CERN-TH.7351/94

LOWER LIMIT ON THE HIGGS MASS IN THE STANDARD MODEL: AN UPDATE

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Abstract

As a consequence of the large value of the top mass, the Higgs particle must be too heavy to be found at LEP if the Standard Model is valid up to a sufficiently large scale Λ . We reanalyze the lower limit in detail as a function of m_t and Λ .

CERN-TH.7351/94
July 1994

It is well known that in the Standard Model with only one Higgs doublet a lower limit on the Higgs mass m_H can be derived from the requirement of vacuum stability [1-5]. The limit is a function of the top quark mass m_t and of the energy scale Λ where the model breaks down and new physics appears [2]. Similarly an upper bound on m_H (with mild dependence on m_t) can be obtained from the requirement that up to the scale Λ no Landau pole appears [4]. The lower limit on m_H is particularly important in view of the search for the Higgs at LEP200. Indeed the issue is whether one can reach the conclusion that if a Higgs is found at LEP200, i.e. with $m_H \leq m_Z$, then the Standard Model must breakdown at some scale $\Lambda > 1$ TeV. In the present article, in view of the recent convergent indications from precision tests of the Standard Model [6] and from the CDF experiment [7], that locate m_t at around 170 GeV, we reexamine and update the status of the lower limit and its implications. With respect to previous analyses (in particular to ref.[5] where the practical consequence of the bounds were most completely addressed) we refine the derivation of the bound by using some recent calculations of loops effects, we discuss in detail the Λ dependence of the bound and we attempt a realistic evaluation of the related theoretical errors.

The possible instability of the Higgs potential $V[\phi]$ is generated by the quantum loop corrections to the classical expression of $V[\phi]$. At large ϕ the derivative $V'[\phi]$ could become negative and the potential would become unbound from below. The one-loop corrections to $V[\phi]$ in the Standard Model are well known [8] and are given by:

$$V_0^{1-loop}[\phi] = -\frac{1}{2}m_0^2\phi^2 + \frac{1}{24}\lambda_0\phi^4 + \frac{1}{16\pi^2} \left[\frac{1}{4}H^2 \left(\ln \frac{H}{\mu_0^2} - \frac{3}{2} \right) + \frac{3}{4}G^2 \left(\ln \frac{G}{\mu_0^2} - \frac{3}{2} \right) + \frac{3}{2}W^2 \left(\ln \frac{W}{\mu_0^2} - \frac{5}{6} \right) + \frac{3}{4}Z^2 \left(\ln \frac{Z}{\mu_0^2} - \frac{5}{6} \right) - 3T^2 \left(\ln \frac{T}{\mu_0^2} - \frac{3}{2} \right) \right], \quad (1)$$

where

$$H = -m_0^2 + \lambda_0\phi^2/2, \quad G = -m_0^2 + \lambda_0\phi^2/6, \quad T = g_0^2\phi^2/2, \quad (2)$$

$$W = g_0^2\phi^2/4, \quad Z = (g_0^2 + g_{20}^2)\phi^2/4,$$

g_{40} ($i = 1, 2$) are the $SU(2) \times U(1)$ gauge couplings, g_{40} is the top quark Yukawa coupling (we neglect the Yukawa couplings of the other quarks) and μ_0 is the renormalization scale. The one-loop expression of $V[\phi]$ provides a reliable approximation only for a limited range of ϕ , up to values where the one-loop corrections become of order of the tree level terms. For the discussion of the stability requirement at large ϕ one makes use of the renormalization group (RG) techniques to obtain an improved form of the potential valid in a wider range of ϕ [2]. A detailed analysis of the RG improved potential has been performed in Ref.[4]. The results of Ref.[4] confirms the earlier result of Cabibbo et al. [1] that the issue of the stability of $V[\phi]$ for $\phi \sim \Lambda \gg m_Z$ practically coincides with the

requirement that the running coupling $\lambda(\Lambda)$ never becomes negative. Indeed for ϕ much larger than all mass scales in the theory the one-loop potential of eq.(1) can be written as:

$$V_0^{1-loop}[\phi] \simeq +\frac{1}{24}\phi^4 \left\{ \lambda_0 + [\beta_\lambda(\lambda_0, g_0) - 4\lambda_0\gamma(\lambda_0, g_0)] \ln\left(\frac{\phi}{\mu_0}\right) + O(\lambda_0^2, g_0^2) \right\}. \quad (3)$$

This is just the expansion of the quartic term in the RG improved potential:

$$V_{RG}^{1-loop}(\phi) \simeq +\frac{1}{24}\lambda(t)[\xi(t)\phi]^4 + O(\lambda(t)^2, g(t)^2), \quad (4)$$

where

$$\xi(t) = \exp\left(-\int_0^t \gamma(\lambda(t'), g(t')) dt'\right), \quad t = \log(\Lambda/\mu_0), \quad (5)$$

$$\frac{d\lambda(t)}{dt} = \beta_\lambda(\lambda, g), \quad \frac{dg_i(t)}{dt} = \beta_i(\lambda, g_i), \quad (6)$$

$$\lambda(0) = \lambda_0, \quad g_i(0) = g_{i0}. \quad (7)$$

In eq.(4) all leading logarithmic terms have been reabsorbed in the definition of the running couplings and in the rescaling of the field. Within the quartic approximation of eq.(4) it is clear that the stability of $V(\phi)$ is equivalent to the positivity of $\lambda(\Lambda)$. Also, our present knowledge of the realistic range of values for m_H and m_t ensures that the instability of $V[\phi]$ can only occur at $\phi \gg m_Z$, thus providing an a-posteriori justification for the quartic approximation in eq.(4) [5].

In general, a more accurate expression for the potential can be obtained by also keeping the non quartic terms in $V[\xi(t)\phi, \lambda(t), g_i(t), \Lambda]$ and replacing the one-loop with the two-loop beta functions in the evolution for $\xi(t)$, $\lambda(t)$ and $g_i(t)$ in eqs.(6). For consistency one must then include in the matching conditions of eqs.(7) the one-loop corrections. In particular the tree level relations $\lambda_0 = 3m_H^2/v^2$ and $g_0^2 = \sqrt{2}m_t/v$ must be replaced by:

$$g_t(\mu_0) = \frac{\sqrt{2}m_t}{v} [1 + \delta_t(\mu_0)], \quad (8)$$

$$\lambda(\mu_0) = \frac{3m_H^2}{v^2} [1 + \delta_\lambda(\mu_0)], \quad (9)$$

with $v = (\sqrt{2}G_F)^{-1/2}$ and μ_0 chosen at values of order m_Z or m_t . The shifts δ_λ and δ_t will be discussed in the following. Similarly one considers the running gauge couplings $\alpha_i(\mu_0) = g_i(\mu_0)^2/4\pi$. The one loop corrections to m , the coefficient of the quadratic term in the potential, are not important because the ϕ^2 terms are themselves a small correction to the quartic approximation at large ϕ .

Starting from the above improved potential we have studied the stability problem and checked that the requirement $dV[\phi]/d\phi > 0$ as a function of Λ is indeed practically equivalent to $\lambda(\Lambda) > 0$, for all values of Λ in the range $1 \text{ TeV} < \Lambda < M_{Planck}$, with an accuracy on the resulting bound of the order $\delta m_H \sim \pm 1 \text{ GeV}$.

As a consequence we restrict our attention to the bound following from the constraint $\lambda(\Lambda) > 0$, with the two-loop beta functions¹ and the one-loop matching conditions, as a functions of Λ and m_t . Given that

$$\frac{d\lambda(t)}{dt} = \frac{1}{16\pi^2} \left[4\lambda^2 + 12\lambda g_t^2 - 36g_t^4 - 9\lambda g_1^2 - 3\lambda g_2^2 + \frac{9}{4}g_2^4 + \frac{27}{4}g_1^4 \right] + O(\lambda^3, \lambda^2 g_t^2, \lambda g_t^4, g_t^6) \quad (10)$$

for small m_H and large m_t , $d\lambda(t)/dt < 0$, thus λ can become negative. Eventually the λ^2 term in the derivative wins and λ increases towards its Landau pole. The general dependence of λ as a function of Λ , for $m_t = 174 \text{ GeV}$ and $m_H = 60 - 150 \text{ GeV}$, is plotted in fig.1.

As already mentioned, the precise bound on m_H depends on the one-loop corrections, given by δ_λ and δ_t , to the matching conditions in eqs.(7). Defining m_t as the pole mass and g_t in the $\overline{MS}$ scheme, the largest contribution of $\delta_t(\mu)$ is the QCD correction [9]:

$$\delta_t^{QCD}(\mu_0) = +\frac{1}{16\pi^2} \left[-\frac{16}{3}g_S^2 + 8g_S^2 \ln \frac{m_t}{\mu_0} \right] \quad (11)$$

which, evaluated at $\mu_0 = m_t$, amounts to $\sim -5\%$. Since this one-loop effect is quite large one can keep also the two-loop QCD correction [5], which evaluated at $\mu_0 = m_t$ amounts to $\sim -1\%$. Then there are the terms proportional to g_t and λ , recently computed in Ref.[11]:

$$\delta_t^{top}(\mu) = +\frac{1}{16\pi^2} \left[\frac{1}{24}\lambda + g_t^2 \left(\frac{13}{8} + \frac{3}{2} \ln \frac{m_t}{m_H} - \frac{9}{2} \ln \frac{m_t}{\mu_0} \right) \right], \quad (12)$$

which amount to $\sim +1\%$ at $\mu_0 = m_t$, so that they almost cancel the two-loop QCD correction. The remaining uncertainty from higher order QCD terms at fixed $\alpha_s(m_Z)$, the not yet computed contribution of electroweak gauge couplings and the ambiguity from choosing the value of μ_0 between m_Z and m_t , can be estimated to $\sim \pm 1\%$. This error implies an uncertainty of $\sim 3 - 4 \text{ GeV}$ on the resulting bound on m_H , for $m_t = 174 \text{ GeV}$ and $\Lambda = M_{Planck}$.

The correction $\delta_\lambda(\mu_0)$ has been studied in detail in ref.[12]. Actually in ref.[12] m_H is not exactly the pole Higgs mass but the second derivative of the potential at the minimum, the correction between the two has been considered in Ref.[3, 5] and is a small effect. However the total effect of $\delta_\lambda(\mu_0)$ on the bound on m_H is quite sizeable, as can be seen from the large scale dependence of λ at small Λ in fig.1. Consequently the ambiguity related to the neglected higher order terms and the residual dependence on the choice of μ_0 is not insignificant and correspond to $\sim 1 - 2 \text{ GeV}$ on m_H .

The resulting lower bound on m_H is plotted in fig.2 as a function of Λ for $m_t = 140 - 210 \text{ GeV}$ and in fig.3 as function of m_t for $\Lambda = 10^3 - 10^{19} \text{ GeV}$. The curves correspond to the values $\alpha_S(m_Z) = .118$, $\alpha_{em}(m_Z)^{-1} = .128$ and $\sin^2_W(m_Z) = .232$. For

¹The complete expressions for the two loop beta functions can be found in Ref.[4].

$m_t \sim 174$ GeV and $\alpha_S(m_Z) = .118$ the results at $\Lambda = 10^{19}$ GeV or $\Lambda = 1$ TeV can be given by the approximate formulae:

$$m_H > 135 + 2.1[m_t - 174] - 4.5 \left[\frac{\alpha_S(m_Z) - .118}{.006} \right], \quad \Lambda \sim 10^{19} \text{ GeV} \quad (13)$$

$$m_H > 72 + 0.9[m_t - 174] - 1.0 \left[\frac{\alpha_S(m_Z) - .118}{.006} \right], \quad \Lambda = 1 \text{ TeV} \quad (14)$$

where the masses are in units of GeV. Concerning the error on the limiting value of m_H from sources other than α_S and m_t , we consider in fig.4 a number of curves obtained for a number of cases where the beta functions are taken at one or two-loops, the correction δ_t^{top} is included or not, the matching scale μ_0 for λ is chosen at m_Z or m_t . We see that clearly the errors increase with the increasing of the range of the extrapolation. The spread of the results can be taken as a rough indication of the errors involved (with the conservative criterium that the error is of the order of the last terms kept in the expansion). Altogether, when the different errors are combined, one obtains a total error of the order of $\sim \pm(5 - 10)$ GeV at the highest values of Λ . Within this accuracy our evaluation of the limit at large Λ in eq.(13) is in good agreement with the result of ref.[5].

In conclusion we see from fig.3 and eqs.(13-14) that indeed for $m_t > 150$ GeV the discovery of a Higgs particle at LEP200 would imply that the Standard Model breaks down at a scale Λ well below M_{GUT} or M_{Planck} , smaller for lighter Higgs. Actually, if the Standard Model is valid up to $\Lambda \sim M_{GUT}$ or M_{Planck} , for $m_t \sim 174$ GeV only a small range of values is allowed: $130 < m_H \leq 200$ GeV (the upper limit arise from avoiding the Landau pole). As is well known [5], the lower limit is not much relaxed even if strict vacuum stability is replaced by some sufficiently long metastability. On the other hand the limit is only valid in the Standard Model with one Higgs doublet: it is enough to add a second doublet with mass lighter than Λ to avoid it. A particularly important example of theory where the bound is avoided is the Minimal Supersymmetric Standard Model. In this model, for $m_t \sim 174$ GeV, the lightest Higgs must necessarily be lighter than ~ 130 GeV and possibly within the LEP200 range for the attractive case where the top quark lies near its fixed point (at small $tg\beta$) [13].

Acknowledgements

We thank R. Barbieri for interesting discussions.

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Figure 1: $\lambda(\Lambda)$ for $m_t = 174$ GeV, $\alpha_S(m_Z) = .118$ and different values of M_H .

Figure 2: Limits on m_H as a function of Λ , for different values of m_t .

Figure 3: Limits on m_H as a function of m_t , for different values of Λ (in units of GeV).

Figure 4: Limits on m_H as a function of Λ , for of $m_t = 174$ GeV and $\alpha_S = .118$. The full curve correspond to two-loop beta functions and matching corrections $\delta_t(\mu_0)$, $\delta_\lambda(\mu_0)$ at $\mu_0 = m_Z$. In the dash-dotted curve the δ_t^{top} correction has been neglected, in the dashed curve δ_λ has been evaluated at $\mu_0 = m_t$, finally in the dotted curve one-loop instead of two-loop beta functions have been integrated.