First Order Inflation in General Relativity

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Abstract

I give a general formulation of the constraints on models of inflation ended by a first order phase transition arising from the requirement that they do not produce too many large (observable) true vacuum voids – the ‘big bubble problem’. It is shown that this constraint can be satisfied by a simple model in Einstein gravity – a variant of ‘hybrid’ or ‘false vacuum’ inflation.

The idea that inflation could be ended by a first order phase transition is often dismissed as too problematic following Guth’s original trouble in finding a graceful exit from the false vacuum [1]. The scalar field, ψ, whose self-interaction potential acts like a cosmological constant driving the expansion, is trapped in a local potential minimum so cannot evolve classically. A fixed tunnelling rate to the true vacuum in a de Sitter universe leads to a time invariant state. One expects the phase transition to complete either at once or not at all.

First order inflation was revived in the guise of extended inflation [2] which invoked an extended gravitational lagrangian. In Brans-Dicke gravity [3], where Newton’s constant is replaced by a field Φ, the expansion rate decreases with time as the Planck mass, m_pl, grows during inflation allowing the phase transition to eventually complete. However this too was shown to produce too many large true vacuum voids in any Brans-Dicke model that was sufficiently close to general relativity to obey other observational limits [4, 5] – the ‘big bubble problem’. Even when the Brans-Dicke parameter ω is allowed to vary in time [6] it is difficult to achieve an acceptable model as the same variation of the Hubble rate required to produce a non-scale-invariant bubble spectrum also produces a non-scale-invariant conventional perturbation spectrum incompatible with models of large-scale structure formation [7].

My aim here is to show that these problems need not necessarily be present if it is the changing shape of the potential that varies during inflation rather than (or in addition to) the Hubble rate. Soon after La and Steinhardt’s work it was pointed out by both Linde [8] and Adams & Freese [9] that a graceful exit from first order inflation can also be achieved within general relativity. All that is required is the presence of a second scalar field which can interact with ψ and slow-rolls while the ψ field is trapped in the false vacuum. It is not obvious whether these models based on two interacting fields in general relativity should fair any better than the extended gravity models in avoiding the ‘big bubble problem’. Indeed, as far as I am aware, this issue has never been seriously addressed before. Here I will quantify the big bubble constraints on the model proposed originally by Linde [8] and give a more general formulation of the big bubble constraint in first order inflation. For a wide range of parameter space, models of first order inflation in general relativity easily satisfy these constraints while producing near-scale-invariant perturbation spectra. These results are based on work presented in [10].
The Model

To give probably the simplest example, consider the first order potential proposed originally by Linde [8] and discussed more recently as a variant of ‘hybrid’ [11] or ‘false vacuum’ [10] inflation.

\[
V(\psi, \phi) = \frac{\lambda}{4} (\psi^4 + M^4) - \frac{\lambda'}{3} M \psi^3 + \frac{\alpha}{2} M^2 \psi^2 + \frac{\lambda'}{2} \phi^2 + \frac{1}{2} m^2 \phi^2.
\]  

(1)

\(\lambda, \lambda', \alpha\) and \(\gamma\) are all dimensionless coupling constants which we can take to be of order unity. Requiring the energy density of the true vacuum (at \(\phi = 0\)) to vanish one can eliminate \(\gamma\) in terms of \(\alpha\), say. I will consider the case where the mass \(M\) is very much greater than \(m\) and \(\psi\) will rapidly roll down to \(\psi = 0\) while \(\phi\) is still large.

The effective potential for the field \(\psi\) at different values of the field \(\phi\) is then that drawn in Figure 1, for \(0 < \alpha < \gamma^2/4\lambda\). The mass of the \(\psi\) field in the false vacuum (\(\psi = 0\)) is

\[
M_0^2(\phi) = \alpha(\phi) M^2 \equiv \alpha M^2 + \lambda' \phi^2.
\]

(2)

Clearly a second order transition to the true vacuum is not possible when \(\alpha > 0\).

A useful parameter describing the ‘shape’ of the first order potential for \(\psi\) is given by

\[
\delta(\phi) = \frac{9 \lambda \dot{\phi}}{\gamma^2}.
\]

(3)

For \(\delta > 9/4\) the false vacuum at \(\psi = 0\) is the only minimum of \(V(\psi)\). As \(\phi\) continues to roll down towards its minimum at \(\phi = 0\), \(\delta\) decreases, and for \(\delta < 2\) the potential of the second minimum, the true vacuum, becomes lower than the false vacuum, although they are still separated by the potential barrier. Eventually, as the energy difference between the two minima increases, if the nucleation rate becomes large enough, the phase transition can complete.

Big bubble constraints

The most important parameter in determining the dynamics of a cosmological first order transition is the percolation parameter

\[
p = \frac{\Gamma}{H^2},
\]

(4)

where the Hubble rate \(H \equiv \dot{a}/a\) and \(\Gamma\) is the physical nucleation rate (i.e. the number of bubbles of the true vacuum nucleated per unit physical volume per unit time).

To produce a successful model of first order inflation the percolation parameter must fulfil two competing requirements. Firstly it must remain small enough for long enough to produce a large, flat, homogeneous, etc. universe solving the horizon, flatness, smoothness, etc. problems with an acceptably small filling fraction of large true vacuum voids nucleated early on during inflation and swept up by the subsequent expansion (\(p \leq p_\text{cr} \sim 1\)). But eventually the percolation parameter must reach its critical value in order to complete the transition to the true vacuum phase (\(p \geq p_\text{cr} \sim 1\)). As remarked earlier, in models based on extended gravity theories this may be achieved by allowing the effective gravitational “constant” to vary producing a decreasing Hubble rate and thus an increasing \(p\). The precise big bubble constraints have been considered in the case of both extended [5] and hyperextended [6] inflation. Here I will approximate these results by the requirement that the filling fraction of voids nucleated \(e^{55}\) expansion times (55 e-foldings) from the end of inflation be less than \(10^{-5}\) (\(p_{55} \leq 10^{-5}\)).

We can give these constraints in terms of the shape of the first order potential using the calculation of the tunnelling rate given originally by Coleman [12]

\[
\Gamma = A \exp(-SE),
\]

(5)
with $A \sim M^4$ and $S_E$ is the Euclidean action of the tunnelling configuration\(^3\). Recently Adams [14] has shown that this can be given in terms of a numerically calculated fitting function of the shape of the potential:

$$S_E = \frac{2 \pi^2}{\lambda} B_4(\delta) .$$  \hspace{1cm} (6)

Thus requiring $p \geq p_{\text{tr}}$ to end inflation places an upper bound on the minimum value of the parameter $\delta$ as $\phi \rightarrow 0$

$$\delta_0 \equiv \frac{9 \lambda \alpha}{\gamma^2} ,$$  \hspace{1cm} (7)

such that

$$B_4(\delta_0) < B_4(\delta_{\text{tr}}) = \frac{\lambda}{2 \pi^2} \ln \frac{\lambda M^4}{4 \rho_{\text{tr}} H^2} .$$  \hspace{1cm} (8)

If the transition is too strongly first order for a given energy scale $M$, i.e. $\delta_0 > \delta_{\text{tr}}(M)$, the transition can never complete and inflation continues forever.

On the other hand requiring $p_{55} \lesssim 10^{-5}$ corresponds to an lower limit on the parameter $\delta$ 55 e-foldings from the end of inflation.

$$B_4(\delta_{55}) \gtrsim B_4(\delta_0) = \frac{\lambda}{2 \pi^2} \left( \ln \frac{\lambda M^4}{4 \rho_{\text{tr}} H^2} + 11.5 \right) ,$$  \hspace{1cm} (9)

where the calculated values of $\delta_{\text{tr}}$ and $\delta_0$ are plotted in Figure 2. Models such as extended inflation which only alter the mass scales must proceed along a horizontal trajectory (i.e. at fixed $\delta$) from outside $\delta_{55}$ to reach $\delta_{\text{tr}}$ during the last 55 e-foldings of inflation, whereas if the shape of the potential changes (changing $\delta$) they can also proceed vertically.

### Conclusions

Returning to our specific model, $\delta_{\text{tr}}$ fixes the value of $\phi$ at the end of inflation, and we also have a minimum value for $\delta$ and thus $\phi$ 55 e-foldings earlier. In the limit where $\lambda M^4 \gg m^2 \phi^2$ the potential energy density remains approximately constant during inflation and the ratio between the value of $\phi$ at the end of inflation and the value $N$ e-foldings earlier is $\exp(N m^2 m_{\text{pl}}^2/2 \pi \lambda M^4)$ so we can translate this big bubble constraint into a bound on the mass scales [10]

$$\frac{m^2 m_{\text{pl}}^2}{M^4} \gtrsim \frac{\pi \lambda}{55} \ln \left( \frac{\delta_0 - \delta_{55}}{\delta_{55} - \delta_0} \right) .$$  \hspace{1cm} (10)

As the expression on the right-hand-side is likely to be $\lesssim 10^{-2}$ for $\lambda \lesssim 1$ (as long as $\delta_{\text{tr}}$ is not too close to $\delta_0$) this bound does not threaten to force us out of the extreme slow-rolling limit. If we demand that it is fluctuations in the $\phi$ field that are the near-scale-invariant seed density perturbations for large-scale structure\(^4\) then this gives us another relation between the mass scales [10] turning the big bubble constraint into a lower bound on both $m$ and $M$ individually. For values of the coupling constants of order unity this constrains $M$, say, to be greater than about $10^{13}$ GeV.

Thus even while the Hubble rate stays very nearly constant (and the perturbation spectrum is very nearly flat [10]) it is possible to ensure a sufficiently rapid growth in the percolation parameter after observable scales start leaving the horizon and bring inflation to a graceful end. Any decrease in the Hubble parameter only accelerates this change in the percolation parameter.

Models which rely solely on a decreasing Hubble rate to increase the percolation parameter tend to run into problems as this also changes the tilt of the conventional perturbation spectrum [7]. Any models based on alternative gravity theories which also change the potential shape may also be

\(^3\)This flat spacetime result is valid except in the extreme thin wall limit which turns out not to be relevant for cosmologically interesting scenarios [13].

\(^4\)We can obtain small quantum fluctuations in the slow-rolling $\phi$ field in this model without introducing small coupling parameters due to the disparate mass scales $m$ and $M$. 

viable [15, 11] or even phenomenologically interesting [16], but invoking an extended gravity theory is not necessary in order to successfully end inflation by a first order phase transition.

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References


Figure captions

*Figure 1.* $V(\psi)$ given in Eq. (1) for $\alpha = \lambda = 1$ at five different values of $\phi$ and thus different $\delta$.

*Figure 2.* $\delta_{cr}$ and $\delta_{*}$ plotted against $\log_{10}(V^{1/4}/m_{pl})$.