BY THEIR ANNHIILATION IN GALACTIC HALO

BOUNDS ON VERY HEAVY RELIC NEUTRINOS

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IN GALACTIC HALO

BOUNDS ON VERY HEAVY RELIC NEUTRINOS BY THEIR ANNIHILATION
ABSTRACT

Taking into account neutrino condensation in the gravitational field of collapsing matter, we investigate the annihilation of heavy relic neutrinos in the Galaxy resulting in the generation of cosmic rays. The main neutrino annihilation processes are considered, i.e. $\nu \bar{\nu} \rightarrow 2\gamma$ and $\nu \bar{\nu} \rightarrow W^+W^-$. The condensation mechanism allows one to get information on the density distribution in the Galaxy halo without any recourse to an explicit dynamical halo model, and the resulting cosmic ray spectrum provides constraints on the heavy neutrino mass. The comparison of the predicted cosmic ray flux with the observed one excludes the heavy neutrino mass range $60$ GeV $< m_\nu < 115$ GeV.

Such a restriction leads to a bound on the present energy density of very heavy neutrinos which is comparable with the corresponding baryonic one in the range $115$ GeV $< m_\nu < 300$ GeV. Our approach is valid for a multicomponent dark matter and can be used for species that give even a negligible contribution to the critical density.

Finally, we briefly discuss also the possibility of searching for heavy neutrinos in accelerator experiments, which allow investigation in the region $m_\nu \sim M_Z^2/2$ where it is difficult to get cosmological constraints on the neutrino mass.

1. INTRODUCTION

By measuring the width and the height of the Z-boson peak LEP experiments [1] tell us that there are three neutrino species. However, this constraint is applied only to light neutrinos with mass $m_\nu < M_Z^2/2$, where $M_Z$ is the mass of the Z-boson, and therefore does not forbid the existence within the framework of the Standard Model of very heavy neutrinos, so that Z-bosons decays into these are prohibited by phase space. In particular, the results of modern experiments are not inconsistent with the existence of heavy Dirac neutrinos with $m_\nu > 44$ GeV [2].

The important additional source of information on neutrino parameters is cosmology. According to the theory of the Hot Universe, the Universe should contain a background of relic neutrinos, whose concentration is related to that of relic photons. The energy density $\rho_\nu$ of massive stable neutrinos should be smaller than the total energy density $\rho$ of the Universe. Assuming that the cosmological density does not exceed the critical one

$$\rho_\nu \lesssim \frac{\rho_\nu}{\rho_c} \lesssim \rho_c,$$

where $H$ is the present Hubble expansion rate in units of $100$ km s$^{-1}$ Mpc$^{-1}$, it was found [3] that the allowable ranges for the neutrino mass are:

$$m_\nu < 30$$ eV and $3$ GeV $< m_\nu < 3$ TeV.

In these calculations the range of large neutrino masses is available due to the rising cross section of neutrino annihilation with the increase of the neutrino mass as $\sigma \sim m_\nu^2$ for $1$ GeV $< m_\nu < M_Z^2/2$. This leads to a decrease of both the residual
neutrino concentration as \[ n_{\nu} = \frac{1}{(m_{\nu} \sigma_{\nu})} \sim m_{\nu}^{-3} \] and of the neutrino energy density in the Universe. However, beyond the Z-pole, the annihilation cross section for the process \( \nu \bar{\nu} \to f \bar{f} \) (where \( f \) is a fermion) starts decreasing as \( m_{\nu}^{-2} \), due to the momentum dependence of the Z-boson propagator. In this region the relic number is proportional to \( m_{\nu} \) and the neutrino energy density increases as \( m_{\nu}^{-2} \), reaching again the critical value \( \rho_c \) for a few TeV.

However these calculations were carried out without taking into account the important annihilation channel \( \nu \bar{\nu} \to W^+ W^- \). As was shown in Ref.[4], in the region of neutrino masses \( m_{\nu} > M_W \) this process defines the fastest rate of the neutrino annihilation with a cross section \( \sigma \sim m_{\nu}^{-2} \). This leads to a corresponding decrease in the neutrino energy density \( \rho_{\nu} \sim m_{\nu}^{-2} \) and therefore there is no longer a cosmological upper bound on the stable neutrino mass based on the consideration of the total energy density in the Universe.

Nevertheless the nonuniform distribution of massive neutrinos in the Universe and the local increase of the neutrino density in the formation of the galactic halo lead to the considerable sensitivity of astrophysical data to the existence of heavy neutrinos in the Universe. As it was shown first in Ref.[5] the condensation of neutrinos in the Galaxy should speed up their annihilation resulting in the generation of cosmic rays. Therefore the comparison of the predicted cosmic ray flux due to such annihilation with the observed flux allows one to obtain nevertheless some restrictions on the heavy neutrino mass. The effect of the neutrino condensation caused by the gravitational binding of heavy neutrinos in the gravitational field of collapsing gas at the stage of Galaxy formation (which was discovered in Ref.[6]) was used in Ref.[8] for the analysis of the implications of weak annihilation of supersymmetric relic particles in the Galaxy.

In the present paper we carry out a detailed analysis of the influence of the effects of the very heavy neutrino annihilation (\( m_{\nu} > 44 \) GeV) on the cosmic ray production in the Galaxy and also investigate the possibility of searching for such neutrinos at accelerators in the reaction \( e^+ e^- \to \nu \bar{\nu} \gamma \). To be explicit, we consider the standard electroweak model, including however one additional family of fermions. Then the heavy neutrino \( \nu \) and a heavy charged lepton \( L \) form a standard \( SU(2)_L \) doublet. In order to ensure the stability of the heavy neutrino \( \nu \) we assume that \( m_{\nu} > M_L \) and that the heavy neutrino is a Dirac neutrino. The organization of the paper is the following. In sect.2 we compute the residual relic concentration of heavy neutrinos taking into account the main processes of their annihilation, i.e. \( \nu \bar{\nu} \to W^+ W^- \), \( \nu \bar{\nu} \to f \bar{f} \).

In sect.3 the distribution of collisionless particles in galaxies is considered. In sect.4 we present the computation of the cosmic ray spectra due to neutrino annihilation and investigate the possible constraints on the mass of stable heavy neutrinos. In sect.5 we analyze the role of Higgs meson in the cosmological constraint on neutrino mass. In sect.6 we discuss the possibility of searching for the heavy neutrinos at accelerators in the reaction \( e^+ e^- \to \nu \bar{\nu} \gamma \). 

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2. THE RESIDUAL RELIC CONCENTRATION OF HEAVY NEUTRINOS
IN THE UNIVERSE

In the early Universe at high temperatures \( T \gg m_\nu \) heavy neutrinos (if they exist) should be in thermal equilibrium with the other kinds of particles and their concentration should be compatible with that of photons. As the temperature in the Universe drops the neutrinos become non-relativistic at \( T = m_\nu \) and their abundance falls of rapidly according to \( n_\nu \sim \exp(-m_\nu/T) \) although they are still in thermal equilibrium with the other particles. However, in the further expansion of the Universe as the temperature drops below the freeze-out temperature \( T_f \), the weak interaction processes become too slow to keep neutrinos in equilibrium with the other particles. The equilibrium is destroyed when the rate of change of the equilibrium concentration due to the temperature decrease turns out to be comparable to the rate of the equilibrium reactions. As a consequence the concentration of heavy neutrinos fails to follow the equilibrium concentration and the exponential drop of the concentration becomes much slower.

The residual relic concentration of heavy neutrinos in the Universe is given, according to Ref.[7], by

\[
n_\nu = \left( \frac{2}{2\pi^2} \right)^2 \frac{1}{m_\nu^2} n_y \rho \left( \frac{T_f}{m_\nu} \right)^{3/2} \exp(-m_\nu/T_f) \quad (1)
\]

where \( n_y = 0.241^{-3} \) is the equilibrium photon concentration; \( T = 2.7K \) is the present photon temperature; the factor \( 2/2\pi^2 \) takes into account the increase of the photon temperature due to the annihilation of the particles after the quenching of heavy neutrinos; \( g_\nu^* \nu(T) = N_y \sqrt{(7/8)} N_f \) is the number of effective degrees of freedom (the photon contribution to \( g_\nu^* \) is 2); quantity

\[
r_f = \left( \frac{g_\nu^*}{4\pi T_f m_\nu} \right)^{-3/2} \exp(-m_\nu/T_f) \quad (2)
\]

is the relative equilibrium concentration of heavy neutrinos at the moment of quenching; \( g_\nu^* \) is the number of particle spin states (for photons and massive fermions \( g_\nu^* = 2 \)). The freeze-out temperature is defined by

\[
\frac{n_\nu}{T_f} = \frac{1}{2} \ln \left( \frac{T_f}{m_\nu} \right) = \frac{1}{2} \ln \left( \frac{40 + \ln(\eta^2 m_p^2) + \ln(n_\nu/m_\nu) + \ln(g_\nu^* g_\nu^{1/2})}{m_\nu} \right),
\]

where \( m_p \) is the proton mass and \( \eta \) is the neutrino velocity (in units of the velocity of light).

Taking into account eq.(3), expression (1) for the relic concentration of heavy neutrinos can be written as

\[
n_\nu \approx 2 \times 10^{-18} g_\nu^{1/2} \left( m_\nu/m_p \right)^2 \left( \eta^2 m_p^2 \right)^{-1} \left( 40 + \ln(\eta^2 m_p^2) + \ln(n_\nu/m_\nu) + \ln(g_\nu^* g_\nu^{1/2}) \right) n_y \quad (4)
\]

where the condition \( T_f \ll m_\nu \) is assumed.

In order to calculate \( n_\nu \) we have to know the total annihilation cross section of neutrinos at freeze-out. At that moment the heavy neutrinos are non-relativistic [7], so in the annihilation their total energy is \( \frac{3}{4} \approx 2m_\nu \). The analysis of the annihilation reactions of heavy neutrinos shows that:

1. In the region of neutrino masses \( m_\nu \gg 100 \text{ GeV} \) the reaction

\[
\nu - \bar{\nu} \rightarrow Z \rightarrow W^+ W^-
\]

through the s-channel Z-boson exchange starts dominating the total cross section of the neutrino annihilation [4] only if the
total energy $\frac{1}{2} \vec{s}$ is not in the vicinity of the resonant Higgs boson peak.

2. The relative contributions to the process (5) of the diagrams with the heavy lepton exchange and of the interference diagrams are suppressed in comparison with the s-channel Z-boson exchange if $(m_{\nu}^2 \gg 10)$ or $(m_{\nu}^2 \gg 1, m_{\ell}^2 \gg 10 m_{\nu}^2)$, where $\tilde{m}_{\nu} = m_{\nu}/m_{W}$.

3. In the region $\tilde{m}_{\nu} > 2$, the annihilation cross section for reaction (5) is given approximately by

$$\sigma \beta \approx 6.8 \frac{-2}{m_{\nu}^2 + 4} \text{ pb}. \quad (6)$$

4. The solution of eq. (3) by an iterative procedure [4] yields for the freeze-out temperature the value $T_f = m_{\nu}/30$.

We note that in eq. (4) the expression in square brackets depends very weakly on the neutrino mass $m_{\nu}$ and can be replaced by a constant value. As a result of this approximation we can write the residual relic concentration of heavy neutrinos in the form

$$n_{\nu} = \frac{2 \times 10^{-9}}{\tilde{m}_{\nu}^2 (m_{\nu}^2 + 4)} \text{ cm}^{-3}. \quad (7)$$

if $\tilde{m}_{\nu} > 2$. Therefore, in the region of large neutrino masses due to the process (5), the residual neutrino concentration decreases fast enough as $n_{\nu} \propto \tilde{m}_{\nu}^{-3}$.

In the region of neutrino masses $m_{\nu} < 100$ GeV the dominant annihilation process is the neutrino annihilation into a fermion pair

$$\nu \bar{\nu} \rightarrow Z \rightarrow f \bar{f}. \quad (8)$$

through the Z-boson exchange in the s-channel. Let us write the total cross section for this process:

$$\sigma = \frac{G^2 N c^2 D_2}{2 \pi} \left[ s \right] \left[ s^2 + g^2 (1 + \beta^2 p^2 / 3) \right] + 2 \left( g_v^2 + g_s^2 \right) \frac{m_{\nu}^2}{s (s - 2m_{\nu}^2)} + 4 \left( g_v^2 + g_s^2 \right) m_{\nu}^2 m_{\nu}^2 m_{Z}^2 (s - m_{Z}^2 - 2)^3,$$

where $\theta_w$ is the Weinberg angle, $G$ is the Fermi constant, $N$ is a color factor (N=1 for leptons and N=3 for quarks), $m_{\nu}$ is the fermion mass, $\beta = (1 - 4m_{\nu}^2 / s)^{1/2}$, $r = (1 - 4m_{\nu}^2 / s)^{1/2}$. $g_v$ and $g_s$ are the standard vector and axial constant, $D_2 = (s - m_{Z}^2 - 2m_{\nu}^2)^{-1}$. $s$ is the Z-boson width. Here we neglected the Higgs boson contribution, that is reasonable either at $\sqrt{s} \approx 2m_{\nu}$ due to the suppression of Higgs exchange, or in the region of large Higgs boson mass $m_{H}^2 > m_{Z}^2$.

The appearance in expression (9) of the term proportional to $m_{\nu}^2$ is inevitable because of the axial part of the weak neutral current. However the role of the additional terms in formula (9) is important only for heavy fermions, which can be produced in the annihilation of neutrinos with large masses. But in this case the dominant reaction of annihilation is the process $\nu \bar{\nu} \rightarrow W^{+} W^{-}$, so that the contribution of these terms to the residual neutrino concentration is not sensible. Thus, in particular, the actual values of the top quark mass and the masses of new quarks of fourth generation are not very important in the calculation of $n_{\nu}$.
The results of the numerical calculations of the residual concentration of heavy neutrinos on the basis of eqs.(3),(4),(9) (and the formulas of Ref.14) for the reaction (5) are presented in Fig.1. The calculations were carried out at the mass of a heavy lepton $N_L = 1$ TeV (however the actual value of $N_L$ is not very important) and at the top quark mass $m_t = 100$ GeV. The masses of the other heavy quarks and the Higgs boson mass were assumed to be above 1 TeV. The annihilation cross section was evaluated at

$$s = 4 \sigma_s = 4 m_{\nu}^2 + 12 m_{\nu} T_F$$

(10)

since the annihilating neutrinos are non-relativistic and constitute a Boltzmann-distributed gas, so that $\langle p^2 \rangle = m_{\nu} T_F$ (the results of the numerical calculations depend very weakly on the value of the second term in eq.(10)).

Qualitatively the behaviour of $n_\nu$ (as shown in Fig.1) is easily understood. In the region $m_{\nu} \sim M_\nu/2$ the residual relic neutrino concentration is small due to the huge value of the cross section at resonance in the $s$-channel. With the increase of the neutrino mass the cross section for neutrino annihilation into fermions drops and this leads to the increase of the residual concentration. But at $m_{\nu} > M_\nu$ the additional annihilation channel $\nu \bar{\nu} \to W^+ W^-$ opens and gradually becomes the dominant one, since its cross section grows like $m_{\nu}^2$ (till $m_{\nu} < M_\nu$) and the residual concentration drops again.

3. DISTRIBUTION OF COLLISIONLESS PARTICLES IN GALAXIES

The very heavy neutrinos decoupled from radiation at very early epochs ($t \sim 10^{-10}$ sec) contrary to baryons which remain in thermal equilibrium with photons till the last scattering at redshift $z \sim 1500$. However, as soon as the cosmological expansion is dominated by matter (for instance by the heavy neutrino or baryonic densities at $z \sim 10^4 - 10^5$), then heavy neutrinos may gravitationally cluster into clouds, thus forming a primordial density seed where galaxy may later condense. After the recombination once baryons become neutral ($z \sim 1500$), also the baryonic matter plays a role in clustering and the baryons "feel" the already existing primordial gravitational seeds in a coupled gravitational system [8]. Such a seed role (of cold dark matter) is able to speed up the baryonic clustering and to reduce the primordial density contrast for baryons up to the observed bounds ($\Delta T/T < 10^{-4} - 10^{-5}$). The subsequent baryonic galaxy formation is due to selfgravity and to energy dissipation. The energy dissipation may amplify the galactic density contrast (with respect to the cosmological one) by many orders of magnitude.

At the stage of the Galaxy formation neutrinos can interact with matter by gravitation only. Therefore no energy dissipation due to radiation takes place in the gas of heavy neutrinos as it is in the case of the ordinary matter. Nevertheless, as it was shown in Ref.15, the motion of neutrinos in the nonstatic gravitational field of ordinary matter, which contracts due to the energy dissipation via radiation, provides an effective mechanism of energy dissipation for neutrinos too. As a consequence the
contracting ordinary matter induces the collapse of the neutrino gas and leads to the following significant increase in the neutrino (antineutrino) density in the Galaxy \(5\):

\[
\rho_{\nu} = n_{\nu} \rho_G / \rho_0, \tag{11}
\]

where \(\rho_G = 10^{-22} \text{g/cm}^3\) is the average density of the matter in the Galaxy, \(\rho_0 = 2 \times 10^{-30} \text{g/cm}^3\) is the density of the matter in the Universe.

For future applications let us consider the mechanism suggested in \(5\), in more detail.

When ordinary matter ("baryons") contracts due to the energy dissipation via radiation, neutral heavy leptons ("neutrinos") move in a potential, which varies with time. Since the energy of a particle moving in a time-variable potential is generally not conserved, neutrinos can reduce their energy and, consequently, increase their density.

To illustrate this mechanism, let's treat, following \(5\), the simplest case of particle motion along radial orbits in the central part of the contracting baryon system, where the density is independent of radius. If \(\rho_B(t)\) and \(\rho_\nu(t)\) are, respectively, the central densities of neutrinos and baryons, the motion of neutrinos is determined by the equation

\[
d^2r/dt^2 = -\omega^2(t) r \tag{12}
\]

where

\[
\omega(t) = \left(\frac{G}{(4\pi/3)} \rho_B(t) + \rho_\nu(t)\right)^{1/2}
\]

Let baryons increase slowly their density. For slowly varying \(\omega\) the amplitude of oscillations is provided by the adiabatic invariant

\[
A^2\omega(t) = A^2_0 \omega(0) \tag{13}
\]

As \(\omega\) grows the oscillation amplitude of neutrinos decreases and their density increases, respectively, according to the equation

\[
\frac{\rho_\nu(t) / \rho_\nu(0)}{\rho_B(t) / \rho_B(0)} = \left(\frac{A^2_0}{A^2} \sqrt{\omega(t) / \omega(0)}\right)^{3/2} = \left(\frac{\rho_\nu(t)}{\rho_B(t) / \rho_B(0) + \rho_\nu(0)}\right)^{3/4}. \tag{14}
\]

Introducing the variable

\[
x(t) = \rho_B(t) / \rho_\nu(0) \tag{15}
\]

one can conveniently rewrite eq.\((14)\) in the following form

\[
(x + 1)^3 = x(t) \rho_B(t) / \rho_B(0) x(0) (1 + x(0))^3. \tag{16}
\]

Two limiting cases are possible. In the first one the expression on the right-hand side of eq.\((16)\) is small, what corresponds to the condition

\[
\rho_B(t) \ll \rho_\nu(t), \tag{17}
\]

In this case the neutrino density grows as

\[
\frac{\rho_\nu(t) / \rho_\nu(0)}{\rho_B(t) / \rho_B(0)} = 1 + 3(\rho_B(t) / \rho_B(0) - \rho_B(0)/\rho_\nu(0)) \tag{18}
\]

whereas baryon density while growing remains still smaller than the initial density of neutrinos. However, due to radiation energy losses, baryon density can grow to make the condition \((17)\) invalid, so that the opposite condition

\[
\rho_B(t) \gg \rho_\nu(t) \tag{19}
\]

holds. Then one obtains from eq.\((16)\) that the density of neutrinos grows with time as

\[
\frac{\rho_\nu(t) / \rho_\nu(0)}{\rho_B(t) / \rho_B(0)} = \frac{\rho_B(t) / \rho_B(0) + \rho_\nu(0)}{\rho_B(t) / \rho_B(0)}. \tag{20}
\]

and therefore
\[ \frac{\rho_b(t)}{\rho_\nu(t)} \sim \rho_b(t)^{1/4}. \quad (21) \]

The same eqs.(20)-(21) hold for the motion of neutrinos along circular orbits, when [5]

\[ r \nu = \text{const} \]
\[ v^2 \equiv G \rho \quad r^2 \]
\[ r^2 \rho^{1/2} = \text{const} \]

and

\[ \frac{\rho_\nu(t)}{\rho_\nu(0)} = \left( \frac{r \nu}{r_0} \right)^3 = \left( \frac{\rho_b(t)}{\rho_\nu(0) + \rho_b(0)} \right)^{3/4}. \]

The analytical treatment, given above, was completed in [5] by numerical models, proving the same law of condensation (eq.(21)) to be valid in the cases of nonradial and noncircular motions and also for inhomogeneous density. The result was shown to be independent of the details of the numerical methods used. So both analytical and numerical calculations [5] prove the conclusion on the condensation of collisionless gas in selfgravitating systems, while contracting due to radiation energy losses.

The considered process of condensation of a collisionless gas may take place in any collapsing system of ordinary matter, provided that at all the stages contraction is dominated by selfgravity. It is generally assumed that this condition is not satisfied at the initial stages of formation of objects smaller than globular clusters, at which the development of the thermal instability and the effects of the outer pressure of the hot gas are dominant. So the considered mechanism should be effective in the course of galaxy formation but does not seem to work in the process of formation of globular clusters and smaller astronomical objects (stars, in particular).

If heavy neutrinos (or some other hypothetical particles) dominate the cosmological density, such a mechanism provides an explanation for the formation of massive halos of galaxies by these particles. It was used e.g. in [9] for the scenario of a neutrino dominated Universe to explain why massive neutrinos remain at the periphery of galaxies and do not contribute much to the density in the central parts of galaxies. In this case, the assumption that the hypothetical particles dominate in the galactic halo allows one, without any recourse to an explicit dynamical model of halo formation, to get estimates for the density distribution in halo, to analyze the particle distribution in the Galaxy and to evaluate possible effects of their weak annihilation (as it was done in [6] for supersymmetric particles). However the universality of the mechanism [5] permits its application to the case of a small contribution of the hypothetical particles to the total density, thus providing a reasonable estimate for the expected distribution of the particles in the Galaxy and their possible effects, even if they do not play any significant role in the dynamics of halo formation. The actual distribution of collisionless particles deserves special analysis for this case, what will be done in a separate work. As an estimate, following [5], we used above for the averaged central density of massive neutrinos eq.(11) linearly relating it to the averaged baryonic density in the Galaxy. One can check that such a relationship corresponds to the law of condensation (21) for the ratio of the central densities of baryons and neutrinos.
4. THE SPECTRUM OF COSMIC RAYS

The condensation of heavy neutrinos in the Galaxy leads to the increase in the rate of the neutrino annihilation, resulting in a copious production of cosmic rays. The most stringent limit on the mass of heavy neutrinos can be obtained by considering the electronic component of cosmic rays. Then, in this section, we shall evaluate the output of relativistic electrons by exploiting the following formula of the flux [5]:

\[ J = \frac{dn}{dt} \frac{1}{T_e} \left( \frac{c}{4} \right) (5/2 \pi) \right) \left( \frac{\text{cm}^{-2} \text{s}^{-1} \text{sr}^{-1}}{\Gamma} \right) \]  \hspace{1cm} (22)

where

\[ \frac{dn}{dt} = n_{\nu}^2 \sigma_{\nu} \Gamma \]  \hspace{1cm} (23)

is the rate of neutrino annihilation in the Galaxy per unit volume, \( \sigma_{\nu} \) is the cross section of the neutrino annihilation in the Galaxy, \( T_e = 10^9 \) years is the life-time of cosmic rays in the Galaxy, \( c \) is the velocity of light, \( \delta \) is the number of relativistic electrons with energy in the interval \( E_e - \Delta E/2 < E_e < E_e + \Delta E/2 \) which are produced in one act of the neutrino annihilation.

Substituting formulae (11), (23) into eq.(22) gives us the general expression for the flux of cosmic rays:

\[ J = 2.8 \times 10^{12} n_{\nu}^2 \sigma_{\nu} \delta (\sigma_{\nu} \beta / \text{pb}) \]  \hspace{1cm} (24)

In order to obtain the constraint on the heavy neutrino mass we consider first the annihilation channel \( \bar{\nu} \nu \rightarrow e^+ e^- \) in the Galaxy. Since heavy neutrinos in the annihilation in the Galaxy are non-relativistic

\[ E_e = m_{\nu} = m_{\nu} (\nu/c)^2/2 \]  \hspace{1cm} (25)

where \( v=300 \text{ km/s} \) is the velocity of neutrinos in the Galaxy, then the ultrarelativistic electrons in the annihilation reaction (6) are produced practically monochromatic with \( E_e = m_{\nu} \) (even for \( m_{\nu} = 1 \text{ TeV} \) the energy spread is \( \Delta E \approx 1 \text{ MeV} \)). The electrons can also be produced in the secondary processes from the decays of \( \mu^- \)-leptons and quarks, but these processes contribute only to the soft part of the cosmic ray spectrum and therefore we shall neglect them.

The annihilation cross section for the process \( \bar{\nu} \nu \rightarrow e^+ e^- \) can be written, according to formula (9), as

\[ \sigma_{\nu} \beta = 2.8 \times n_{\nu}^2 \frac{3}{2} \Delta E \]  \hspace{1cm} (26)

and therefore we have for the flux of cosmic electrons

\[ J_e = 8.1 \times 10^{12} n_{\nu}^2 \frac{3}{2} \Delta E \]  \hspace{1cm} (27)

where \( n_{\nu} \) is calculated at \( \delta \), see eq.(10) and \( \Delta E \) is used. The experimental energy spectrum of cosmic electrons (10) integrated over the energy resolution \( \Delta E \) of the detector is given by

\[ J_{e}^{\text{exp}} = 1.16 \times 10^{-2} n_{\nu}^2 e^{-2.6 \Delta E} \]  \hspace{1cm} (28)

where \( 3 \text{ GeV} < E_e < 300 \text{ GeV} \) and \( \Delta E < 4 \text{ GeV} \).

The results of the numerical calculations of the electron flux (27) are presented in Fig.2. Also the experimental flux (28) is shown in this figure for the electron energy \( E_e = m_{\nu} \) and the energy resolution \( \Delta E = 1 \text{ GeV} \). As we can see in this case the existence of heavy neutrinos is forbidden in the mass range

\[ 80 \text{ GeV} < m_{\nu} < 115 \text{ GeV} \]  \hspace{1cm} (29)

The absence of the constraint in the mass range 44 GeV - 80 GeV is due to the smallness of the relic neutrino concentration (the annihilation cross section of neutrinos in the early Universe is very large in the vicinity of the Z-boson peak). The constraint in the region of large neutrino masses is a consequence of the
rapid decreasing of the flux \( J_e \sim m_{\nu}^{-1}(m_{\nu}^2 + 4)^{-2} \) in comparison with the experimental flux \( J_{e}^{\exp} \sim m_{\nu}^{-2.6} \).

Let us notice that any bound on the neutrino mass does imply corresponding limit on the present energy density in the Universe. This point can be seen from Fig.3, where the results of numerical calculations of the dimensionless neutrino energy density \( \dot{\rho}_\nu = \dot{\rho}_\nu/\rho_0 \) at \( \rho_0 = 10^{-29} \text{ g/cm}^3 \) are plotted. In the neutrino mass region below 80 GeV the relic neutrino energy density \( \dot{\rho}_\nu \sim 2\alpha_{\nu}/\rho_0 \) is too small (below the luminous baryonic density \( \dot{\rho}_b \sim 3 \times 10^{-3} \)) to be of observational interest. Only in the region above 115 GeV we could expect significant contributions and cosmological implications of the neutrino component (comparable to the baryonic one).

We should note that the constraint (29) is very sensitive to the value of the energy resolution \( \Delta E \). In particular, the improvement of the resolution from 1 GeV to 1 MeV \( \Delta E = 1 \text{ MeV} \) is the width of the energy distribution of electrons produced in neutrino annihilation in the Galaxy) would give a relative gain of the order \( 10^3 \), thus allowing to remove the constraint on \( m_{\nu} \) in the region of 1 TeV and leading to a relic neutrino energy density below the baryonic one.

The other possibility to obtain a constraint on \( m_{\nu} \) is to consider the process of neutrino annihilation \( \nu \bar{\nu} \rightarrow W^+W^- \) and the electron production from \( W \)-boson decays \( W^{-} \rightarrow e^{-}\bar{\nu}_e \). However, in this case there is a strong suppression factor \( \delta \sim \Delta E/(\sqrt{q}) \) (30) of the flux of cosmic rays. The appearance of this factor is a consequence of the flat energy distribution of the electrons due to the relativistic motion of \( W \)-bosons in the reaction (5)

\[
(q_0 - q)/2 \leq E_e \leq (q_0 + q)/2
\]

where \( q_0, q \) are the energy and the momentum of the \( W \)-boson. The coefficient \( 1/9 \) in eq. (30) is the branching ratio into the channel \( W^- \rightarrow e^-\bar{\nu}_e \).

So we obtain the following approximate expression for the flux of cosmic rays (electrons) from the cascade \( \bar{\nu} \nu \rightarrow W^+W^- \rightarrow e^-\bar{\nu}_e \) :

\[
J_e \approx 2.8 \times 10^{11} \frac{m_{\nu}}{Q} \left( m_{\nu}^2 + 4 \right)^{-1} \left( \Delta E/\text{GeV} \right) \approx m_{\nu}^{-1} \quad (32)
\]

The result of the numerical calculations of the flux \( J_{we} \) is also presented in Fig.2 at \( \Delta E = 1 \text{ GeV} \). As one can see from Fig.2 the flux (32) is lower than the experimental flux (28) due to the fast decreasing of \( J_{we} \sim m_{\nu}^{3-2} \left( m_{\nu}^2 + 4 \right)^{-1} \) and therefore in this case there is no additional constraint on \( m_{\nu} \).

Nevertheless, we note that, at least in principle, the annihilation reaction \( \nu \bar{\nu} \rightarrow W^+W^- \) with positron production from \( W \)-boson decays \( W^- \rightarrow e^-\bar{\nu}_e \) could give an important constraint on \( m_{\nu} \). The flux of positrons in this case would be exactly the same as for the electron production (32). However, the energy resolution of the positron detection is very poor (for example at the positron energy \( E_e \approx 30 \text{ GeV} \) the resolution is \( \Delta E \approx 20 \text{ GeV} \) [11]) but the experimental flux \( J_{e}^{\exp} \) as well as the flux \( J_{we} \) (in contrast with \( J_e \) for the reaction \( \bar{\nu} \nu \rightarrow e^{-}\bar{\nu}_e \)) are proportional to the energy resolution \( \Delta E \) and their relative value does not depend on \( \Delta E \) (at \( \Delta E \ll E_e \)). In the energy region where separate measurements of electrons and positrons are available \( (E_e > 50 \text{ GeV}) \) a significant excess of electrons was found [11]. At higher energies we can use only theoretical predictions. If we assume, for instance, the validity of the model of dynamical halo, then it
gives us the ratio \( N(e^+) / N(e^-) \approx 10^{-2} \) at \( E_e > 50 \text{ GeV} \) [10]. It follows from Fig.2 that in this case the limit on the heavy neutrino mass would be \( m_\nu > 500 \text{ GeV} \).

5. THE ROLE OF HIGGS MESON IN THE COSMOLOGICAL CONSTRAINT ON NEUTRINO MASS

It was assumed in previous sections that the Higgs meson is heavy \( (m_H \gg m_\nu) \) so that the contribution of Higgs meson exchange

\[ \nu \tilde{\nu} \rightarrow H \rightarrow \nu^* \tilde{\nu} \]  

(33)
to the process of neutrino annihilation can be neglected.

In this section we will take into account the finite mass of the Higgs meson and show how the reaction (33) modifies the restriction (29) which was obtained by us in the limit case \( m_H \rightarrow 0 \). In the case of a finite mass of the Higgs meson, the cross section for the process (33) is given by

\[ \sigma_H = \sigma_{\text{HH}} + \sigma_{\text{LH}} = \frac{G_F^2 m_H^2}{16\pi} \frac{\beta_\nu}{s} \sqrt{s} \cdot \frac{1}{D_H} \left[ G_{\text{HR}} + 2 \left( \frac{s - m_H^2}{s} \right) G_{\text{LH}} \right] \]  

(34)

\[ G_{\text{HR}} = \sqrt{s} - 4 \alpha_\nu \left( s^2 - 4 \hat{s} + 12 \right) \]  

(35)

\[ G_{\text{LH}} = - \frac{m_\nu^2}{2} \left( \frac{s + 2}{s} \right) + 2 \left( \frac{s + 2}{s} \right) \frac{\hat{m}_\nu^2 - \hat{s}^2 + 2 \hat{s} - 8 + \hat{s} - 2}{s^2 \beta_\nu} \]  

(36)

\[ \frac{\hat{m}_\nu^2}{s^2 \beta_\nu} \left( \frac{s - 2}{s} \right) + \hat{m}_L^2 \left[ 2 \left( \frac{s + 2}{s} \right) \frac{\hat{m}_\nu^2 - \hat{s}^2 + \hat{s} - 2}{s^2 \beta_\nu} \right] \]

\[ - \frac{\hat{m}_\nu^2}{s^2 \beta_\nu} \left( \frac{s + 2}{s} \right) + \hat{m}_L^2 \left[ \left( \frac{s - 2}{s} \right) \frac{\hat{m}_\nu^2 - \hat{s}^2 + \hat{s} + 4}{s^2 \beta_\nu} \right] \]

\[ \ln \left[ \frac{\beta_\nu \beta_{\text{LH}}}{-s(m_\nu^2 s^2 + 2(1 + m_\nu^2 m_H^2)} \right] \]

where \( \beta_\nu = (1 - 4 \frac{m_H^2}{s})^{1/2} \), \( D_H = \left[ (-s - m_H^2) + \Gamma_H^2 \right]^{1/2} \). \( \Gamma_H \) is the width of the Higgs meson, \( s = s/m_\nu^2 \), \( m_L = m_L/m_\nu \). The second term \( (\sigma_{\text{LH}}) \) in eq.(34) is the result of the interference between the diagrams of the s-channel H boson exchange and the t-channel charged L-lepton exchange. The interference between the diagrams of Z as-'H boson exchanges in the total cross section does not occur. The cross section \( \sigma_H \) was calculated for the first time in Ref.[41] but we note that there is a difference between the expression for \( G_{\text{LH}} \) in formula (36) and the corresponding expression of Ref.[41].

As the temperature in the Universe drops very heavy neutrinos quickly become non-relativistic and we can put \( \hat{m}_\nu^2 \approx \hat{s} \).\( \hat{\Delta} \ll 4 \hat{s} \). Moreover we assume that the masses of neutrino and L-lepton are large (as numerical calculations show, the residual concentration of very heavy relic neutrinos and the output of cosmic rays from neutrino annihilation depend weakly on \( m_L \)) and besides that \( m_L \gg m_\nu \) \( \gg H^2 \). Then

\[ \beta \sigma_H \approx \frac{G_F^2 m_H^2}{16\pi} \frac{\hat{m}_\nu^2}{s} \left( 6 - \frac{m_H^2}{m_\nu^2} \right) \]  

(37)

where \( D_{\text{LH}} = D_H m_\nu^4 \). We note that this expression can be negative as well as positive since \( \sigma_H \) is only a part of the total cross section \( \nu \tilde{\nu} \rightarrow \nu \tilde{\nu} \).

We should compare expression (37) with the formula for the cross section of the process (5) in the approximation under consideration

\[ \beta \sigma_H \approx \frac{G_F^2 m_H^2}{16\pi} \frac{\hat{m}_\nu^2}{s} \]  

(38)

and thus

\[ \sigma_H/\sigma_V \approx \frac{2 m_\nu^2 \hat{s}^2}{\sqrt{s} \hat{s}^2 s \hat{s}^2} \left( 6 - \frac{m_H^2}{m_\nu^2} \right) \]  

(39)
We see that at $m_H \gg m_\nu$ or $m_H \ll m_\nu$, the ratio $\sigma_\nu/\sigma_W$ is small and only in the region of resonance $m_H \approx 2m_\nu$, the ratio $\sigma_\nu/\sigma_W \approx m_\nu^2/T_H^2$ can be large. Indeed, calculating the residual concentration of the relic neutrinos we evaluate the annihilation cross section at $s = <q_f>$ (according to expression (10) this corresponds to $\Delta = 2m_\nu/T_H$) and at the width value $\Gamma_H = 61 (m_H/500\text{GeV})^3$ GeV in the case $m_H^+ m_Z^- \ll m_H$, so that $\sigma_\nu/\sigma_W \approx 0.4m_\nu^2/T_H^2 \gg 1$ at least for $250 \text{GeV} < m_H < 500 \text{GeV}$.

In the region of neutrino masses below the mass $m_\nu$ the dominant annihilation channel of heavy neutrinos is in the reaction (8) and

$$\nu \bar{\nu} \rightarrow H \rightarrow f \bar{f},$$

where $f$ denotes a fermion. The formula for the cross section of the process (40) is given in Ref.14 and using also the result (9) we obtain (at $m_\nu < m_W$)

$$\sigma_\nu = 3 \frac{m_\nu^4}{m_W^4} \frac{D_\nu}{D_\gamma}.$$  \hspace{1cm} (41)

Far from the $H$-resonance expression (41) is small. However, at the $H$-resonance ($m_H = 2m_\nu$) and far from the $2$-resonance this expression takes the form $\sigma_\nu/\sigma_W \approx 0.1(m_\nu/T_H^2)^2$ due to the smallness of the width of the Higgs meson $\Gamma_H = 0.45(m_H/100\text{GeV})$ GeV in this region (here we put the value $m_f$ equal to the mass of the $b$-quark which is the heaviest fermion in this region since the $t$-quark mass $m_t > m_W$).

Thus if the neutrino mass is around the resonance one for the Higgs meson exchange the annihilation cross section of very heavy neutrinos sharply increases, and this leads to a sharp decreasing in the residual ( relic) concentration of such neutrinos in the Universe. On the other side, in the calculations of the annihilation cross section of very heavy neutrinos in the halo of the Galaxy the effect of Higgs meson is not important. This is due to the fact that such neutrinos in the Galaxy are strongly non-relativistic (25) and therefore the value $\Delta$ in this case is very small ( $\Delta \approx 4 \cdot 10^{-6} m_\nu^2$). Also an additional suppression factor in eq.(41) will appear due to the smallness of the electron mass when we consider the most interesting channel of the annihilation in the Galaxy $\nu \bar{\nu} \rightarrow e^+e^-$ which produces a practically monochromatic line in the spectrum of cosmic electrons at energies $E_e \approx m_\nu$.

The results of the numerical calculations of the electron flux from the very heavy neutrino annihilation in the galactic halo are presented in Fig.4. In these calculations the residual concentration of relic neutrinos was evaluated by formula (4) and the flux of relativistic electrons was evaluated according to expression (22). For the averaging of the cross section over the temperature we used the formula of Ref.15

$$<\sigma v> = \int_0^{m_\nu} \frac{d \sigma}{ds} \frac{d^3 \mathbf{p}}{p^2} \left(2\pi \alpha \right)^2 K_0(s^1/2/T) \sigma (\alpha).$$ \hspace{1cm} (42)

where $K_0(z)$ is the MacDonald function.

The experimental spectrum of cosmic electrons integrated over the energy resolution of the detector $\Delta E = 1 \text{GeV}$ is also shown in Fig.4. As it follows from Fig.4, the Higgs meson exchange modifies significantly in the resonance region the constraint (29) (which was obtained assuming $m_H \gg m_\nu$) to the form $60 \text{GeV} < m_\nu < 80 \text{GeV}$ at $m_H = 200 \text{GeV}$.

In this section we have shown that the Higgs meson
exchange affects significantly the constraint on the mass of very heavy neutrinos in the case when the neutrino mass is around the resonance value \( m_N = 2 m \). The numerical calculations and the comparison with the experimental data show that, in this case, the constraint on the neutrino mass is absent due to the smallness of the residual ( relic) concentration of neutrinos in the neutrino mass range \( \Delta m_N \sim 10 \Gamma_H \). If below the threshold of \( \psi \)-boson pair production this value is not large (\( \Delta m_N \lesssim 0.1 \text{ GeV} \)), then above the threshold the width of Higgs meson is large and the constraint on neutrino mass is eliminated in the region \( \Delta m_N \) of the order of 10 GeV.

6. ON THE POSSIBILITY OF SEARCHING FOR HEAVY NEUTRINOS AT ACCELERATORS

The search for stable heavy neutrinos is a difficult problem. It seems that, to this aim, the process

\[ e^+ e^- \rightarrow \nu \bar{\nu} \gamma \]  

(43)

with the detection of a single photon could be more attractive. This process was suggested a long ago [14] as a neutrino counting experiment. There are five Feynman diagrams corresponding to the process (43) and they are shown in Fig.5. If the final state of the reaction (43) contains heavy neutrinos of the new generation then this process is described only by the two Feynman diagrams a) and b) corresponding to the \( s \)-channel \( Z \)-boson exchange.

Using standard techniques we calculate the differential cross section for the process (43) with the heavy neutrinos in the final state:

\[
\frac{\Delta \sigma}{\Delta x \Delta z} = \frac{a}{6 \pi} \frac{G_F^2 (\frac{q^2}{2} + \frac{v^2}{2})}{x (1 - z^2)} \frac{[1 - 4 \frac{m_N^2}{q^2}]^{1/2} (\frac{\alpha}{2} - \frac{\beta^2}{2} + \frac{\gamma^2}{2})^{-1}}{1 - 4 \frac{m_N^2}{q^2}} \frac{1}{(1 - x)^2}
\]

(44)

where \( \alpha = 1/137 \), \( x = 2v^2/\sqrt{s} \), \( \beta \) is the photon energy, \( z = \cos \theta \), \( \theta \) is the photon angle with respect to the direction of the electron beam, \( q^2 = s(1 - x) \).

In order to evaluate the total cross section we integrate eq.(44) over \( z \) and \( x \):

\[
- \frac{\beta}{v_0} \leq z \leq \frac{\beta}{v_0} \quad \frac{2v_0}{\sqrt{s}} \leq x \leq 1 - 4 \frac{m_N^2}{s} \quad (45)
\]

where \( v_0 \) (and \( \beta_0 \)) is the experimental cut-off.

The results of numerical calculations of the total cross section for different neutrino masses at \( v_0 = 1.5 \text{ GeV} \) and \( \beta_0 = 30^\circ \) are shown in Fig.6. We plotted also the results for the process (43) with the light \( \nu_e \)-\( \nu_e \)-neutrinos in the final state (the calculation for \( \nu_e \) takes into account the charged current as well as the neutral one). As we see from Fig.6 the background process with the light neutrinos dominates over the process (43) with heavy neutrinos due to the charged current and therefore we can hope only for the investigation in the vicinity of the \( Z \)-boson peak: \( m_N \approx 50 \pm 100 \text{ GeV} \). In this mass region at a luminosity of the accelerator \( L = 10^{32} \text{cm}^{-2} \text{c}^{-1} \) we could expect \( 30 \sim 300 \) events/year with heavy neutrinos. We note that experiments of this kind allow the investigation in the region \( m_N \sim M_Z/2 \) where it is extremely difficult to obtain cosmological constraints on \( m_N \).
The above analysis is based on the assumption of absence of mixing between the hypothetical heavy neutrino and the light neutrinos. The existence of such a mixing (without a special suppression) would lead to the instability of the heavy neutrinos and in this case it would be possible the direct detection of neutrinos by their decay products.

We note also that renormalization-group considerations restrict the number of quarks flavours to 16, since in this case QCD remains asymptotically free. Therefore it is possible to introduce only five additional neutrino generations within the framework of the Standard Model without violation of the asymptotic freedom. The analysis of the effective potential on the vacuum stability also allows for additional neutrino generations with masses of the order of a few hundred GeV (if the origin of this mass is not connected with the Higgs mechanism of symmetry breaking).

7. CONCLUSION

In this paper, using the idea of Ref. [5] about neutrino condensation in the gravitational field of collapsing matter at the stage of Galaxy formation, and analyzing the processes of cosmic rays production due to the relic neutrino annihilation in the Galaxy, we excluded the possibility of existence of heavy stable neutrinos in the mass range $60 < m_{\nu} < 115$ GeV. These bounds on neutrino masses imply also a corresponding bound on the total energy density of the Universe. We have shown that in the allowed neutrino mass range $44$ GeV $< m_{\nu} < 60$ GeV the neutrino energy density is less than the luminous baryonic density and only in the mass range $115$ GeV $< m_{\nu} < 300$ GeV the relic neutrino density can be large enough to be of observational interest (comparable to that of baryons), i.e., \( 3 \times 10^{-3} < \rho_{\nu}/\rho_c < 10^{-2} \), where \( \rho_c = 10^{-29} \text{ g/cm}^3 \). These constraints could be considerably extended by improving the precision of measurements of electron and positron spectra and their energy resolution, and by the separate measurements of electron and positron spectra at high energies. It seems that the study of photon production in the Galaxy could reduce the relic neutrino density to a negligible value even in comparison with the luminous baryonic one and, thus leading at least to a severe restriction on the role of such cold dark matter particles in solving the dark matter puzzle in the galactic halo.

Our treatment can be easily extended to any other weak-interacting stable (neutral) particles, and, what is more, the weaker is their interaction the larger is their residual relic concentration in the Universe and the more stringent constraints could be obtained on the parameters of these particles from astrophysical experiments. It is also applicable to the analysis of the expected distribution and effects in the Galaxy of hypothetical strongly interacting massive particles (SIMPs), assumed to form collisional, but nonradiating gas [15], which may also condense via the considered mechanism [5]. It is also important to emphasize that our approach is valid in the case of a multicomponent dark matter and may be used for species that give even a negligible contribution to the total energy density of the Universe.

The search for heavy neutrinos at accelerators in the reaction
\( e^- e^- \rightarrow \nu \nu \) could give the possibility to analyse the mass region \( m_\nu \sim \frac{\mu}{2} \), which is difficult for an astrophysical investigation.

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Fig. 1 The relic concentration of heavy neutrinos.

Fig. 2 The flux of cosmic electrons from the neutrino annihilation in the Galaxy as function of the neutrino mass.

Fig. 3 The relic neutrino energy density versus neutrino mass, where $\rho_c = 10^{-20} \text{g/cm}^3$ and LBM is the luminous baryonic matter contribution ($Q_b = 3 \times 10^{-3}$).

Fig. 4 The flux of cosmic electrons from the neutrino annihilation in the Galaxy as function of the neutrino mass for three values of Higgs meson mass.

Fig. 5 Feynman diagrams for the process $e^+ e^- \rightarrow \nu \overline{\nu}$.

Fig. 6 The total cross sections for the process $e^+ e^- \rightarrow \nu \overline{\nu}$; All values are in GeV. The value $M$ corresponds to the cutoff in photon energy $\omega < (1-4M^2/s)^{1/2}$ for the background process with massless neutrinos.