PHOTON STRUCTURE: QCD TREATMENT AND PARTON DENSITIES

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The QCD treatment of the photon structure beyond the leading order is discussed with emphasis on a proper choice of the factorization scheme and for the boundary conditions. Recent parametrizations of the photon’s parton content are examined and compared. The sensitivity of the photon structure function on the QCD scale parameter is reconsidered.

1. Introduction

The hadronic structure of real photons [1], see also [2], is most easily illustrated by considering inclusive electron-photon deep-inelastic scattering (DIS). This process is completely analogous to the usual lepton-nucleon DIS. Consequently, the unpolarized cross section for $e\gamma \rightarrow e\gamma'\gamma \rightarrow eX$ can be expressed in terms of two structure functions $F_1^\gamma(x,Q^2)$, where $x$ denotes the Bjorken scaling variable and $Q^2$ is the virtuality of the probing off-shell photon. Only $F_1^\gamma$ has been measured so far. The main issues addressed in this talk can already be introduced at the parton model level. Here, two different contributions occur, which originate in point-like and non-pointlike photon-quark couplings.

The pointlike contribution derives from the lowest order QED process (‘Born-Box’) $\gamma^*\gamma \rightarrow q\bar{q}$ [3], and is for $Q^2 \gg m_q^2$ given by

$$\frac{1}{x} F_{2,B}^\gamma = \frac{\alpha}{2\pi} \sum_q 2\epsilon_q^4 [k_q^{(1)}(x) \ln \frac{Q^2}{m_q^2} + C_q(x)] . \quad (1)$$

Effective masses $m_q$ of the quarks with charges $\epsilon_q$ had to be introduced in order to avoid collinear singularities. Contrary to all hadrons the $x$-dependence of $F_2^\gamma$, governed by $k_q^{(1)}(x)$ and $C_q(x)$, is calculable and the structure function rises linearly with $\ln Q^2$. Analogous to the usual $\overline{\text{MS}}$ treatment of hadronic DIS, the quark densities in the photon can be defined by absorbing the universal mass singularity,

$$q^\gamma_{B,\overline{\text{MS}}} = \frac{\alpha}{2\pi} \epsilon_q^4 \frac{3}{2} \left( x^2 + (1-x)^2 \right) \ln \frac{Q^2}{m_q^2} , \quad (2)$$

where the concrete form of $k_q^{(1)}$ has been put in. The $C_q$-term, not absorbed into the parton distributions in (2), then acts as the subleading, ‘direct’ contribution to $F_2^\gamma$. However, this separation creates problems in the next-to-leading order QCD treatment, mainly since $C_\gamma$ is negative and divergent at $x \rightarrow 1$. Solutions to this problem, either by introducing a different factorization scheme (‘DIS’$^*$) or by adding some ‘technical’ $\overline{\text{MS}}$ input for the evolution equations, are examined in section 2.

The hadronic, non-pointlike contribution to $F_2^\gamma$ can be illustrated by the well-known coupling of the photon to the vector mesons $\rho$, $\omega$ and $\phi$ (vector meson dominance, VMD) [1], resulting in

$$F_{2,\text{VMD}}^\gamma = \left( 4\pi a/\alpha_s^2 \right) F_2^\rho + \ldots . \quad (3)$$

This part is completely analogous to the behaviour of hadrons: there is no rise with $\ln Q^2$ and the $x$-shape is not calculable in perturbation theory. Hence unknown input parton distributions enter the description just as in hadron structure. In section 3 recent parametrizations of $q^\gamma$ and the less well known gluon density $G^\gamma$ are discussed and compared.

The dependence of $F_2^\gamma$ on the unknown effective quark masses is removed by the inclusion of the dominant QCD corrections, leading to the famous large-$x$ large-$Q^2$ parameter-free asymptotic QCD predictions [4,5]. These results, however, are not appropriate at scales accessible at present or in the near future, $Q^2 \lesssim 300 \text{ GeV}^2$ [6]. The original hope on an especially clean $a_s$ determination from $F_2^\gamma$ has therefore been dampend. Some debate has arisen on how much sensitivity survives in a proper, non-asymptotic treatment including the uncalculable hadronic boundary conditions, see e.g. [7]. A short note is added to this discussion in section 4.
2. QCD: Next to Leading Order

The generalized evolution equations for the $O(\alpha)$ quark and gluon content of the photon are [5,8]

$$\frac{d\bar{q}^\gamma}{d\ln Q^2} = \frac{\alpha}{2\pi} \xi^2 k_\xi + \frac{\alpha_s}{2\pi} [P_{gq} \otimes \bar{q}^\gamma + \ldots]$$

$$\frac{dG^\gamma}{d\ln Q^2} = \frac{\alpha}{2\pi} k_\alpha + \frac{\alpha_s}{2\pi} [P_{GG} \otimes G^\gamma + \ldots] \quad .$$

(4)

Here $\otimes$ denotes the usual convolution, and the normal hadronic part has not been written out completely. The photon distribution in the photon, $\Gamma^\gamma = \delta(1-x)-O(\alpha)$, has dropped out at the order of the electromagnetic coupling $\alpha$ considered here, resulting in the characteristic inhomogeneous $k$-terms. A salient consequence of (4) is the absence of a momentum sum rule interrelating $\bar{q}^\gamma$ and $G^\gamma$. Hence an important constraint on the gluon density, present in the hadronic case, is missing here. The splitting functions $P_{ij}$ and $k_i$ are up to now known to next-to-leading order (NLO) accuracy in the strong coupling $\alpha_s \equiv \alpha_s(Q^2)$, see [9] and [10,12], respectively.

The singlet distributions can be decomposed as

$$q^\gamma = \left\{ \frac{2}{G^\gamma} \sum_q \bar{q}^\gamma \right\} = q_{2L}^\gamma + q_{bad}^\gamma \quad ,$$

(5)

where $\bar{q}^\gamma = \bar{q}^\gamma$ has been used. The well-known homogeneous hadronic solution $q_{bad}^\gamma$ contains the perturbatively uncalculable boundary conditions $q^\gamma(Q_0^2)$. The inhomogeneous (‘pointlike’, PL) solution then satisfies $q_{2L}^\gamma(Q_0^2) = 0$.

An analytical solution of (4) is possible for Mellin-moments. The NLO pointlike part reads [11,12]

$$q_{2L}^\gamma = \left\{ \frac{1}{\alpha_s + \hat{U}} \left\{ 1 - \left[ \alpha_s/\alpha_s(Q_0^2) \right]^{1+d} \right\} \frac{1}{1+d} \right\} a$$

$$+ \left\{ 1 - \left[ \alpha_s/\alpha_s(Q_0^2) \right]^{1+d} \right\} \frac{1}{d} b + O(\alpha_s) \quad ,$$

(6)

with $a$, $b$, $d$ and $\hat{U}$ being known Mellin-dependent combinations of the splitting functions. The $x$-dependent distributions are obtained by a numerical Mellin-inversion of (6) and the corresponding hadronic and non-singlet expressions. A major advantage of this procedure is that inconsistent higher order $O(\alpha_s)$ terms, which would lead to spurious $x \rightarrow 1$ singularities, can be trivially omitted. In a direct NLO $x$-space calculation, their cancellation requires somewhat cumbersome algorithms [13,14].
These problems are avoided by working in the DIS$_\gamma$ factorization scheme introduced in [12]. Here, the $C_\gamma$ term is (re-)absorbed in the quark densities, i.e., the constant and the logarithmic parts of (1) are not, as in (2), separated for the definition of $q^\gamma$. The connection to the usual $\overline{\text{MS}}$ scheme is

$$q^\gamma_{\text{DIS}} = \frac{q^\gamma_{\text{MS}}}{2\pi} + \alpha \epsilon_q C_{\gamma,\text{MS}}, \quad C_{\gamma,\text{DIS}} = 0,$$

and the coefficient functions $C_\gamma$ and $C_G$ as well as the definition of the gluon density remain unchanged, in contrast to the hadronic DIS scheme [17]. Thus in DIS, $F^\gamma_q$ retains the usual hadronic $\overline{\text{MS}}$ form without a direct term also at NLO, resulting in a good LO/NLO stability of $F^\gamma_q_{NLO}$ (Fig. 1). Hence in this scheme perturbatively stable, physically motivated inputs for the photonic parton densities, e.g., from VMD as in [18,19], can be used in NLO.

Due to the non-universality of the coefficient function $C_{\gamma}$, $F^\gamma_q$ has been assigned a special role in the redefinition (9) of the quark distributions. The first (pragmatic) motivation for doing so is that $F^\gamma_q$ is the only photonic DIS structure function measured so far and in the near future. The second (theoretical) reason stems from the analogy with the hadronic DIS scheme, where $F^\gamma_q$ is special because of its connection with the charge sum rules [17].

An additional advantage of the DIS$_\gamma$ scheme, improving the perturbative stability of the evolution, is that the strongest $\overline{\text{MS}}$ poles for $x \to 0$ in the NLO photon-parton splitting functions $k^{(1)}_q$ disappear:

$$k^{(1)}_q \sim \begin{cases} \ln^2 x + \ldots & \overline{\text{MS}} \\ 2ln x + \ldots & \text{DIS}_\gamma \end{cases},$$

A corresponding cancellation takes place in the analogous timelike case, where it is even more important: besides the corresponding structure functions, also the photonic gluon fragmentation function $D^\gamma_{\gamma,\text{PL}}$ is strongly negative at small and medium $x$ in the $\overline{\text{MS}}$ scheme [20]. These problems are removed by using the DIS$_\gamma$ factorization [21].

An equivalent $\overline{\text{MS}}$ formulation of the above solution to the $C_\gamma$-problem is of course possible and has been approximately employed in [22]. It can be written as a modification (\text{\textit{\text{PL}}}) of the pointlike part in (5) by an additional 'technical' NLO quark input,

$$q^\gamma_{\text{PL}}(Q^2_0) = -\frac{\alpha}{2\pi} \epsilon_q C_{\gamma,\text{MS}}.$$

This leads to $F^\gamma_{\text{PL}}(Q^2_0) = 0$ and thus allows for similar 'physical' inputs to $q^\gamma_{\text{had}}$ at LO and NLO. The resulting quark densities, however, depicted in Fig. 2 together with their pointlike LO and DIS$_\gamma$ counterparts, exhibit a rather unphysical shape.

In a consistent NLO calculation, where e.g., the input (11) is evolved and convoluted only with LO quantities, the above $\overline{\text{MS}}$ procedure is strictly equivalent to the DIS$_\gamma$ treatment for all physical quantities. On the other hand, as soon as not all higher order $\mathcal{O}(\alpha_s)$ terms are discarded, the $\overline{\text{MS}}$ factorization turns out to be much less well-behaved than the DIS$_\gamma$ scheme. This is obvious from Fig. 3, where the results of a 'slightly' inconsistent calculation, in which only the additional $\mathcal{O}(\alpha_s)$ terms arising from the convolutions like $\alpha_s C_q \otimes q^\gamma$ in (7) have not been omitted, are compared to the consistent result for both schemes. Consequently, in particular for applications in the $\overline{\text{MS}}$ scheme, parametrizations of NLO photonic parton densities should, as in [18], supply also the leading part of $q^\gamma_{\text{NLO}}$ in (6) ($\neq q^\gamma_{\text{LO}}$) necessary for consistent calculations.
3. Photonic Parton Densities

In order to specify the photon’s parton distributions, one has to choose a reference scale \( Q_0^2 \) and to fix the perturbatively uncancellable boundary conditions \( q^\gamma(Q_0^2), \ G^\gamma(Q_0^2) \) for the evolution equations (4). Experimentally, \( q^\gamma \) (mainly \( u^\gamma \)) is known at 0.05 \( \lesssim x \lesssim 0.8 \) from \( F_2^\gamma \) measurements [24–30] with an accuracy of about \( 10 \pm 20\% \). On the other hand, little is known about \( G^\gamma \) up to now: there is evidence for \( G^\gamma \neq 0 \), and an extremely huge and hard gluon (LAC 3, see below) has been ruled out by jet production data [31–34], see also [35]. The prospects on a measurement of the gluon density are discussed in [36]. This lack of experimental information is especially serious here since, as explained above, there is no energy-momentum sum rule relating \( q^\gamma \) and \( G^\gamma \).

VMD provides a connection between the quark and gluon inputs by the hadronic momentum sum rule. Moreover, by SU(6) arguments, \( G^\gamma(Q_0^2) \) can then be estimated from the pionic gluon density \( G^\gamma \), which has been determined from direct photon production \( \pi p \rightarrow \gamma X \) [37,38]. However, for the traditionally chosen input scales, \( Q_0^2 \geq 1 \text{ GeV}^2 \), a pure VMD ansatz is known to be insufficient. An additional hard quark component has to be supplemented in order to achieve agreement with the \( F_2^\gamma \) data at larger \( Q^2 \) [6,7]. In contrast to the quarks, where the Born-Box (1) can provide a natural additional input, a corresponding natural choice beyond VDM does not exist for the gluon density.

In view of this situation two approaches have been used. First, one can keep \( Q_0^2 \geq 1 \text{ GeV} \), fit the quark densities to \( F_2^\gamma \) data and ‘guess’ the gluon input. This method has been adopted in [39] and, more recently, in [22,40]. The second possibility is to try to maintain the VMD idea and to start the perturbative evolution at a very low scale \( Q_0 < 1 \text{ GeV} \) [18,23,41,42]. In the following, the recent parametrizations are discussed. All of them use a QCD scale parameter of \( \Lambda \equiv 200 \text{ MeV} \) for four active flavours in LO and NLO. The PEP, PETRA and TRISTAN data on \( F_2^\gamma \) available when most of these parametrizations were done are shown in Fig. 4. Since then, also first results from LEP have been published [30], see also [43]. The quark and gluon densities are compared in Fig. 5 and 6, respectively.
Fig. 4. The 1991 world data on $F_2^\gamma$ [24–29] compared to the results obtained from the LO and NLO GRV parametrizations, where the charm contribution has been calculated via the lowest order Bethe-Heitler process [18]. The controversial [44] low-$Q^2$ large-$x$ TPC/2γ data points close to the resonance region (open circles) have not been used for the determination of $\kappa$ in (13).

**LAC (only LO) [40]:**

The input scales are $Q_0^2 = 4$ GeV$^2$ for sets 1, 2 and $Q_0^2 = 1$ GeV$^2$ for set 3. The boundary conditions have been fitted to the data of Fig. 4 at $Q^2 > Q_0^2$ without any recourse to physical constraints on the quark flavour decomposition and on the gluon density. This has partially led to rather unphysical results even in the quark sector, e.g. $u^\gamma = 1.2\alpha$, $d^\gamma = 0.95\alpha$, $s^\gamma = 2.8\alpha$ and $c^\gamma = 1.6\alpha$ for LAC 1 at $x = 0.1$ and $Q_0^2 = 10$ GeV$^2$. Moreover, such a procedure gives rise to wild reactions of the fitted gluon density on fluctuations and offsets in the data: in LAC 1 and LAC 2, $G^\gamma$ is very large at $x < 0.1$ due to the small-$x$ interplay of PLUTO data [24] at low $Q^2$ with JADE [25] and preliminary CELLO [28] results at larger $Q^2$ in the fit. Likewise, the extremely huge and hard LAC 3 gluon is driven by the inclusion of the large-$x$ small-$Q^2$ TPC/2γ data [27], which lie close to the resonance region and are therefore subject to potentially large experimental correction factors and higher twist contributions [44]. In any case the LAC gluons dearly illustrate that present $F_2^\gamma$ data do not provide useful constraints on $G^\gamma$.

**GRV (LO/NLO) [18]:**

The gluon and sea quark densities of the proton and the pion have been generated from a valence-like structure at a common, very low resolution scale, $\mu \approx 0.55 (0.5)$ GeV in NLO (LO) [45,46]. First, this procedure has the advantage that few parameters in the gluon/sea sector are needed. The pionic gluon density, being a parameter-free prediction in the simplest version of this approach, shows very good agreement with the results from direct photon production [37,38]. Secondly, predictions at small $x$ arise, where the distributions are determined by the QCD dynamics and do virtually not depend on ambiguous input assumptions. HERA results on $F_2^\gamma$ [47,48] exhibit excellent agreement with these predictions, especially when the charm contribution is properly taken into account [49].

Therefore it is natural to utilize these low scales with $\alpha_S/\pi \approx 1/4$ also in the photon case for a pure VMD ansatz in LO and NLO (DIS$_\gamma$):

$$ (q^\gamma, G^\gamma)(\mu^2) = \kappa (4\pi f_\rho^2(q^\gamma, G^\gamma, \mu^2)) , \quad (13) $$

with the coupling $f_\rho^2/(4\pi) = 2.2$ and $1 \leq \kappa \leq 2$ [1].
Fig. 5. Photonic $\bar{c}$-quark densities at LO and NLO. The LO distributions [18,22,40] have been fitted to 1991 $E_2^\gamma$ data, for the different assumptions and data sets used see the discussions in the text. The NLO results [18,22,23] are presented in the $\overline{MS}$ scheme. Here only the GRV distribution has been directly fitted to data.

Fig. 6. Photonic gluon distributions in LO and NLO. The spread of the LO curves [18,22,40] exaggerates the present experimental uncertainty on $G^\gamma$, since IAC3 has been ruled out [31,32]. On the other hand, the similarity of the NLO results [18,22,23] is due to common VMD prejudices and not enforced by data.
In (13) the pion distributions of [46] have been used instead of the experimentally unknown \( \rho \) densities. \( \kappa \) accounts in a simple way for the higher mass vector mesons \( \rho, \omega \) and \( \phi \). Since this parameter is not sufficiently specified by VMD, it has been fitted to the \( F_2^\gamma \) data in Fig. 4. Good fits are obtained for the very reasonable values \( \kappa = 1.6 (2.0) \) in NLO (LO). Data does not favour neither an additional hard input at \( \mu^2 \) nor a later onset of the pointlike contribution. Thus a pure VMD input at \( Q_0^2 = \mu^2 \) is in fact successful and allows for an approximate prediction of \( G_\gamma \) in this ‘dynamical’ approach. Uncertainties stem from the \( \rho \to \pi \) substitution and from the pion-valence normalization [38,50] entering the determination of \( \kappa \). While the former is difficult to quantify, the latter amounts to about 20%.

It is interesting to note that the same scale \( Q_0 = 0.5 \) GeV for a LO VMD input has been obtained from quite different \( \gamma p \) total cross section considerations [42]. Consequently, the resulting distributions resemble the ones of [18]. In order to remain flexible on the VMD part, the LO pointlike solution (5) has been parametrized separately in [42].

**GS (LO/NLO) [22]:**

The input scale is \( Q_0^2 = 5.3 \) GeV\(^2\), the average of the low-\( Q^2 \) PLUTO data set [24]. The \( F_2^\gamma \) measurements at \( Q^2 > Q_0^2 \) in Fig. 4 have partially been used to fit the free parameters indicated below. In view of the gluon ambiguity discussed above, two sets are provided with different assumptions on \( G_\gamma \). At LO, the inputs of sets 1 and 2 are parametrized as

\[
q_{1,2}^\gamma = \text{VMD} + \text{QPM}(m_q)
\]

\[
G_{1,2}^\gamma = \text{VMD} + \frac{2}{3} \beta_0 P_{g}^{(1)} \otimes \Sigma^\gamma \, , \, G_2^\gamma = \text{VMD} .
\]

The VMD part is treated as in (13), with the old pion distributions from [51] being used. The effective quark masses \( m_{u,d,s} \) in the parton model (QPM) expression (1) and \( \kappa \) in (13) are determined from data. The difference between \( G_1^\gamma \) and \( G_2^\gamma \) presented in Fig. 5 is about twice as big as the LO pointlike contribution evolved from the start scale \( Q_0 = 0.5 \) GeV employed in [18,42]. Corresponding NLO (\( MS \)) densities have been constructed without a new fit by enforcing the same \( F_2^\gamma(Q_0^2) \) as in LO together with assumptions on the flavour decomposition.

**AFG (only NLO) [23]:**

These distributions supersede the NLO ones of [41], where a pure VMD input was employed at LO and NLO (\( MS \)) at \( Q_0 = 0.5 \) GeV. Here, the technical input (12) is supplemented by a VMD ansatz at \( Q_0 = 0.7 \) GeV \( \approx m_\rho \). No fit to data is performed. Somewhat different from (13), a coherent sum of \( \rho, \omega \) and \( \phi \), is used, where identical valence and gluon distribution in all three vector mesons and the pion have been assumed:

\[
u^\gamma = K_0 \left[ \frac{4}{5} u_{val}^\gamma + \frac{2}{3} u_{sea}^\gamma \right] \ldots , \quad G_\gamma = K_0 \frac{2}{3} G^\gamma (15)
\]

The pion densities are adopted from [38] with the normalization \( K_0 \) left free in the parametrization. The standard choice reads \( K_0 = 1 \), with a 50% variation allowed by high-\( Q^2 \) data on \( F_2^\gamma \).

### 4. \( \alpha_s \) and the photon structure

The sensitivity of \( F_2^\gamma \) to the QCD scale parameter \( \Lambda \) considerably decreases with increasing scale \( Q_0^2 \), at which the evolution (4) of the photonic partons is started [7,52,53]. For \( Q_0^2 \approx 5 \) GeV\(^2\), as in [22], only a marginal effect is left. It interesting, however, to reconsider this issue employing the ‘dynamical’ approach, where the parton densities of nucleon, pion and photon are related by valence-like inputs and VMD at a common, very low resolution scale, \( \mu \approx 0.55 \) GeV for \( \Lambda_{MS}^{(4)} = 0.2 \) GeV [18,45,46]. Therefore, this procedure, carried out only for this fixed \( \Lambda \) so far, has been repeated for different values of \( \Lambda_{MS}^{(4)} \) with \( \mu(\Lambda) \) fixed by proton structure information. The free parameter in the photon sector, \( \kappa \) in (13), has been fitted to the published results on \( F_2^\gamma \) with a cut of \( Q^2 > 4 \) GeV\(^2\) in order to suppress possible higher-twist contributions. These fits exhibit a rather good \( \alpha_s \)-sensitivity in this dynamical scenario,

\[
\Lambda_{MS}^{(4)} = (210 \pm 50_{\text{exp.}}) \text{ MeV} ,
\]

where the \( 1 \sigma \) error given in (16) arises from the total experimental uncertainties. A more detailed analysis including theoretical uncertainties will be presented in a separate paper [54]. Since more precise data are expected, especially from LEP 200, prospects do not seem to be too dim for a competitive \( \alpha_s \) determination from \( F_2^\gamma \).
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