ON HADRON MULTIPLICITY IN e^+e^- EVENTS

INDUCED BY MASSIVE QUARKS

V. A. PETROV

Division of Theoretical Physics, IHEP, 142 284 Protvino, Russia
and
Theory Division, CERN, 1211 Geneva 23, Switzerland

A. V. KISSELEV

Division of Theoretical Physics, IHEP, 142 284 Protvino, Russia

ABSTRACT

We analyse the difference in the energy behaviour of hadron multiplicities associated to e^+e^- events induced by light and heavy quarks in the framework of two approaches: the so-called Naive Model and the QCD-motivated one. Expectations of both approaches are presented in comparison with the existing data (LEP, SLC).

INTRODUCTION

One of the many spectacular features related to heavy (c, b) quarks their essential hardness when produced in high energy collisions. For example, it is well established experimentally [1] that b-quarks produced in e^+e^- annihilation have a characteristic fragmentation function, strongly peaked at high momenta (see Fig. 1) and that they carry away nearly 70% of the total energy. (The first theoretical ideas on this phenomenon were put forward more than 15 years ago [2], but a fully satisfactory explanation and description of the heavy-quark fragmentation are still on order [3].) It is clear that this behaviour must be reflected in the energy dependence of the multiplicity of hadrons accompanying "primary" heavy quarks (in addition to their hadronic decay products).

Below we discuss two approaches to the analysis of the average multiplicity associated with heavy-quark production in e^+e^- annihilation: the so-called "naïve model" and the "more scientific" approach based on perturbative QCD (supplied with some additional hypotheses). Predictions of the two approaches are compared to recent experimental data from LEP (DELPHI, OPAL) and SLC (SLD).

THE NAIVE MODEL (NM)

The NM [4] (in a modified form, which accounts more properly for the energy carried away by the primary quarks) is based on the assumption that the multiplicity of hadrons produced in addition to the possible decay products of the primary quark-antiquark pair (which induce all the process of e^+e^- annihilation to hadrons) depends on the available invariant mass M only and is described by a universal function $n_0(M)$ (see Fig. 2), which does not depend on the type of primary quark. Accordingly, the average multiplicity (in the events triggered by the primary quarks of type q), $N_q(W)$, takes the form:

$$N_q(W) = 2n_q + \int_0^1 dx dy (d\sigma_q/\sigma_q dx dy) n_0(M), \quad (1)$$

where

$$M^2 = W^2(1-x)(1-y), \quad \int_0^1 dx dy (d\sigma_q/\sigma_q dx dy) = 1,$$

W is the total c.m.s. energy of the collision, $x(y)$ is the energy fraction carried away by the primary (i.e. containing primary (anti)quark) hadron, n_q is the multiplicity of the primary hadron decay products, and $d\sigma_q/\sigma_q dx dy$ is the inclusive joint density of two primary hadrons.

This is an almost trivial statement (see e.g. ref. [5]), by which Eq.(1) can be rewritten (assuming that the correlation between the primary hadrons is negligible) in the form:

$$N_q(W) = 2n_q + n_0(W(1 - \bar{x}_q)), \quad (2)$$

where $\bar{x}_q$ is some (not necessarily unique) number between 0 and 1, which has no direct physical meaning and depends on W in general.

A non-trivial statement (and this is the main assumption of the NM) is that $\bar{x}_q$ can be approximated with high enough accuracy by $\langle x_q \rangle$, i.e. by the average fraction of the total energy carried away by the primary hadron.

We therefore obtain the relation

$$N_q(W) = 2n_q + n_0(W(1 - \langle x_q \rangle)), \quad (3)$$

which is the essence of the NM.

Some useful consequences can be obtained from Eq. (2) even without knowledge of the functional form of n_0 . For instance average multiplicities in two processes initiated by quarks Q and q are related as follows:

$$N_Q(W) = 2(n_Q - n_q) + N_q(W(1 - \langle x_Q \rangle)/(1 - \langle x_q \rangle)), \quad (4)$$

Mathematically Eq. (3) means that all moments $\langle x^n \rangle$ are very close to $\langle x \rangle^n$, quite a rare property indeed. So it is very amusing that Eq. (3) can pretend to describe the real data [6, 7, 8, 9] at all. However, it does, and not without some success, as can be observed in Fig. 3.

THE QCD-BASED APPROACH

In the framework of perturbative QCD it is impossible to calculate the average multiplicity of hadrons that are of a non-perturbative origin. One cannot think either of the mean multiplicity of gluons or massless quarks which is just infinite. This gap between two impossibilities was partially overcome with the help of the observation [10] that colourless clusters of (virtual) QCD partons have, on average, asymptotically finite masses in contrast with the coloured clusters, the average masses of which grow with energy. A physical picture of the multiple production in pQCD was explained in detail in ref. [11]. Practically this looks as a calculation of the off-shell (i.e. at the virtuality Q_0 such that $\alpha_s(Q_0^2)$ is still "reasonably small") parton multiplicity, which is believed to coincide with the hadronic multiplicity up to some energy-independent phenomenological factor. There is however some trouble because, in non-Abelian gauge theory, off-shell quanta lead generally to the gauge dependence, which is surely inadmissible for the observed quantities. There is nonetheless evidence [11] that gauge-dependent terms are negligible for the leading energy dependence of the average multiplicity of the off-shell gluons. When proceeding from this framework we readily get the following expression for the average multiplicity:

$$N_q(W) = 2n_q + \int dk^2/k^2 (n_g(k^2)(C_F \alpha_s(k^2)/\pi) E_q(W^2, k^2)), \quad (5)$$

where the first term in the r.h.s. is the same as in Eq. (1), $n_g(k^2)$ stands for the average number of "decay products" of the gluon jet with a virtuality up to k^2 ; E_q is defined by the discontinuity of the two-gluon irreducible $\gamma^*(Z)g^*$ amplitude integrated with d^4k/dk^2 and normalized to the total e^+e^- rate. It can be vaguely (no evident positivity) interpreted as a kind of inclusive density of gluon jets in their invariant masses. So its integral in k^2 (down to the permitted value Q_0^2) can be thought of as the average number of gluon jets. These, however, are not all observable as no cuts are imposed to distinguish them in real space. From the spectral condition it follows that $E_q = 0$ at $k^2 \geq (W - 2m)^2$, where m is the mass of the primary quark (we neglect additional quark loops and scale dependence of the

quark mass). The function E_q contains mass singularities, but it has a finite limit at $m \rightarrow 0$ and $k^2 \neq 0$.

Let us consider $N_Q(W)$ in the e^+e^- events induced by the quark Q with mass m and $N_q(W)$ defined by the massless quark q . What is the difference in energy dependence of both quantities? It was argued in refs [12, 13] that at $W \rightarrow \infty$

$$\delta_{Qq} \equiv N_Q(W) - N_q(W) \rightarrow 2n_Q - N_q(m\sqrt{e}), \quad \ln e = 1. \quad (6)$$

The comparison of this result with the data for $Q = b$ [6, 7, 8, 9] can be seen in Fig. 4

It is clear that one can hardly be completely content when Eq. (6) is compared to experiment (assuming that 91 GeV is already "infinite" energy).

Does it mean that pQCD fails to explain the observed value of δ_{bq} ?

WHY $\sqrt{e}$?

First of all let us clarify the origin of the mysterious $\sqrt{e}$ in Eq. (6). In what follows, we shall deal with the first order in $\alpha_s(k^2)$ (i.e. E_q is of zeroth order in $\alpha_s(k^2)$; the arguments in favour of precisely this choice of scale in α_s can be found in ref. [14]). As to the higher orders we can only assume that the cancellation of the "real" gluon radiation against the virtual one proceeds essentially as in QED, i.e. $E_{q,N}$ corresponding to the contribution of $(N+1)$ gluon jets is of the form:

$$(C_F \alpha_s(k^2)/\pi) E_q(W^2, k^2) K N \exp(-K)/N!,$$

where

$$K = \int (C_F \alpha_s(k^2)/\pi) E_q(W^2, k^2) dk^2/k^2.$$

With this assumption $\alpha_s E_q$ gives the total inclusive gluon-jet distribution. We note, however, that this hypothesis is far from evident [15].

Proceeding in this way one can be convinced that the difference

$$\Delta N(W) = \int^{W^2} (dk^2/k^2) n_g(k^2) (C_F \alpha_s(k^2)/\pi) [E_q(W^2, k^2) - E_q(W^2, k^2)],$$

approaches a constant value, ΔN , indeed at $W = \infty$, and

$$\Delta N \approx \int_{-\infty}^{Y_m} v(Y_m - y) \Delta E(y) dy, \quad (7)$$

where $v \equiv n_g C_F \alpha_s/\pi$, $y = \ln(m^2/k^2)$. The function ΔE may be calculated exactly and reads as follows ($y \equiv \ln(1/\rho)$):

$$\Delta E(\rho) = (1 - 3\rho + 3.5\rho^2) \ln(1/\rho) + \rho(7\rho - 20)J(\rho) + 20(1 - J(\rho))/(\rho - 4) + 7\rho + 4.5,$$

where $J(\rho) = [\rho/(\rho - 4)]^{1/2} \ln[(\rho^{1/2} + (\rho - 4)^{1/2})/2]$ (at $\rho < 4$ $\ln$ changes for $\text{arctg} [(4 - \rho)/\rho]^{1/2}$; 4 corresponds to the quark-antiquark pair in the g and $\gamma^*(Z)$ -channels; $J(4) = 1$). The integral in Eq. (7) is convergent, because

$$\Delta E \approx (11/3)e^{-|y|}, \quad y \rightarrow -\infty.$$

The upper limit, $Y_m = 2\ln(m/Q_0)$, is set by the condition $k^2 \geq Q_0^2$.

One can also cast the expression for the "light" average multiplicity into a form similar to Eq.(7):

$$N_q(W) = 2n_q + \int_0^Y v(Y - y) E(y) dy, \quad (8)$$

where v is the same as in Eq.(7), $Y = 2\ln(W/Q_0)$, while $E(y)$ reads ($y \equiv \ln(1/\rho)$):

$$E(\rho) = (1 + 2\rho + 2\rho^2) \ln(1/\rho) - (1.5 + 3.5\rho)(1 - \rho) - \rho(1 + \rho) [\ln(1/\rho)]^2 + 4\rho(1 + \rho) I(\rho),$$

$$I(\rho) = \int_\rho^1 \ln(1/z) dz/(1+z).$$

Now let us look at the asymptotic behaviour of both functions, ΔE and E , at large y :

$$\Delta E(y) \approx y - 1/2 \equiv \Delta E^{\text{as}}(y)$$

$$E(y) \approx y - 3/2 \equiv E^{\text{as}}(y).$$

One easily observes that $\Delta E_{as}(y) = E_{as}(y+1)$ and, assuming that the contribution of the region $y < -1$ to ΔN is negligible, arrives at Eq. (6).

DECISIVE ROLE OF "NONLEADING TERMS"

A more attentive look at Fig. 5, where both functions, E and ΔE , are displayed with their asymptotes, E_{as} and ΔE_{as} , reveals however that the use of the latter when estimating ΔN is hardly justified. In fact, it is seen at first glance that $\Delta E(y)$ is far from being just $E(y+1)$. Moreover the following inequality holds in the whole region of integration in Eq. (7):

$$\Delta E(y) > E(y + \Delta y),$$

where Δy solves the equation:

$$\Delta E(2\ln(m_Q/Q_0)) = E(2\ln(m_Q/Q_0) + \Delta y). \quad (9)$$

Making use of the positivity of ΔE we get the upper bound:

$$\delta_{Qq}(\infty) < 2n_Q - N_q(m_Q e^{0.5\Delta y}). \quad (10)$$

In the case of the b-quark ($\Delta y = 1.6$) inequality (10) leads to the bound:

$$\delta_{bq} < 3.7 + 4.1$$

for $m_b = 5.3$ GeV + 4.7 GeV [16], respectively, and $Q_0 \approx 1$ GeV, which lies comfortably below the central value (≈ 5.5) of the "MLLA LPHD" result [12, 13]. For the above estimates we have used $2n_b = 11$ [6] and the parametrization for N_q , the multiplicity in the events induced by the light quarks, from ref. [17]:

$$N_q(W) = 2.554 + 0.1252 \exp(2.317\sqrt{\ln W}). \quad (11)$$

For the case of the charm quark the NM (see Eq.(4)) gives the following estimates:

$$\delta_{cq}(29 \text{ GeV}) = 1.2, \delta_{cq}(35 \text{ GeV}) = 1.1, \delta_{cq}(91 \text{ GeV}) \approx -0.5, \quad (12)$$

where we assumed $\langle x_c \rangle = 0.5$, $\langle x_q \rangle (29 + 35 \text{ GeV}) = 0.32$, $\langle x_q \rangle (91 \text{ GeV}) = 0.27$, $2n_q = 2.4$ (see ref.[8] and references therein) and $2n_c = 5.2$ [17]. In the QCD-based approach we get:

$$\delta_{cq}(\infty) < 0.3 + 0.7 \quad (13)$$

for $m_c = 1.7 + 1.3$ [16], respectively, if we take use of the parametrization (11). It is, however, mentioned in ref.[17] that it apparently deviates from the experimental values of the "light" multiplicity at 3 + 7 GeV. Note that effective energies in N_q in the r.h.s. of the inequality (9) are of the order of 4 + 5 GeV, i.e. namely in this region. When using values of N_q which correspond to the experimental ones we get instead of the previous inequality the following one:

$$\delta_{cq}(\infty) < 1.3 + 1.7. \quad (14)$$

In fact the "rapidity" shift between ΔE and E is locally significantly larger than Δy from Eq.(9), as is well seen in Fig. 5. Therefore one should expect values for δ_{Qq} even lower than the upper bounds given by Eq. (10).

A more exact estimate of δ_{Qq} asks for an accurate enough expression for the gluon jet multiplicity, n_g , which we did not yet have. For this purpose we choose the following "QCD-motivated" expression:

$$n_g(k^2) \approx 1.014 (1 + \ln k^2)^{-1.945} \exp[1.642\sqrt{\ln k^2}] \quad (15)$$

With this input we reproduce quite accurately the values of the multiplicity in the events induced by the light quarks [17].

The result is shown in Fig. 6. At LEP energy we get (for $m_b = 4.8$ GeV)

$$\delta_{bq}(\text{LEP}) = 3.68$$

in rather good agreement with the average value [8] (OPAL, DELPHI and SLD):

$$\langle \delta_{bq} \rangle = 3.16 \pm 0.47.$$

As is seen from Fig. 6 $\delta_{bq}(W)$ decreases very slowly with energy and goes quite close to the upper bound (which does not depend on the exact form of n_g , and is therefore less model dependent). The limiting value of $\delta_{bq}(W)$ at $W = \infty$ is 3.32.

For completeness we have calculated also the energy behaviour of $\delta_{cq}(W)$ with the following representative predictions ($m_c = 1.5$ GeV):

$W, \text{ GeV}$	29	35	91	170	...	∞
$\delta_{cq}(W)$	1.02	1.02	1.01	0.99	...	0.81

We see that the energy dependence is almost negligible. These values are in evident contradiction with the upper bound (13) and in full conformity with the upper bound (14). The NM predictions, as well as those of the QCD-based approach, are fairly compatible with (quite scarce) experimental data [13]:

$$\delta_{cq}(29 \text{ GeV}) = 2.2 \pm 2.1, \quad \delta_{cq}(35 \text{ GeV}) = 2.8 \pm 2.1,$$

which are, certainly, too rough for making definite conclusions.

SUMMARY AND CONCLUSIONS

Let us summarize and discuss the results presented.

1. The NM as seen from Fig. 3, does quite well (at worst cannot be ruled out) when considering the whole set of measured values of δ_{bq} . It seems to work worse at the highest (LEP, SLC) energies. This means that the distribution of the leading hadrons becomes not only softer (general QCD expectation) but also gets broader with energy, and identification of $\bar{x}_q$ with $\langle x_q \rangle$ is less justified. One could surely try to "improve" the NM by refusing of such an identification. In this case the NM immediately loses all its predictive power and reduces to the extraction of the (energy-dependent) parameter $\bar{x}_q$ from the data, hardly an interesting exercise. In spite of its appealing simplicity and naturalness, the NM is too phenomenological and so does not (and cannot) say much about the underlying dynamics of the particle production.

2. The QCD-based approach (which seems to be quite successful with its predictions for δ_{bq}) is evidently more attractive and more promising being related to many fundamental issues. One should notice, however, that the prediction of, say, δ_{bq} close to the experimental data (which are not accurate enough, especially at lower energies) cannot be considered as a clean and decisive test of pQCD. In this respect we cannot, for instance, agree with the statement, made in ref. [7], that the (would be) agreement of the "MLLA LPHD" prediction for δ_{bq} with the data "supports the notion that QCD remains asymptotically free down to the scale M_b^2 ". This (quite ambiguous) statement looks almost like the most important scientific output of the data obtained by authors of ref. [7]. In fact, the existence of a nonzero limit of $N_Q - N_q$ at $W \rightarrow \infty$ (which is really the actual finding of "MLLA LPHD") is not due to asymptotic freedom. This phenomenon is quite indifferent to the behaviour of the gluon jet multiplicity. As to the very value of δ_{Qq} it includes the integration in gluon virtualities down to Q_0 , the "preconfinement scale", which is normally smaller than, say, m_b .

There are still serious problems (partly mentioned in the text) to be clarified. One of them deserves to be mentioned explicitly. Eq.(5) is, strictly speaking, incomplete. In fact, one more term should be present which is the counterpart to the higher-twist terms in DIS. Usually this term is considered to be negligible in comparison with the leading behaviour of the second term in the r.h.s. of Eq.(5). Nonetheless when considering the difference of two large quantities, N_q and N_Q , we have no solid arguments that the term in question cannot contribute significantly to δ_{Qq} . Besides the conceptual issues it is important, for instance, to estimate accurately enough the energy dependence of δ_{Qq} at lower energies, etc. Certainly, this would make sense should more accurate data became available. The case of the charmed quark is of particular interest but unfortunately it is still out of thorough experimental investigation at higher energies.

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FIGURE CAPTIONS

- Figure 1. Fragmentation function of the b-quark (reproduced from the second of refs. [3]).
- Figure 2. Hadron production pattern in the NM.
- Figure 3. Modified Naive Model in comparison with the data on the difference $\delta_{bq} \approx N_b(W) - N_q(W)$ [6, 7, 8, 9]. (This figure and Figs 4 and 6 were kindly prepared by Jan Chrin).
- Figure 4. "MLLA LPHD" prediction for δ_{bq} [12] versus the data.
- Figure 5. Functions $E(y)$, $\Delta E(y)$ and their asymptotes, $Eas(y)$ and $\Delta Eas(y)$.
- Figure 6. "QCD" expectations for the energy behaviour of $\delta_{bq}(W)$ and the upper bound for $\delta_{bq}(\infty)$. Upper and lower lines in both cases correspond to $m_b = 4.7$ GeV and 5.3 GeV, respectively.

FIG. 1

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