots, N$, are significant only for