\textbf{\textsuperscript{6}Li Inelastic Form Factors in a Cluster Model}

S. Weber, M. Kachelrieß, M. Unkelbach and H.M. Hofmann

\textit{Institut für Theoretische Physik, Universität Erlangen-Nürnberg, Staudtstr. 7 D-91058 Erlangen, Germany (July 8, 1994)}

Longitudinal and transverse formfactors are calculated for the transition into the low lying excited T=0 and T=1 states of \textsuperscript{6}Li in the framework of the resonating group model. All formfactors are reproduced simultaneously using three cluster-wavefunctions. Meson exchange currents yield only minor corrections and do not lead to any specific structures at large momentum transfers.

The structure and properties of the \textsuperscript{6}Li nucleus are experimentally and theoretically well studied (especially formfactors are of particular interest). For such a light nucleus microscopic calculations, starting from a nucleon-nucleon force, are feasible for a large variety of different models. Special interest is devoted to three-body \(\alpha\)-np models often in the framework of Faddeev equations [1-4], but also shell model [5,6] and cluster model [7,8] calculations are affluent. In many cases only the groundstate properties of \textsuperscript{6}Li are studied. We report here of an extension of ref. [8] to all low lying excited states of \textsuperscript{6}Li in the framework of the Resonating-Group-Model (RGM). We use completely antisymmetrised RGM cluster-wavefunctions in the form \(\alpha\)-np and an effective nucleon-nucleon potential [9].

In ref. [8,9] the groundstate wavefunction of \textsuperscript{6}Li was calculated using Ritz variational principle, allowing all possible combinations of S- and D-waves on the intercluster coordinates between neutron and proton and center-of-mass of n-p and \(\alpha\)-particle. For the T=0 spin-orbit triplet \((3^+, 2^+, 1^+)\) such a variation isn’t possible anymore, because the lowest energy for this partial wave is just the \(\alpha\)-deuteron threshold. Therefore we used the parameters of the model-space of our groundstate and diagonalized the Hamiltonian in the corresponding spaces. The excitation energies are given in table I. Obviously the agreement with experiment [10] is only fair due to the fixed width parameters. We refrained from changing the width parameters in order to reproduce the data, because there is no controllable way to do that.

In an \(\alpha\)-deuteron scattering calculation the phaseshifts for the \(3^+, 2^+\) and to lesser extend \(1^+\) vary rapidly in the neighborhood of the experimental energies (see ref. [11,12] for potentials similar to ours). For T=1 the situation is different, since the decay into the \(\alpha\)-deuteron channel is forbidden by isospin. The \(0^+\)T=1 state is bound, relativ to the \(\alpha\)-n-p threshold. Therefore we tried again Ritz variational principle to determine the wavefunction. Besides the obvious spin zero and pure S-wave component, we allowed P-waves on the intercluster coordinates coupled to 1. For this model space we found a rather stable local minimum above the \(3\)-body breakup threshold with the parameters \(\beta = 0.2838, \gamma = 0.1620, \delta_1 = 0.2609\) and \(\delta_2 = 0.03624\) (see ref. [9] for an explanation of these parameters). For the \(2^+\)T=1 state these parameters were also used and we allowed all combinations of S-,P- and D-waves which could contribute.

The energies are given below (table I). Since the calculated level order is not correct, we allowed also configurations of the \textsuperscript{6}Li-n and \textsuperscript{6}He-p in the \(0^+\)T=1 wavefunction, gaining 0.7 MeV additional binding energy. As will be shown later the effects of the wavefunction modification on the formfactors had been quite small. So we won’t give any details of the complicated wavefunction.

With these wavefunctions we calculated now electromagnetic transition formfactors. For one-body operators we used the standard expressions for the charge density, convection current and magnetisation density [8]. For the meson exchange currents we used the prescription of Ohta [13]; details of the calculation in the RGM framework are given in ref. [14].

Due to the finite size of the nucleons the formfactors must be modified by the single nucleon formfactor. This is done by multiplying all matrix elements with the nucleon formfactors attained by the well-known dipole formula [15] (This expression is misprinted in ref. [8]).

\[ f(k) = \left(1 + \frac{1}{k^2 A_1^2} \right)^2 \text{ with } A_1 = 0.71 (\text{GeV}/\text{fm})^2 \]

All details of the calculation are given in [14]. We mention in passing that the groundstate wavefunction reproduces the tiny quadrupole moment (see [9] but also the discussion in [7]) and the elastic formfactors quite well only if the wavefunction is properly antisymmetrised.

In Fig. 1 and 2 the calculated inelastic longitudinal formfactors are displayed. The formfactors for the \(T=0\) states are of similar magnitude and k-dependence since in all cases the C2 is the dominant contribution. For the \(2^+\)T=1 our result is some orders of magnitude smaller, for the \(3^+\)T=0 state the calculation agrees well with the data [16]. For all other resonances there exists no data, either due to the width of the \(T=0\) states or the smallness of the longitudinal
formfactors for the $2^+ T=1$ state. For the $T=0$ tripplet states our results are similar to those of ref. [3], whereas for the $2^+ T=1$ we disagree in magnitude and form.

In Fig. 3 and 4 the calculated transverse formfactors are displayed together with the multipoles. $E2$- and $M3$-transition are the dominant ones. Whereas the absolute magnitude agrees well with the findings of ref. [3], the results for the various multipoles are quite different. Unfortunately there is no data to compare with. Note that in our model meson exchange contributions are not possible for this transition due to the isospin zero of initial and final state.

For the $T=1$ states however such contributions are possible. In Fig. 5 we compared the calculation with experimental data for the $0^+ T=1$ state [16,17]. The one-body M1 operator reproduced the data quite nicely. The MEC's are only of minor importance, but reducing the calculated results and thus yielding a somewhat less agreement. Therefore we allowed for additional $^6$Li-n and $^8$He-p configurations in the $0^+ T=1$ wavefunction. The result was a further small reduction in the formfactor similar to the MEC effects, and even less, but still satisfactory agreement with data (see Fig. 5). The reduction can be easily explained by the reduced overlap of the orbital wavefunction for the excited state with the groundstate one due to structures missing in the groundstate. The inclusion of these structures into the groundstate resulted in a totally wrong quadrupolemoment. Since both wavefunctions however reproduced the data much better than ref. [3] we didn’t pursue this issue further (as pointed out in ref. [18] the inclusion of additional structures may in general influence substantially the formfactors; here it resulted only in a totally wrong quadrupolemoment). In Fig. 6 we compare transverse formfactors of the $2^+ T=1$ state with data [16,17]. Again $E2$ and $M3$ are the dominant contributions, with MEC’s playing only a minor role, but reducing the calculation by a few percent. Both calculations agree well with data.

In conclusion one can say that our calculation reproduced consistently the measured elastic [8] and inelastic longitudinal and transverse formfactors for all low lying states of $^6$Li. Meson exchange contributions yielded only minor effects and introduced no structure at larger momentum transfers.

Discussions with A. Booten are gratefully acknowledged.


FIG. 1. Longitudinal formfactors for the $1^+ T=0$ and $2^+ T=0$ state

FIG. 2. Longitudinal formfactors for the $2^+ T=1$ and $3^+ T=0$ state data: Bergstrom, J.C., Nucl. Phys. A327 (1979) 439

FIG. 3. $F_2^n$ for the $1^+ T=0$ and $3^+ T=0$ state

FIG. 4. $F_2^n$ for the $2^+ T=0$ state

2
FIG. 5. $F^2_\pi$ for the $0^+T=1$ state
full line: impulse-approximation
dotted line: including MEC's
data:

FIG. 6. $F^2_\pi$ for the $2^+T=1$ state
<table>
<thead>
<tr>
<th>State:</th>
<th>Calculated:</th>
<th>Experimental</th>
</tr>
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<tr>
<td>$1^+ T-0^a$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$3^+ T-0$</td>
<td>3.33</td>
<td>2.18</td>
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<tr>
<td>$0^+ T-1$</td>
<td>6.16, 4.46$^b$</td>
<td>3.56</td>
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<td>$1^{++} T-0$</td>
<td>5.37</td>
<td>5.65</td>
</tr>
</tbody>
</table>

* TABLE I. $^6$Li Energies (in MeV) *

$^a$Ground state  
$^b$Complex wavefunction