A Bohr-Mottelson model of nuclei at finite temperature*

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Abstract

Starting from Boltzmann-Langevin equation and performing a fluid dynamical reduction, a system of coupled Langevin equations is derived for a multipolar velocity field. This system is then applied to monopole and β -, γ -quadrupole resonances, making a link between microscopic approaches and phenomenological macroscopic ones.

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1 Introduction

In the last decade, collective motions of hot nuclei have been investigated intensively by heavy ion reactions. Especially the existence of giant dipole resonances (GDR) has been identified through γ -ray yields and their angular distributions [1, 2, 3, 4]. From them, information on shapes of rotating hot nuclei are extracted and successfully compared with the prediction of rotating liquid drop model. Another interesting quantity is the width of the resonances which reflects damping mechanisms of the collective motion. An energy dependence of the width is still under discussion, concerning how to extract it from the data and how to explain it theoretically.

As a phenomenological approach, Landau theory is found to be useful for describing stable configuration of rotating hot nuclei and Langevin equation for the dynamics of the shapes. Such a model has been applied to calculations of GDR widths in order to take into account fluctuations of quadrupole deformation at finite temperature, i.e. the motional narrowing of the widths [5, 6]. Microscopic theories have also been applied to the problem: the cranked Nilsson approach to calculate the free energy at finite rotational frequency and finite temperatures [7], thereby to find equilibrium shapes; the thermal random phase approximation [8] as well as other approaches to calculate the energy dependence of GDR widths. The Boltzmann transport equation has also been applied to estimate the widths [9, 31]. But there is no consistent theory which connects phenomenological models and microscopic ones. It is therefore quite meaningful to develop a theoretical framework to derive a macroscopic model from a microscopic level which describes collective motions with dissipation and fluctuation.

Under certain conditions, fluid dynamics description is successful in describing collective motions of many body systems [11], and very convenient because it is essential to deal with a small number of variables for complex phenomena. This kind of approaches have been widely used in the context of nuclear dynamics at sufficiently high temperature to neglect quantum effects and justify a semi-classical approach. Usual fluid dynamics descriptions, however, deal only with average quantities and do not take into consideration fluctuations. For some specific cases, fluctuations should be incorporated in order to describe the phenomena. This is for example the case for the width of GDR, as stated above. In the case of induced fission too, thermal fluctuations allow some nuclei to pass over the potential barrier [12].

It is thus necessary to develop a stochastic extension of usual fluid dynamical approaches, leading to a Langevin type equation for the collective variables. This has been done in ref. [13], assuming one collective variable. This method should be extended to a general framework to describe collective motions of rotating hot nuclei, including the monopole, dipole, quadrupole... vibrations as well as rapid rotation. The framework could be taken as a Bohr-Mottelson model for hot nuclei. This model is limited to sufficiently high energies where the collective states are built on excited states.

In this article we propose a microscopic derivation of the stochastic equation of motion for a multi-component isoscalar velocity field without rotation, as a first step. A generalization to the isovector dipole mode and to the case of rapid rotations will be given in forthcoming papers. In section 2, we briefly discuss the Boltzmann-Langevin equation, which incorporates the higher order correlations into the fluctuating collision term, in

addition to the usual collision term. They are shown to satisfy the fluctuation-dissipation relation. After a projection onto collective variables, which is made in section 3, we end up with a system of generalized Langevin equations that are applied to monopole and f -, γ -quadrupole vibrations in section 4.

2 The Boltzmann-Langevin approach

Many aspects of nuclear collisions dynamics can be described by a one-body transport model (for a review, see ref. [14]), in which only average effects of two-body collisions are retained. Higher order correlations are eliminated through the Stoßzahlansatz (molecular chaos assumption) in the derivation of the Boltzmann equation and therefore, fluctuations are lost. In some specific situations (near a threshold, for example), it may be necessary to take into consideration the fluctuations. Recently, an extension of these one-body approaches have been proposed in the semi-classical limit by incorporating dynamical fluctuations due to high order correlations into the equation of motion [15, 16]. This yields the Boltzmann-Langevin equation (BLE) for the fluctuating phase-space density $f(\mathbf{r}, \mathbf{p}, t)$ [15],

$$\left(\frac{\partial}{\partial t} + \frac{\mathbf{p}}{m} \cdot \nabla_{\mathbf{r}} - \nabla_{\mathbf{r}} U(f) \cdot \nabla_{\mathbf{p}} \right) \hat{f}(\mathbf{r}, \mathbf{p}, t) = K(\hat{f}) + \delta K(\mathbf{r}, \mathbf{p}, t). \quad (1)$$

Here, $K(\hat{f})$ is the Ühling-Uhlenbeck collision term which only contains average effects of two-body collisions,

$$K(\hat{f}_1) = \frac{g}{(2\pi\hbar)^3} \int d^3p_2 d^3p_3 d^3p_4 W(12, 34) ((1 - \hat{f}_1)(1 - \hat{f}_2)\hat{f}_3\hat{f}_4 - \hat{f}_1\hat{f}_2(1 - \hat{f}_3)(1 - \hat{f}_4)), \quad (2)$$

where $\hat{f}_i = \hat{f}(\mathbf{r}_i, \mathbf{p}_i, t)$. In eq. (2), $W(12, 34)$ denotes the spin-isospin averaged transition rate,

$$W(12, 34) = \frac{4}{m} \frac{d\sigma}{d\Omega} \delta(\mathbf{p}_1 + \mathbf{p}_2 - \mathbf{p}_3 - \mathbf{p}_4) \delta(\epsilon_1 + \epsilon_2 - \epsilon_3 - \epsilon_4), \quad (3)$$

expressed in terms of the nucleon-nucleon cross section, $d\sigma/d\Omega$. In eq. (1), the additional term, δK , is referred to as the fluctuating collision term, and it arises from correlations not included into the collision term $K(\hat{f})$. Calculating it explicitly would mean to solve the N -body problem exactly. In analogy with the Brownian motion, it is assumed that eq. (1) describes a stochastic process, in which the entire phase-space density is a stochastic function and δK acts as a random force. In such a stochastic description, the fluctuating collision term is characterized by a zero mean value, $\overline{\delta K(\mathbf{r}, \mathbf{p}, t)} = 0$ ¹ and a correlation function,

$$\overline{\delta K(\mathbf{r}, \mathbf{p}, t) \delta K(\mathbf{r}', \mathbf{p}', t')} = C(\mathbf{p}, \mathbf{p}') \delta(\mathbf{r} - \mathbf{r}') \delta(t - t'), \quad (4)$$

which is Markovian and local in space, coherently with the structure of the collision term, eq. (2). In the weak coupling limit, $C(\mathbf{p}, \mathbf{p}')$ can be evaluated explicitly,

$$C(\mathbf{p}_1, \mathbf{p}_2) = \int d^3p_3 d^3p_4 W(12, 34) (f_1 f_2 \hat{f}_3 \hat{f}_4 + \hat{f}_1 \hat{f}_2 f_3 f_4)$$

¹The bar denotes here a *local* average or alternatively an average taken over a time interval short enough to accomodate only small amplitude fluctuations.

$$-2 \int d^3p_3 d^3p_4 W(13, 24) (f_1 f_3 \hat{f}_2 \hat{f}_4 + \hat{f}_1 \hat{f}_3 f_2 f_4) + \delta(\mathbf{p}_1 - \mathbf{p}_2) \int d^3p_2 d^3p_3 d^3p_4 W(12', 34) (f_1 f_2' \hat{f}_3 \hat{f}_4 + \hat{f}_1 \hat{f}_2' f_3 f_4). \quad (5)$$

This correlation function is closely related to the collision term, and entirely determined by the locally averaged single-particle density, $f(\mathbf{r}, \mathbf{p}, t)$, and the N-N cross section. This close relation between the correlation function and the collision term is a consequence of the fluctuation-dissipation-theorem associated with the stochastic evolution of the phase-space density.

The BLE provides a probabilistic description for nuclear collisions, in contrast to the deterministic description of the BUU model. A given initial density, $f(\mathbf{r}, \mathbf{p}, 0)$, evolves into a bundle of different histories, $\{\hat{f}(\mathbf{r}, \mathbf{p}, t)\}$, each conserving the total energy and momentum. In particular, when the system is going through instability, different histories can lead to widely different configurations. As a result, using the BLE, we can study catastrophic processes such as phase transitions and nuclear multifragmentations. Of course, the ultimate test of the model will be provided by a comparison with the experimental data. Unfortunately, up to now, such an extensive investigation has not been done. Nevertheless, some investigations, which have been carried out thus far, give promising results [17, 18].

The BLE, also provides a useful basis for studying nuclear collective vibrations (i.e. giant resonance excitations) and induced fission. Employing a fluid dynamical treatment and using the linearized limit of the BLE, it is possible to deduce collective transport models and calculate transport coefficients associated with collective variables. For a single collective variable, we have carried out such a reduction [13], and applied the resulting generalized Langevin equation to investigate dynamics of the induced fission [19]. Here, we propose an extension of this previous work to N collective variables and compressible fluid and propose possible applications to collective vibrations of hot nuclei.

3 Fluid dynamical reduction of BLE over a multi-component velocity field

In this section we will generalize the work performed in ref. [13] to a N -dimensions case. For that purpose we will follow exactly the same path as in [13] and try to keep the same notations. A similar derivation has already been done for such a velocity field [11], but it only considers average quantities; we propose here a stochastic extension.

3.1 Fluid dynamical reduction

In fluid dynamics, one deals with the macroscopic quantities, such as the local density,

$$\rho(\mathbf{r}, t) = \int g \frac{d^3p}{(2\pi\hbar)^3} \hat{f}(\mathbf{r}, \mathbf{p}, t), \quad (6)$$

and the current density, $\mathbf{u}$, defined by,

$$m\rho\mathbf{u}(\mathbf{r}, t) = \int g \frac{d^3p}{(2\pi\hbar)^3} \mathbf{p} \hat{f}(\mathbf{r}, \mathbf{p}, t). \quad (7)$$

where g denotes here and in the following the degeneracy parameter. In order to derive dynamical equations for these two quantities from BLE, we need to carry out the first two $\mathbf{p}$ -moments of the single-particle density in the momentum space. This yields the continuity equation

$$\frac{\partial}{\partial t}\rho + \nabla(\rho\mathbf{u}) = 0, \quad (8)$$

and the Euler equation,

$$m\rho\left(\frac{\partial}{\partial t}\mathbf{u} + (\mathbf{u}\cdot\nabla)\mathbf{u}\right) + \nabla\Pi + \rho\nabla U(\rho) = 0. \quad (9)$$

Here, the mean-field is related to the potential part of the energy density, $U(\rho) = \delta E_{\text{pot}}(\rho)/\delta\rho$ and the momentum flow tensor is defined by

$$\Pi_{ij}(\mathbf{r}, t) = \int g \frac{d^3p}{(2\pi\hbar)^3} Q_{ij} \hat{f}(\mathbf{r}, \mathbf{p}, t), \quad (10)$$

with

$$mQ_{ij} = (p_i - mu_i)(p_j - mu_j), \quad (11)$$

where the indexes i, j label cartesian components. The gradient of this tensor is,

$$(\nabla\Pi)_j = \sum_{i=1}^3 \frac{\partial}{\partial r_i} \Pi_{ij}. \quad (12)$$

We consider a large amplitude collective motion in a diabatic approximation and assume that the motion is characterized by a set of N independant collective variables, $\{q_n(t), n = 1, \dots, N\}$ and a set of associated irrotational, local velocity fields $\{\phi_n(\mathbf{r}), n = 1, \dots, N\}$. In terms of the velocity fields, the flow velocity can be expressed as

$$\mathbf{u}(\mathbf{r}, t) = \sum_{n=1}^N \dot{q}_n \nabla\phi_n. \quad (13)$$

For simplicity, we suppose that the set of velocity fields is orthogonal,

$$\sum_{n=1}^N \phi_n(\mathbf{r})\phi_n(\mathbf{r}') \propto \delta(\mathbf{r} - \mathbf{r}'), \quad (14)$$

and satisfies the completeness relation,

$$\int d^3r \phi_n(\mathbf{r})\phi_m(\mathbf{r}) \propto \delta_{nm}. \quad (15)$$

In equation (13), n can be a double indice, $n = (l, m)$ as for example the double index of a multipole field. We keep this compact notation n for sake of simplicity.

We will also assume that the time dependence of the density is determined in terms of the instantaneous values of the collective variables, $\rho(\mathbf{r}, t) \simeq \rho_0(\mathbf{r}, q_1(t), \dots, q_N(t))$. In this case, the continuity equation reads

$$\sum_{n=1}^N \dot{q}_n \left(\frac{\partial\rho_0}{\partial q_n} + \nabla(\rho_0\nabla\phi_n) \right) = 0, \quad (16)$$

and it gives a scaling condition in each independent direction,

$$\frac{\partial\rho_0}{\partial q_n} + \nabla(\rho_0\nabla\phi_n) = 0, \quad (17)$$

in agreement with the results in ref. [11]. In order to derive an equation of motion for the collective variables, we multiply Euler equation (eq. 9) by $\nabla\phi_p$ and integrate over $\mathbf{r}$ -space. Using the scaling condition and performing some partial integrations, we find a set of N coupled Langevin equations for the collective variables,

$$\sum_{n=1}^N M_{pn} \ddot{q}_n + \frac{1}{2} \sum_{n,m=1}^N \dot{q}_n \dot{q}_m \frac{\partial M_{nm}}{\partial q_p} + \frac{\partial V}{\partial q_p} = F_p(t) \quad \forall p. \quad (18)$$

In this expression, M_{nm} denotes the elements of the inertial tensor,

$$M_{nm}(q_1, \dots, q_N) = m \int d^3r \rho_0(\mathbf{r}, q_1, \dots, q_N) \nabla\phi_n \cdot \nabla\phi_m, \quad (19)$$

V is the potential energy,

$$V(q_1, \dots, q_N) = \int d^3r E_{\text{pot}}(\rho_0(\mathbf{r}, q_1, \dots, q_N)), \quad (20)$$

and $F_p(t)$ denotes a dynamical force due to the deformation of the Fermi sphere and the compressibility,

$$F_p(t) = \int g \frac{d^3r d^3p}{(2\pi\hbar)^3} \hat{F}_p(\mathbf{r}, \mathbf{p}) \hat{f}(\mathbf{r}, \mathbf{p}, t), \quad (21)$$

with

$$\hat{F}_p = \partial_i \partial_j \phi_p Q_{ij}. \quad (22)$$

For a given velocity field, mass parameters can easily be evaluated. The potential can be taken from the Liquid Drop Model or calculated from a Skyrme force, for example. Up to that stage, the major assumptions for deriving eq. (18) are the diabatic and quasistatic approximations, provided one assumes a velocity field of the form eq. (13). Eq. (18) is however not yet a closed equation for the collective variables, as $\hat{f}(\mathbf{r}, \mathbf{p}, t)$ entering the dynamical force, eq. (21), should be chosen as a solution of the BLE. One could hence consider situations very far from equilibrium provided one is able to evaluate $\hat{f}$. Therefore, the problem, now, is to evaluate the dynamical force: in order to obtain a tractable solution, we shall assume small departures from equilibrium. This will allow us to evaluate $\hat{f}(\mathbf{r}, \mathbf{p}, t)$ in an approximative way.

3.2 Calculation of the dynamical force

For a compressible fluid, the dynamical force given by eq. (21), can be separated into two parts,

$$F_n(t) = C_n(t) + Q_n(t), \quad (23)$$

where $C_n(t)$ denotes the force due to incompressibility,

$$C_n(t) = \frac{1}{3m} \int g \frac{d^3r d^3p}{(2\pi\hbar)^3} (\Delta\phi_n)(\mathbf{p} - m\mathbf{u})^2 \hat{f}(\mathbf{r}, \mathbf{p}, t), \quad (24)$$

and $Q_n(t)$ is the force due to the deformation of the Fermi surface,

$$Q_n(t) = \int g \frac{d^3r d^3p}{(2\pi\hbar)^3} \partial_i \partial_j \phi_n \tilde{Q}_{ij} \hat{f}(\mathbf{r}, \mathbf{p}, t), \quad (25)$$

with

$$\tilde{Q}_{ij} = Q_{ij} - \frac{1}{3m} \delta_{ij} (\mathbf{p} - m\mathbf{u})^2. \quad (26)$$

The force $C_n(t)$ can be expressed as,

$$C_n(t) = \frac{2}{3} \int d^3r \Delta\phi_n \epsilon \rho_0(\mathbf{r}, t), \quad (27)$$

where ϵ is the kinetic energy per particle. It should be noted that C_n disappears for an incompressible fluid (for which $\Delta\phi_n = 0$). This was, for example, the case considered in ref. [13]. In order to calculate the dynamical force $Q_n(t)$, eq. (25), we assume that the system is close to a local equilibrium and linearize the BLE around local equilibrium,

$$\hat{f}(\mathbf{r}, \mathbf{p}, t) = n(\mathbf{r}, \mathbf{p}, t) + n\tilde{n}\chi(\mathbf{r}, \mathbf{p}, t), \quad (28)$$

where $n(\mathbf{r}, \mathbf{p}, t)$ is the Fermi-Dirac distribution at temperature T and $\tilde{n} = 1 - n$. Using a Chapman-Enskog-like method (see, e.g. ref. [20]) and neglecting the gradients of the chemical potential and of the temperature in n , the linearized BLE reads,

$$\frac{\partial}{\partial t}(n\tilde{n}\chi) - \frac{\partial n}{\partial \epsilon} \left(\sum_{p=1}^N \dot{q}_p \hat{F}_p - \frac{2}{3} \epsilon_F \nabla \mathbf{u} \right) = I_0 \chi + \delta K_0, \quad (29)$$

where I_0 denotes the linearized collision operator,

$$I_0 \chi = \int g \frac{d^3p_2 d^3p_3 d^3p_4}{(2\pi\hbar)^3} W(12, 34) n_1 n_2 \tilde{n}_3 \tilde{n}_4 \Delta \chi, \quad (30)$$

with $\Delta \chi = \chi_4 + \chi_3 - \chi_2 - \chi_1$. Here, the correlation function of δK_0 has the same form as in eq. (5), but it is evaluated with the local equilibrium distribution n . From the shape of eq. (29) it is natural to look for a solution of the form

$$\chi = \sum_{p=1}^N \alpha_p(t) \left(\hat{F}_p - \frac{2}{3} \epsilon_F \Delta \phi_p \right). \quad (31)$$

Using this ansatz in $Q_n(t)$, we find,

$$Q_n(t) = \sum_{p=1}^N \alpha_p(t) T \Gamma_{np}, \quad (32)$$

where

$$\Gamma_{np} = \frac{g}{T} \int \frac{d^3r d^3p}{(2\pi\hbar)^3} (\hat{F}_n(\mathbf{r}, \mathbf{p}) - \frac{2}{3} \epsilon_F \Delta \phi_n) (\hat{F}_p(\mathbf{r}, \mathbf{p}) - \frac{2}{3} \epsilon_F \Delta \phi_p) n(\mathbf{r}, \mathbf{p}) \tilde{n}(\mathbf{r}, \mathbf{p}). \quad (33)$$

One can then evaluate the α_p 's as functions of Q_p 's by inverting the matrix (Γ) . For a spherical Fermi distribution Γ_{np} can easily be evaluated analytically,

$$\Gamma_{np} = \frac{4A}{5} < \epsilon_F (\partial_i \partial_j \phi_n \partial_i \partial_j \phi_p - \frac{1}{3} \Delta \phi_n \Delta \phi_p) >, \quad (34)$$

where A is the total number of nucleons, ϵ_F , the Fermi energy and where $< >$ denotes the average over real space. We obtain a Langevin equation for the dynamical force $Q_n(t)$ by multiplying eq. (29) by $\hat{F}_n - \frac{2}{3} \epsilon_F \Delta \phi_n$ and integrating over the phase space. This gives,

$$\frac{dQ_n}{dt} + \sum_{p=1}^N \Gamma_{np} \dot{Q}_p = - \sum_{p=1}^N \frac{Q_p}{\tau_{pn}} + \delta K_{F_n}. \quad (35)$$

In this expression, τ_{pn} denotes the elements of the relaxation times matrix,

$$\frac{1}{\tau_{pn}} = \sum_{q=1}^N \frac{(\Gamma^{-1})_{pq}}{T} I_{qn}, \quad (36)$$

with

$$I_{qn} = g^2 \int \frac{d^3r d^3p_1 d^3p_2 d^3p_3 d^3p_4}{(2\pi\hbar)^6} W(12, 34) n_1 n_2 \tilde{n}_3 \tilde{n}_4 \frac{\Delta \hat{Q}_n}{2} \frac{\Delta \hat{Q}_q}{2}, \quad (37)$$

where $\Delta \hat{Q}_n = \hat{Q}_n(\mathbf{r}, \mathbf{p}_1) + \hat{Q}_n(\mathbf{r}, \mathbf{p}_3) - \hat{Q}_n(\mathbf{r}, \mathbf{p}_2) - \hat{Q}_n(\mathbf{r}, \mathbf{p}_4)$ and $\hat{Q}_n(\mathbf{r}, \mathbf{p}) = \partial_i \partial_j \phi_n \tilde{Q}_{ij}$. In the Fermi gas model, for low temperatures ($T \ll \epsilon_F$) and isotropic N-N cross-sections, it is possible to evaluate the relaxation time matrix, $1/\tau_{pn}$, analytically, for given multipole modes [9]. Using $C(\mathbf{p}_1, \mathbf{p}_2)$, eq. (5), the correlation function of the random force δK_{F_n} can be calculated explicitly, and we find,

$$\overline{\delta K_{F_n}(t) \delta K_{F_p}(t')} = 2I_{pn} \delta(t - t'). \quad (38)$$

The width of the fluctuating force is directly connected to the dissipation part of the equation, in agreement with the fluctuation-dissipation theorem. In the Langevin eq. (35) for the dynamical force, the second term of the l.h.s. describes a driving force due to distortion of Fermi surface, which arises from coherent coupling between single-particle and collective motion. On the other hand, the r.h.s. describes two competing effects of decay of distortions by two-body collisions and enhancement of distortions due to fluctuations. Note that the dissipative part of eq. (35) only contains two-body damping and directly results from the collision integral of BLE. With the choice of the velocity field, eq. (13), and the diabatic approximation, one loses one-body damping.

3.3 Memory effects

Formally integrating eq. (35) as,

$$Q_n(t) = - \int_{-\infty}^t dt' \sum_{p=1}^N \left(\Gamma_{np} \dot{Q}_p(t') + \frac{Q_p(t')}{\tau_{pn}} (1 - \delta_{pn}) \right) e^{-(t-t')/\tau_{nn}} + \delta K_n, \quad (39)$$

and inserting it into eq. (18) yields a new set of Langevin equation for the collective variables,

$$\begin{aligned} \sum_{n=1}^N M_{pn} \ddot{q}_n + \frac{1}{2} \sum_{n,m=1}^N \dot{q}_n \dot{q}_m \frac{\partial M_{nm}}{\partial q_p} + \frac{\partial V}{\partial q_p} - C_p(t) \\ = - \int_{-\infty}^t dt' \sum_{n=1}^N \left(\Gamma_{np} \dot{q}_n(t') + \frac{Q_n(t')}{\tau_{pn}} (1 - \delta_{pn}) \right) e^{-(t-t')/\tau_{rp}} + \delta K_p \quad \forall p. \end{aligned} \quad (40)$$

Using the result of eq. (38) the correlation matrix of the random forces δK_p can be calculated as

$$\overline{\delta K_n(t) \delta K_p(t')} = 2 \frac{\tau_{nn} \tau_{pp}}{\tau_{nn} + \tau_{pp}} I_{pn} \left(\exp\left(\frac{t-t'}{\tau_{pp}}\right) \theta(t' - t) + \exp\left(\frac{t'-t}{\tau_{nn}}\right) \theta(t - t') \right). \quad (41)$$

In eq. (40), the coefficients M_{np} , Γ_{np} and τ_{np} are respectively given by eqs. (19), (34) and (36).

The set of Langevin eqs. (40) describes a relaxation process with memory effects due to the non-local structure of the friction kernel. Due to coherent coupling between single-particle and collective motion, the collective kinetic energy is initially stored as dynamical potential energy and subsequently dissipated into coherent excitation energy by two-body collisions. As a result, the dynamical force appears as a restoring friction force on the collective variables [11, 13]. In the limit of very slow collective motion as in induced fission, the memory effects can be neglected by approximating,

$$\exp(-(t-t')/\tau_{np}) = \tau_{np} \delta(t-t'), \quad (42)$$

and the dynamical force in eq. (40) appears as a local friction force.

For a single collective variable q , eq. (40) reduces to a simple form,

$$M \ddot{q} + \frac{\dot{q}^2}{2} \frac{\partial M}{\partial q} + \frac{\partial V}{\partial q} = C(t) - \int_{-\infty}^t dt' \Gamma \dot{q}(t') e^{-(t-t')/\tau} + \delta K. \quad (43)$$

with,

$$\overline{\delta K(t) \delta K(t')} = T \Gamma e^{-(t-t')/\tau}. \quad (44)$$

This is an equation similar to the one obtained in ref. [13], except for the additional force, $C(t)$, due to compressibility. Eq. (43) is a generalization of the Lagrange equation,

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}} \right) - \frac{\partial L}{\partial q} = - \frac{\partial \mathcal{F}}{\partial \dot{q}} + \delta K, \quad (45)$$

where the Lagrangian reads,

$$L = \frac{1}{2} M \dot{q}^2 - V(q) + \mathcal{C}(t), \quad (46)$$

with

$$\mathcal{C}(t) = \frac{2}{3} \int d^3 \mathbf{r} \Delta \phi \epsilon \int dq p_0(\mathbf{r}, q), \quad (47)$$

and the Rayleigh's dissipation function [21],

$$\mathcal{F} = \int_{-\infty}^t dt' \Gamma \dot{q}(t) \dot{q}(t') e^{-|t-t'|/\tau}, \quad (48)$$

is memory dependent. The additional force, δK , comes from the interaction of the collective variable with the heat bath and is directly coming from the microscopic derivation. The l.h.s. of eq. (45) represents the initial Bohr-Mottelson model and the r.h.s. contains additional terms for hot nuclei, arising from two-body dissipation and written in terms of nucleon-nucleon cross section.

The set of Langevin eqs. (40), which are usually called generalized Langevin equations because of the memory effects, provides a closed set of equations for the collective variables. All the parameters entering into these equations can be determined in terms of standard nuclear matter properties. It is a quite complicated set of equations, especially because there is a coupling between the stochastic forces, eq. (41), but we can expect some simplifications in particular cases. In the next section, we will apply eq. (40) to investigate the monopole and quadrupole nuclear vibrations.

4 Application to giant resonances

4.1 Application to monopole resonance

First, we only consider the monopole vibration, which corresponds to a breathing mode. To describe this mode we take a velocity field of the form,

$$\mathbf{u} = \dot{q}_0 \nabla \left(\frac{r^2}{2} \right). \quad (49)$$

The equation of motion for the collective variable q_0 becomes,

$$M \ddot{q}_0 + \frac{1}{2} \frac{\partial M}{\partial q_0} \dot{q}_0^2 + \frac{\partial V}{\partial q_0} = C(t), \quad (50)$$

Since there is no deformation of the Fermi surface, the only contribution to the dynamical force is due to compressibility, $C(t)$. (Note that deformation of the Fermi sphere of the momentum space is induced by deformation of the configuration space, as already seen in eq. (25)). Therefore, the dynamical equation for the monopole vibration is deterministic one, there are no fluctuation because there is no damping. This is inherent to our derivation where we have neglected the gradients of temperature and Fermi energy, which means neglecting fluctuations in real space.

To evaluate the coefficients entering eq. (50), we need to evaluate integrals of the following form,

$$A_\alpha(t) = \int d^3 \mathbf{r} \rho_0^\alpha(\mathbf{r}, t). \quad (51)$$

Using the scaling condition given by eq. (17) and carrying out some partial integrations, we obtain,

$$\dot{A}_\alpha(t) = -3(\alpha - 1) \dot{q}_0 A_\alpha(t), \quad (52)$$

which yields,

$$A_n(t) = A_n(0)e^{3(n-1)q_0}. \quad (53)$$

For a degenerate Fermi gas, the average kinetic energy is given by $\epsilon = \frac{3}{5}\epsilon_F(\rho)$. With the help of eq. (53), we calculate the dynamical force given by eq. (27) due to the compressibility and find,

$$C(t) = \frac{6A}{5}e^{5q_0}\epsilon_F(\rho_s), \quad (54)$$

where $\epsilon_F(\rho_s)$ is the Fermi energy at the saturation density ρ_s . In order to calculate the potential energy, we use a Skyrme force of the form,

$$U(\rho) = a\frac{\rho}{\rho_s} + b\left(\frac{\rho}{\rho_s}\right)^\sigma, \quad (55)$$

which gives,

$$V(q_0) = A\left(\frac{a}{2}e^{-3q_0} + \frac{b}{\sigma+1}e^{-3\sigma q_0}\right). \quad (56)$$

Eventually, the inertia parameter can be calculated in a similar way, and we obtain,

$$M(q_0) = mA < r^2 >_s e^{2q_0}, \quad (57)$$

where $< . >_s$ means average value over density at saturation.

After linearization, the equation of motion for the collective variable q_0 becomes,

$$m < r^2 >_s \ddot{q}_0 + K_\infty q_0 = 0, \quad (58)$$

where K_∞ is the compressibility of nuclear matter,

$$K_\infty = 9(a + \sigma b + \frac{2}{3}\epsilon_F(\rho_s)). \quad (59)$$

Eq. (58) is exactly the same as the one obtained with a hydrodynamical model in ref. [22], and leads to the same frequency,

$$\Omega_0 = \sqrt{\frac{K_\infty}{m < r^2 >_s}}, \quad (60)$$

which is larger by a factor $\sqrt{\pi^2/15}$ compared to the exact frequency obtained with a velocity potential $\phi_0 \propto j_0(kr)$ [23]. Nevertheless, in the case of monopole vibrations where there is no deformation of Fermi sphere, hydrodynamical and fluid dynamical approaches are identical.

4.2 Application to quadrupole vibrations

In this section, we apply the derived Langevin equations to a very simple two-dimensions case of β - and γ -quadrupole vibrations relevant for the physics of hot nuclei. The parameters, β and γ are the Hill and Wheeler coordinates, which, for small amplitude oscillations, can be easily related to spherical harmonics representation [24],

$$R(\theta, \varphi) = R_0 \left(1 + \beta(\cos \gamma) Y_2^0(\theta, \varphi) + \frac{1}{\sqrt{2}} \sin \gamma (Y_2^2(\theta, \varphi) + Y_2^{-2}(\theta, \varphi)) \right). \quad (61)$$

This representation is more useful to define a parametrization which satisfies eq. (13). In the following, we call the β -quadrupole oscillation the one that corresponds to a deformation proportional to $Y_2^0(\theta, \phi)$ and the γ -one, the one that is a superposition of $Y_2^2(\theta, \phi)$ and $Y_2^{-2}(\theta, \phi)$ deformations. Therefore, for these two oscillations, velocity fields read,

$$\mathbf{v}_\beta = \nabla \phi_\beta \quad \text{with} \quad \phi_\beta = \frac{1}{2}(2z^2 - x^2 - y^2), \quad (62)$$

$$\mathbf{v}_\gamma = \nabla \phi_\gamma \quad \text{with} \quad \phi_\gamma = \frac{\sqrt{3}}{2}(x^2 - y^2), \quad (63)$$

and the corresponding collective variables, q_β and q_γ , can be expressed in terms of Hill and Wheeler coordinates as

$$q_\beta = \sqrt{\frac{5}{16\pi}}\beta \cos \gamma, \quad (64)$$

$$q_\gamma = \sqrt{\frac{5}{16\pi}}\beta \sin \gamma. \quad (65)$$

This allows us to calculate all coefficients entering Langevin equation, eq. (40), and compare the frequencies and widths of these resonances with experimental data.

4.2.1 Single resonance

As a first step we consider both resonances separately, which reduces our problem to twice a one dimension study. Since these quadrupole vibrations are incompressible, the dynamical forces $C_\beta(t)$ and $C_\gamma(t)$ vanish, and the equation of motion given by eq. (40) can be written in a more convenient form,

$$\ddot{q} + \frac{1}{\tau}\dot{q} + \frac{\Gamma}{M}q + \frac{1}{\tau M}\frac{\partial^2 V}{\partial q^2}\dot{q} + \frac{1}{\tau M}\frac{\partial V}{\partial q} = \sqrt{\tau\Gamma T}w(t), \quad (66)$$

with

$$\overline{w(t)} = 0 \quad \text{and} \quad \overline{w(t)w(t')} = 2\delta(t - t'). \quad (67)$$

In eq. (66), we have neglected second order term. To get it, we have derived eq. (18) and combined it with eq. (35). Eq. (66) is linear, therefore, the equation of motion of the average value of the deformation is the same as eq. (66) without the stochastic force. For a single resonance case, fluctuations do not affect the mean value evolution and a deterministic equation is sufficient.

For quadrupolar resonances, it is well known that for isoscalar vibrations the potential force is much smaller than the one arising from the deformation of Fermi surface, therefore we can safely neglect it, as a first approximation. We will see further that the potential becomes essential in case of coupled modes. Then, in the small amplitude limit, eq. (66) describes a damped harmonic oscillator with the mean amplitude given by

$$\dot{q}(t) = \dot{q}_0 e^{-t/2\tau} \cos(\Omega t + \varphi). \quad (68)$$

Here, the frequency of the vibration determined by the inertia parameter M and the stiffness Γ , $\Omega^2 = \Gamma/M$, the width is equal to $\hbar/\tau$, and $\dot{q}_0$ and φ are fixed by initial

conditions. The fact that we obtain an equation over $\dot{q}$ is directly linked to the choice of the collective velocity field, eq. (13). In the small amplitude limit, i.e. at the zeroth order in q , the inertia and the stiffness are the same for the β - and γ - vibrations, and given by,

$$M = 2mA < r^2 >, \quad (69)$$

$$\Gamma = \frac{24A\epsilon_F}{5}. \quad (70)$$

As a result, the frequency of the collective vibrations in these modes becomes,

$$\Omega^2 = \frac{12\epsilon_F}{5m < r^2 >}, \quad \text{ie,} \quad \hbar\Omega = 66A^{-1/3} \text{ MeV}. \quad (71)$$

It is possible to calculate analytically the relaxation time τ associated with quadrupole vibrations at low temperatures, $T \ll \epsilon_F$, and for heavy nuclei, $\hbar\Omega \ll \epsilon_F$, using the methods of refs. [9, 25] (for details, see [26]). We find the same relaxation time for both the β - and γ -vibrations,

$$\frac{1}{\tau} = 8\sigma v_F \rho \left(\frac{T}{\epsilon_F} \right)^2. \quad (72)$$

Frequency is in good agreement with experimental data and the width is far too small. Actually, it has been shown that memory effects in collision term of the Boltzmann equation significantly increase the width [9], in relation (72) they multiply T^2 by $(1 + (\Omega/2\pi T)^2)$. With this improvement, τ is not diverging anymore when temperature vanishes and analytical width is about 1/3 of experimental one.

4.2.2 Coupling between β - and γ -oscillations

We now discuss the general case of coupled β - and γ -vibrations. First, we consider the linear order in q . In this limit, the inertia, stiffness and relaxation matrices can easily be evaluated from eqs (19, 34, 36). In the zeroth order in q_β, q_γ , all the coupling parameters are vanishing and these matrices are diagonal,

$$(M) = \begin{pmatrix} M & 0 \\ 0 & M \end{pmatrix}, \quad (73)$$

$$(\Gamma) = \begin{pmatrix} \Gamma & 0 \\ 0 & \Gamma \end{pmatrix}, \quad (74)$$

$$(1/\tau) = \begin{pmatrix} 1/\tau & 0 \\ 0 & 1/\tau \end{pmatrix}, \quad (75)$$

where the parameters, M, Γ and τ are given by eqs. (69,70,72). Therefore, the linear equations of motion for these two coupled modes becomes,

$$M\ddot{q}_\beta + \frac{\partial V}{\partial q_\beta} = F_\beta, \quad (76)$$

$$M\ddot{q}_\gamma + \frac{\partial V}{\partial q_\gamma} = F_\gamma. \quad (77)$$

Here, the dynamical forces are determined by,

$$\frac{dF_\beta}{dt} = -\Gamma\dot{q}_\beta - \frac{F_\beta}{\tau} + \delta K_\beta, \quad (78)$$

$$\frac{dF_\gamma}{dt} = -\Gamma\dot{q}_\gamma - \frac{F_\gamma}{\tau} + \delta K_\gamma. \quad (79)$$

with,

$$\overline{\delta K_\beta(t)\delta K_\gamma(t')} = 0, \quad (80)$$

and,

$$\overline{\delta K_\beta(t)\delta K_\beta(t')} = \overline{\delta K_\gamma(t)\delta K_\gamma(t')} = 2\frac{T\Gamma}{\tau}\delta(t-t'). \quad (81)$$

We eventually obtain a very simple set of two coupled GLE, especially because stochastic forces are not correlated. In the linear limit of $\beta - \gamma$ vibrations, the only coupling arises from the potential term. All parameters are set which describe dissipation and fluctuation of the system. It is worth noticing here that they are all calculated from the nucleon-nucleon elementary cross-section. Given a potential map, they are easy equations to solve numerically. One possible application would be to study the motional narrowing of GDR, in the line of the work done in refs. [5, 6]. However, eqs. (76,77) cannot be directly applied to describe isovector vibrations. In order to describe the isovector vibrations, the model must be improved further by distinguishing the motion of protons and neutrons. Nevertheless, as a first step, one could use the same coupling between dipole and quadrupole modes as the one of refs. [5, 6].

In the case of large amplitude motion, such as thermal fission, we need to keep higher order terms. In the one dimension case, it has been shown that second order term can increase up to 40% the fission rate [19]. The stiffness and the relaxation matrices are still diagonal due to the fact that their derivation is based on the small amplitude deformations in the momentum space. The inertia tensor is non-diagonal and its elements can be given as,

$$M_{\beta\gamma}(q_\beta, q_\gamma) = 2\frac{\sqrt{3}}{3}Am < r^2 > e^{-2q_\beta} \sinh(2\sqrt{3}q_\gamma), \quad (82)$$

$$M_{\beta\beta}(q_\beta, q_\gamma) = \frac{2}{3}Am < r^2 > (e^{-2q_\beta} \cosh(2\sqrt{3}q_\gamma) + 2e^{4q_\beta}), \quad (83)$$

$$M_{\gamma\gamma}(q_\beta, q_\gamma) = 2Am < r^2 > e^{-2q_\beta} \cosh(2\sqrt{3}q_\gamma), \quad (84)$$

As a result, the equations of motion for the large amplitude $\beta - \gamma$ vibrations become,

$$M_{\beta\beta}\ddot{q}_\beta + M_{\beta\gamma}\ddot{q}_\gamma + \frac{1}{2} \left(\frac{\partial M_{\beta\beta}}{\partial q_\beta} \dot{q}_\beta^2 + \frac{\partial M_{\beta\gamma}}{\partial q_\beta} \dot{q}_\gamma^2 + 2\frac{\partial M_{\beta\gamma}}{\partial q_\beta} \dot{q}_\beta \dot{q}_\gamma \right) + \frac{\partial V}{\partial q_\beta} = F_\beta, \quad (85)$$

$$M_{\gamma\gamma}\ddot{q}_\gamma + M_{\beta\gamma}\ddot{q}_\beta + \frac{1}{2} \left(\frac{\partial M_{\beta\beta}}{\partial q_\gamma} \dot{q}_\beta^2 + \frac{\partial M_{\gamma\gamma}}{\partial q_\gamma} \dot{q}_\gamma^2 + 2\frac{\partial M_{\beta\gamma}}{\partial q_\gamma} \dot{q}_\beta \dot{q}_\gamma \right) + \frac{\partial V}{\partial q_\gamma} = F_\gamma, \quad (86)$$

where the dynamical forces F_β and F_γ are determined by eqs. (78,79). These equations can be applied to study thermal fission, in the line of the work done in refs. [27, 28, 29].

5 Concluding remarks

We have carried out a fluid dynamical reduction of the BLE to a situation where the velocity field can be described by a set of N collective variables. We obtained a set of N coupled Langevin equations for the collective variables, which constitute a generalization of the Bohr-Mottelson model for hot nuclei. In these equations, the collisional dissipation and the associated fluctuations are incorporated in terms of appropriate transport coefficients, which are determined in terms of nuclear matter properties without any free parameters. However, we should remark that the deduced transport coefficients are valid only in the classical limit, i.e. at sufficiently high temperatures and low frequencies of the collective motion. In the quantal regime, i.e. at high frequencies and low temperatures, these expressions for the transport coefficients are not realistic, due to the fact that the underlying BLE does not contain any quantal effects, aside from the Pauli blocking factors. Recently, an improvement of the BLE has been proposed by including the memory effects into the collision term as well as its fluctuating part in a consistent manner [30]. A treatment, similar to the one carried out here, but based on the memory incorporated BLE will allow for more accurate determination of transport coefficients in the quantal regime of collective motion. In recent publications, the quantal effects associated with the mean-field evolution on the low-energy nuclear collisions has been studied in ref. [31], and the quantal effects on dynamics of the induced fission has been investigated in ref. [32].

We should also point out that in this work, we consider only the volume dissipation due to incoherent binary collisions and neglect entirely the surface dissipation mechanism. The surface dissipation, in general, involves two different contributions: one part is associated to the Landau damping or the one-body dissipation and the other part is due to coupling to the low-lying collective vibrations. In the case of high frequency collective vibrations in medium weight and heavy nuclei, for both isoscalar and isovector modes, the collisional damping accounts about 1/3 of the spreading width [9]. The large part of the spreading width arises from the coupling to low-lying collective modes as illustrated in ref. [33]. In the case of the monopole mode, the collisional damping vanishes due to symmetry and hence its spreading width is entirely due to the coupling with low-lying modes. It will be very useful, using the memory incorporated BLE, to extract transport coefficients for the collective variables corresponding to various dissipation mechanisms [30].

In this work, we have then investigated the simple cases of monopole oscillation and β - and γ -quadrupolar oscillations. For such cases, the Langevin equations become simple and, especially for the last case, the only coupling left is the one due to the potential. For applications to realistic cases such as the motional narrowing of GDR widths, we have to extend the framework to the isovector mode, taking into account rapid rotations of nuclei as well. This line of development is under way. A validity of the assumption of irrotational flow should also be investigated, because the vorticity is not conserved in the case of a viscous, i.e., dissipative fluid.

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References

- [1] F.S. Dietrich, J.C. Browne, W.J. O'Connell and M.J. Kay, Phys. Rev. **C10** (1974) 795
- [2] J.O. Newton, B. Herskind, R.M. Diamond, E.L. Dines, J.E. Draper, K.H. Lindenberg, C. Schüick, S. Shih and F.S. Stephens, Phys. Rev. Lett. **46** (1981) 1383
- [3] K.A. Snover, Annu. Rev. Nucl. Part. Sci. **36** (1986) 545 and references therein
- [4] J.J. Gaardhoje, Annu. Rev. Nucl. Part. Sci. **42** (1992) 483 and references therein
- [5] Y. Alhassid and B. Bush, Nucl. Phys. **A531** (1991) 1
- [6] Y. Alhassid and B. Bush, Phys. Rev. Lett. **63** (1989) 2452; Nucl. Phys. **A514** (1990) 434
- [7] M. Gallardo, M. Diebel, T. Dossing and R.A. Broglia, Nucl. Phys. **A443** (1985) 415
- [8] K. Sugawara-Tanabe and K. Tanabe, Phys. Lett. **B192** (1987) 268
- [9] S. Ayik and D. Boilley, Phys. Lett. **B276** (1992) 263; Phys. Lett. **B284** (1992) 482E
- [10] A. Bonasera et al., Gull Lake Conf. on Giant Resonances (Aug. 1993), to be published in Nucl. Phys. **A**
- [11] B. Morgenstern and W. Nörenberg, Nucl. Phys. **A492** (1989) 93
- [12] Y. Abe, C. Grégoire and H. Delagrange, J. Phys. (Paris) **C4**, 47 (1990) 413
- [13] S. Ayik, E. Suraud, J. Stryjewski and M. Belkacem, Z. Phys. **A337** (1990) 413
- [14] G.F. Bertsch and S. Das Gupta, Phys. Rep. **160** (1988) 190
- [15] S. Ayik and C. Grégoire, Phys. Lett. **B212** (1988) 269; Nucl. Phys. **A513** (1990) 187
- [16] J. Raudrup and B. Remaud, Nucl. Phys. **A514** (1990) 339
- [17] E. Suraud, S. Ayik, M. Belkacem and J. Stryjewski, Nucl. Phys. **A542** (1992) 141
- [18] P. Chomaz, G.F. Burgio and J. Randrup, Phys. Lett. **B254** (1991) 340

- [19] D. Boilley, E. Suraud, Y. Abe and S. Ayik, Nucl. Phys. **A556** (1993) 67
- [20] K. Huang, *Statistical mechanics* (J. Wiley & Sons, New-York, 1963)
- [21] H. Goldstein, *Classical mechanics* (Addisson-Wesley, Reading, 1980)
- [22] G.F. Bertsch, Nucl. Phys. **A249** (1975) 253
- [23] J.P. Blaizot, Phys. Rev. **64** (1980) 171
- [24] P. Ring and P. Schnuck, *The Nuclear Many-Body Problem* (Springer-Verlag, New-York, 1980)
- [25] G. Bertsch, Z. Phys. **A289** (1978) 105
- [26] D. Boilley, PhD thesis, Université de Caen - GANIL T93 04 (1993)
- [27] T. Wada, N. Carjan and Y. Abe, Nucl. Phys. **A538** (1992) 283c
- [28] T. Wada, Y. Abe and N. Carjan, Phys. Rev. Lett. **70** (1993) 3538
- [29] P. Fröbrich and S. Y. Xu, Nucl. Phys. **A477** (1988) 143; P. Fröbrich, Nucl. Phys. **A545** (1992) 87c
- [30] S. Ayik, submitted to Z. Phys. **A** (1994) and in preparation for publication
- [31] A. Bonasera, V.N. Kondratyev, A. Smerzi and E.A. Remler, Phys. Rev. Lett. **71** (1993) 505
- [32] H. Hofmann, G.-L. Ingold and M.H. Thoma, Phys. Lett. **B317** (1993) 489
- [33] G.F. Bertsch, P.F. Bortignon and R.A. Broglia, Rev. of Mod. Phys. **56** (1983) 287