An updated analysis of NN elastic scattering data to 1.6 GeV

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Abstract

An energy-dependent and set of single-energy partial-wave analyses of $NN$ elastic scattering data have been completed. The fit to 1.6 GeV has been supplemented with a low-energy analysis to 400 MeV. Using the low-energy fit, we study the sensitivity of our analysis to the choice of $\pi NN$ coupling constant. We also comment on the possibility of fitting $np$ data alone. These results are compared with those found in the recent Nijmegen analyses.

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I. INTRODUCTION

This analysis of elastic nucleon-nucleon scattering data updates the content of Ref. [1]. In the intervening period, a substantial amount of new \( np \) data has been accumulated. These additions to the database are the subject of Section II. In Section III, we give the results of our analyses and compare with our previous solutions [1–3] and those produced by the Nijmegen group [4].

The Nijmegen group has continued to analyze data in the low-energy region and now [5] claims the ability to fit both the \( I=0 \) and \( I=1 \) phases using \( np \) data alone. In order to explore the low-energy region more closely, an analysis to 400 MeV (VZ40) was carried out. Using VZ40 we considered the sensitivity of our fits to the choice of pion-nucleon coupling constant, and carried out fits to the \( pp \) and \( np \) data separately. We have also studied the effect of pruning high-\( \chi^2 \) data from the database. Our finding are summarized in Section IV.

II. THE DATABASE

Our previous \( NN \) scattering analyses [1] were based on 11,880 \( pp \) and 7572 \( np \) data. In Ref. [1] the \( pp \) analysis extended up to a laboratory kinetic energy of 1.6 GeV; the \( np \) analysis was truncated at 1.1 GeV. The present database is considerably larger due both to an expanded energy range for the \( np \) system (up to 1.3 GeV) and the addition of new data. The distribution of recent (post-1991) \( pp \) and \( np \) data is given in Fig. 1. The total database has doubled over the last decade (see Table I). New \( np \) data, mainly produced by LAMPF and Saclay since 1991, have resulted in a better balance between \( pp \) and \( np \) datasets. In fact, the \( np \) database has increased by a factor of 1.3 since 1991. Unfortunately, we cannot extend our analysis of the \( I = 0 \) system up to a nucleon kinetic energy of 1.6 GeV, due to the lack of \( np \) data between 1.3 GeV and 1.6 GeV.

Since most of the new data [6–45] are from high-intensity facilities, they have added weight against the older data. Most of new \( pp \) data were produced between 500 and 800 MeV.
LAMPF, for instance, has produced differential cross sections [36], polarization variables P [31], and correlation parameters $A_{zz}$, $A_{zx}$ [19], and $A_{yy}$ [41]. Excitation measurements of P were carried out at KEK [42] for $37 \pm 2^\circ$ between 491 MeV and 1600 MeV and Saclay [15] for $43^\circ$ between 523 MeV and 708 MeV.

Most of new $np$ were either measured below 100 MeV or between 350 MeV and 1100 MeV. The sources of low energy data are TUNL, PSI, and Uppsala which gave $d\sigma/d\Omega$ [23]; P [13], [17], [24]; $A_{zz}$ [9]; $A_{yy}$ [43]; and $D_t$ [11]. LAMPF has completed a 10 year $np$ program, producing data for $d\sigma/d\Omega$ [34]; $A_t$, $A'_t$, $R_t$, $R'_t$ [10] and [20]; $A_{yy}$, $A_{xx}$, $A_{zx}$ and $A_{zz}$ [18], [35], and [45]; $D_t$ and P [33] and [31]. A detailed study of $np$ polarization quantities was carried out at Saclay, producing data for P [25], [26]; $A_{yy}$ [27]; $A_{zz}$ [28]; $A_{zx}$ [29]; $A_t$, $N_{onkk}$, $D_{on\omega}$, $R_t$, $N_{on\alpha}$, D, and $D_t$ [39]. Total $np$ cross sections in pure spin states were also measured [6], [8], [30], [32], [37], [40].

### III. PARTIAL-WAVE ANALYSIS

As mentioned in the introduction, this analysis extended to 1.6 GeV, with an $np$ component up to 1.3 GeV. The energy-dependent solution required 77 isovector and 44 isoscalar parameters. The solution (FA91) described in Ref. [1] had 123 free parameters. The present energy-dependent solution gives a $\chi^2$/datum of 22371/12838 for $pp$ data and 17516/10918 for $np$ data. A comparison with several of our previous solutions is given in Table I. In addition to the energy-dependent analysis, single-energy fits of the $pp$ and $np$ data were obtained up to 1.25 GeV. Two further analyses of $pp$ data alone were added at 1.3 GeV and 1.6 GeV. These are described in Table II, where we list the number of varied parameters in each single-energy fit and compare with the $\chi^2$ found in the energy-dependent solution. These single-energy results are plotted with uncertainties in Fig. 2.

The most significant changes to FA91 [1] were made in the parameterization of the S-waves and in the tuning of the deuteron pole parameters. The solutions FA91 and SM94 differ little in the isovector partial-waves; only the isoscalar waves are plotted in Fig. 2. Here
we have displayed both SM94 and FA91 for the purpose of comparison. Large variations are seen in the $^3D_2$ partial-wave, and at low-energies in $\epsilon_1$. In Fig. 3, some prominent partial-waves are plotted in an Argand diagram [46].

In order to ascertain that the full fit to 1.6 GeV (1.3 GeV for np) was not seriously degraded at low energies, a $0-400$ MeV fit was also developed. The resultant solution, VZ40, used 26 $I=1$ and 27 $I=0$ variable parameters to give a $\chi^2$/datum of 3098/2170(pp), and 4595/3367(np). The global fit, SM94, produced, for the same energy range, a $\chi^2$/datum of 3443/2170(pp) and 5290/3367(np). We consider this quite reasonable given that the number of variable parameters per datum is nearly twice as large for VZ40 as it is for SM94. A comparison of selected phases is given in Fig. 4. Here we have also compared with the Nijmegen analysis [4]. Note that while substantial differences are seen between the Nijmegen and SM94 results for the $^1P_1$ and $^3P_0$ phases, the VZ40 and Nijmegen results are quite consistent. The most noticeable disagreement is seen in $\epsilon_1$.

To illustrate the stability of our solution (either VZ40 or SM94) against pruning of the database, we performed the following exercise with VZ40. The dataset was first pruned by discarding all data with $\chi^2$ contributions greater than 9; this resulted in the removal of 74 data (27 pp and 47 np) with a consequent decrease in $\chi^2$ of about 1000. The solution was then searched and $\chi^2$ decreased by a mere 45. When we further pruned data giving $\chi^2$ contributions in excess of 7, 71 more points were removed with a reduction of 590 in $\chi^2$. Further searching reduced $\chi^2$ by only 14. The resultant, pruned fit gave a $\chi^2$/datum of 2397/2112(pp) and 3643/3280(np) with virtually no detectable change in the resultant phases. Our $\chi^2$ values are clearly dependent upon the existence of poorly fitted data. However, the solution itself appears quite insensitive to the removal of high-$\chi^2$ data.

In joint analyses of $pp$ and $np$ data, it is commonly assumed that the $I=1$ phases are essentially determined by the $pp$ data. If $I=1$ phases could be determined directly from the $np$ data, this would provide an interesting check on charge independence. Until recently, this was not possible. However, the Nijmegen group claims [5] to have succeeded in an analysis of the $np$ data alone, and have compared their results to those coming from analyses of the
pp data alone. We have attempted this using our VZ40 solution and find that it is indeed possible to fit the np data separately.

We first attempted to fit the pp data alone, in order to determine the effect of np data on the I=1 phases. The np data were removed and the solution adjusted to best fit the pp measurements. The $\chi^2$ for the pp data dropped from 3098 to 3083. This is what one would expect if only the pp data were significantly influencing the I=1 phases. More surprising was the effect of removing all pp data from the 0–400 MeV database. A stable solution was found with $\chi^2$ changing from 4595 to 4422 for the np data. The small decrease in $\chi^2$ suggests that charge independence is a reasonable assumption in joint analyses of np and pp data.

Sensitivity to the pion-nucleon coupling, $g^2/4\pi$ was probed by mapping $\chi^2$ versus $g^2/4\pi$ for the solution VZ40. The results are illustrated in Fig. 5 where we have plotted the changed in $\chi^2$ for pp, np and combined data. The resulting parabola for combined data shows a consistency with our chosen value (13.7), but with a rather weak sensitivity. We do not consider this to be a reliable determination of $g^2/4\pi$ because it is dependent upon the particular way in which we account for the one-pion-exchange in our representation. In Fig. 5, H-waves and higher were treated in a one-pion-exchange approximation. Purely for comparison purposes, we have included in Fig. 5 the parabola which resulted from a $\chi^2$ mapping in our pion-nucleon analysis [49] to 2.1 GeV in the pion laboratory kinetic energy. This analysis was based on more data (by a factor of 4) than were used in the VZ40 fit, but the sensitivity is clearly much greater in our pion-nucleon analysis.

IV. RESULTS AND COMPARISONS

We have incorporated a large new set of NN elastic scattering data into our analyses. This set was mainly comprised of np measurements, and these produced noticeable changes in some isoscalar partial-waves. The isovector waves remained fairly stable. At low energies, $\epsilon_1$ changed significantly from the FA91 results. Also at low energies, apart from $\epsilon_1$,
comparisons between VZ40 and the Nijmegen results show good agreement.

In other tests with VZ40, we found that our solution was quite stable to the removal of high-$\chi^2$ data. We also verified that the $np$ data could be analyzed separately. The $\chi^2$ values for separate fits of the $pp$ and $np$ data were not very different from results found in combined analyses. We also demonstrated that a value for the $\pi NN$ coupling, consistent with our $\pi N$ elastic scattering results, could be determined from VZ40. We should emphasize that this was a consistency check and not a determination of the coupling.

Some new TUNL measurements [50] of the $P$ parameter for the $np$ elastic scattering at 8 and 12 MeV will soon be available. While only a few polarization quantities have been measured at medium energies, some new PSI measurements [51] of $RT$ and $DT$ between 260 and 550 MeV and a few new Indiana data [52] of $P$ and $AYY$ at 180 MeV will soon be available.

This reaction is incorporated into the SAID program [53], which is maintained at Virginia Tech. Detailed information regarding the database, partial-wave amplitudes and observables may be obtained either interactively, through the SAID system (for those who have access to TELNET), or directly from the authors.

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† On leave from St. Petersburg Nuclear Physics Institute, Gatchina, St. Petersburg, 188350 Russia.


[44] W. Kretschmer, J. Albert, M. Clajus, P.M. Egun, A. Glombik, W. Gröbler, P. Hautle,


[46] Compare the Argand plots from $\pi d \rightarrow pp$ [47] and $\pi d$ elastic scattering [48].


[50] W. Tornow (private communication, 1994).


[53] Those with access to TELNET can run the Saida program with a link to VTINTE.PHYS.VT.EDU (128.173.176.61). The login (password) is: PHYSICS (QUANTUM). The user may view the current database and compare our solutions to those of other groups.
Figure captions

Figure 1. Energy-angle distribution of recent (post-1991) (a) $pp$ and (b) $np$ data. $pp$ data are [observable (number of data)]: $d\sigma/d\Omega$ (81), P (383), D (8), R (6), A (2), $A_{yy}$ (10), $A_{zz}$ (147), and $A_{zx}$ (64). $np$ data: $d\sigma/d\Omega$ (221), P (924), D (30), $D_t$ (128), $A_{yy}$ (389), $A_{zx}$ (159), $A_{zz}$ (415), $A_{zx}$ (425), $R_t$ (103), $R'_t$ (80), $A_t$ (142), $A'_t$ (85), $N_{nsp}$ (20), $D_{0s'0k}$ (20), $N_{nkk}$ (20), $\Delta\sigma_T$ (31), $\Delta\sigma_L$ (23), and other (5). Total cross sections are plotted at zero degrees.

Figure 2. Isoscalar partial-wave amplitudes from 0 to 1.2 GeV. Solid (dashed) curves give the real (imaginary) parts of amplitudes corresponding to the SM94 solution. The real (imaginary) parts of single-energy solutions are plotted as filled (open) circles. The previous FA91 solution [1] is plotted with long dash-dotted (real part) and short dash-dotted (imaginary part) lines. The dotted curve gives the value of $\text{Im } T - T^2 - T_{sf}^2$, where $T_{sf}^2$ is the spin-flip amplitude. All amplitudes have been multiplied by a factor of $10^3$ and are dimensionless.

Figure 3. Argand plot of the $NN$ partial-wave amplitudes $^1D_2$, $^3P_2$, $^3F_3$, and $^1G_4$. (Compare Figures 7 of references [47] and [48]). The “X” points denote 100 MeV steps. All amplitudes have been multiplied by a factor of $10^3$ and are dimensionless.

Figure 4. Phase-shift parameters from 0 to 400 MeV. The SM94 and VZ40 solutions are plotted as solid and dash-dotted curves, respectively. Single-energy solutions are given by filled circles. A recent solution from the Nijmegen group [4] is plotted as a dashed curve.

Figure 5. A plot of $\chi^2$ versus $g^2/4\pi$. $\chi^2$ values are plotted as deviations from the $pp$ and $np$ minima. The open squares (triangles) give the VZ40 results of $pp$ ($np$) data. The black circles give the result of a combined fit to both $pp$ and $np$ data.
Solid lines drawn are to guide the eye. For the purpose of comparison, a $\chi^2$ map for the recent FA93 $\pi\N$ solution [49] has been added as a dashed curve.
Table I. Comparison of present (SM94, VZ40) and previous (FA91, SM86, and SP82) energy-dependent partial-wave analyses. The $\chi^2$ values for the previous FA91, SM86, and SP82 solutions correspond to the published results([1]-[3]).

<table>
<thead>
<tr>
<th>Solution</th>
<th>Range (MeV)</th>
<th>$\chi^2/pp$ data</th>
<th>Range (MeV)</th>
<th>$\chi^2/np$ data</th>
<th>Ref.</th>
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<td></td>
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<td>(0 − 400)</td>
<td>5290/3367</td>
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<td>0 − 400</td>
<td>4595/3367</td>
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<td>8871/5474</td>
<td>[2]</td>
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<tr>
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<td>9199/5207</td>
<td>0 − 1100</td>
<td>9103/5283</td>
<td>[3]</td>
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Table II. Single-energy (binned) fits of \(pp\) data (Pxxx) and combined \(pp\) and \(np\) data (Cxxx), and \(\chi^2\) values. \(\chi^2_E\) is given by the energy-dependent fit, SM94, and \(N_{\text{prm}}\) is the number parameters varied in the fit.

<table>
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