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Per Carlqvist

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**Division of Plasma Physics
Alfvén Laboratory**
Royal Institute of Technology
S-100 44 Stockholm, Sweden

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Division of Plasma Physics
Alfvén Laboratory
Royal Institute of Technology
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Per Carlqvist

Division of Plasma Physics, Alfvén Laboratory,
Royal Institute of Technology, S-100 44 Stockholm, Sweden

Abstract

Multicomponent double layers, defined as layers composed of more than two kinds of charged particles, are supposed to constitute the predominant type of double layer in cosmic plasmas. A model of a steady and strong multicomponent double layer is studied in both the non-relativistic and relativistic approximations. In particular such properties of the layer as the structure, potential drop, and current composition are investigated.

It is demonstrated that the density distribution of each kind of the positive and negative particles in the non-relativistic multicomponent layer is of the same shape as the density distribution of the ions and electrons, respectively, in Langmuir's two-component layer. Also the shape of the distribution of the electric field corresponds to that of the two-component layer. In the relativistic layer the charges are distributed among two very thin layers of high positive and negative charge density close to the anode and cathode boundaries of the layer, respectively, and a constant but low charge density in the rest of the layer.

It is shown that the potential drop across the multicomponent layer is proportional to the thickness of the layer raised to $4/3$ in the non-relativistic case while it is directly proportional to the thickness in the relativistic case.

Current conditions prescribing the allowed sets of current densities of the various kinds of particles in the layer are derived in the non-relativistic and relativistic approximations. Correspondingly abundance conditions for the particles accelerated through the layer are also obtained. The abundance of the particles accelerated by the double layer is expected generally to be different from the abundance of the ambient plasma. It is suggested that the abundance of the accelerated particles might serve as a means to detect double layers, especially in space plasmas.

1. Introduction

For many years electrostatic double layers have been thoroughly studied in laboratory plasmas of various kinds [see eg. reviews by *Torvén*, 1979, 1993; *Sato*, 1982; *Hershkowitz*, 1985; *Raadu*, 1989]. The experiments show that double layers can exist under fairly different conditions of the plasma. On the basis of the experimental results the idea that double layers might exist also in cosmic plasmas was early raised [*Alfvén*, 1958; *Alfvén and Carlqvist*, 1967; *Block*, 1969]. The laboratory plasmas do not in general differ very much from many cosmic plasmas as regards density and temperature. Hence, the conditions for formation of cosmic double layers seem to be favourable.

There is, however, one prominent difference between the cosmic plasmas and most of the laboratory plasmas devoted to double layer studies. The laboratory plasmas, on the one hand, have mostly consisted of only one kind of ionized gas (at least if unintentional impurities are disregarded). An important reason for this limitation is that the double layer represents a very complex phenomenon which one has intended to study under as simple and pure conditions as possible. Cosmic plasmas, on the other hand, are composed of a great number of chemical elements being more or less ionized. Generally one speaks about the cosmic abundance of elements although the abundance may vary substantially from one site to another. Double layers that are formed in cosmic plasmas have therefore to be of the *multicomponent* type implying that they consist of more than two kinds of charged particles.

Many of the properties of the multicomponent double layer are expected to depend on the abundance of the charged particles in the layer. Recently some aspects of the non-relativistic multicomponent layer have been considered [*Carlqvist*, 1992]. In the present paper we shall study in some more detail the properties of a few simple types of multicomponent double layers. Both non-relativistic and relativistic layers will be treated. Moreover, we shall discuss the important question concerning the abundance of the charged particles accelerated by the double layer. The possibility to detect double layers by means of the particles they accelerate will also be discussed.

2. Model of the multicomponent double layer

For our study of the multicomponent double layer we shall consider a model of the layer that is one-dimensional, steady, and strong. The layer, which sustains a potential drop V_{DL} , ranges from the anode boundary A to the cathode boundary C and has the thickness d . The potential is defined to be $V=0$ at C and $V=V_{DL}$ at A. We introduce an x -axis perpendicular to the boundaries of the layer directed from A towards C. Outside the layer there is plasma. Some of the charged plasma particles are accelerated through the potential drop of the layer while others are reflected back into the plasma again. Since the layer is assumed to be strong the particles enter the layer with kinetic energies that are small compared with the energy associated with the potential drop across the layer. This means that the reflected particles can be left out of account. The layer is thus built up by positive and negative particles of various kinds which enter the layer with negligible velocity at A and C, respectively. Inside the layer these positive and negative particles are accelerated in opposite directions by an electric field E . No collisions or other losses are supposed to take place in the

layer. The electric field is consistently generated by the charged particles present in the layer. Since the layer borders on plasma at A and C we prescribe the electric field to be zero there. This implies that the layer is electrically neutral as a whole.

3. The current condition

The double layer considered is built up by several different kinds of positive particles (index $p\mu$ where $\mu=1, 2, 3, \dots$) and negative particles (index nv where $v=1, 2, 3, \dots$) with rest masses $m_{p\mu}$ and m_{nv} and charges $q_{p\mu}$ and q_{nv} . From the conservation of energy we obtain the velocities of the particles

$$u_{p\mu} = \frac{c(V_1^2 + 2V_{p\mu}V_1)^{1/2}}{V_{p\mu} + V_1} \quad (1)$$

and

$$u_{nv} = - \frac{c(V^2 + 2V_{nv}V)^{1/2}}{V_{nv} + V} \quad (2)$$

where $V_{p\mu} = m_{p\mu}c^2/q_{p\mu}$ and $V_{nv} = -m_{nv}c^2/q_{nv}$ are the volt-equivalents of the rest masses of the particles and $V_I = V_{DL} - V$. The current densities carried by the positive and negative particles are

$$i_{p\mu} = n_{p\mu} q_{p\mu} u_{p\mu} \quad (3)$$

and

$$i_{nv} = n_{nv} q_{nv} u_{nv} \quad (4)$$

respectively, where $n_{p\mu}$ and n_{nv} denote the particle densities. Poisson's equation gives

$$\frac{d^2V}{dx^2} = - \frac{1}{\epsilon_0} \left(\sum_{\mu} q_{p\mu} n_{p\mu} + \sum_v q_{nv} n_{nv} \right) \quad (5)$$

Inserting equations (1) to (4) into (5) we obtain

$$\frac{d^2V}{dx^2} = - \frac{1}{\epsilon_0 c} \left[\sum_{\mu} \frac{i_{p\mu} (V_{p\mu} + V_1)}{(V_1^2 + 2V_{p\mu}V_1)^{1/2}} - \sum_v \frac{i_{nv} (V_{nv} + V)}{(V^2 + 2V_{nv}V)^{1/2}} \right] \quad (6)$$

After multiplying by $2 dV/dx = -2 dV_I/dx$ and integrating we get

$$\left(\frac{dV}{dx}\right)^2 = \frac{2}{\epsilon_0 c} \left[\sum_{\mu} i_{p\mu} (V_1^2 + 2V_{p\mu}V_1)^{1/2} + \sum_{\nu} i_{n\nu} (V^2 + 2V_{n\nu}V)^{1/2} \right] - C_1 \quad (7)$$

where C_1 is a constant. Remembering that the electric field $-dV/dx$ is zero at the boundaries of the layer where $V = 0$ and $V = V_{DL}$ we find from (7)

$$C_1 = \frac{2}{\epsilon_0 c} \sum_{\mu} i_{p\mu} (V_{DL}^2 + 2V_{p\mu}V_{DL})^{1/2} = \frac{2}{\epsilon_0 c} \sum_{\nu} i_{n\nu} (V_{DL}^2 + 2V_{n\nu}V_{DL})^{1/2} \quad (8)$$

yielding

$$\sum_{\mu} i_{p\mu} (V_{DL}^2 + 2V_{p\mu}V_{DL})^{1/2} = \sum_{\nu} i_{n\nu} (V_{DL}^2 + 2V_{n\nu}V_{DL})^{1/2} \quad (9)$$

This equation constitutes the *general current condition* prescribing the infinite number of sets of current densities, $i_{p\mu}$ and $i_{n\nu}$, that are permitted for the multicomponent double layer.

Non-relativistic case: In the non-relativistic case, $V_{DL} \ll V_{p\mu}$ and $V_{DL} \ll V_{n\nu}$, (9) may be written as

$$\sum_{\mu} i_{p\mu} V_{p\mu}^{1/2} = \sum_{\nu} i_{n\nu} V_{n\nu}^{1/2} \quad (10)$$

which represents the *current condition for the non-relativistic multicomponent double layer*.

For a two-component double layer consisting of only electrons and singly charged positive ions (10) is reduced to the well-known Langmuir condition [Langmuir, 1929]

$$\frac{i_{p1}}{i_{n1}} = \frac{i_i}{i_e} = \left(\frac{m_e}{m_i}\right)^{1/2} \quad (11)$$

which may thus be considered a special case of (10).

The current condition (10) is illustrated in Figure 1 by two examples. The first example deals with a three-component double layer composed of e^- , H^+ , and O^+ which may be of interest in ionospheric and magnetospheric physics. The second one concerns a four-component layer consisting of electrons, one kind of negative ions, and two kinds of positive ions.

Relativistic case: In the relativistic case, $V_{DL} \gg V_{p\mu}$ and $V_{DL} \gg V_{n\nu}$, where all the charged particles in the double layer are accelerated to relativistic energies the general current condition (9) is reduced to

$$\sum_{\mu} i_{p\mu} = \sum_{\nu} i_{n\nu} \quad (12)$$

in the first approximation and to

$$\sum_{\mu} i_{p\mu} (V_{DL} + V_{p\mu}) = \sum_{\nu} i_{n\nu} (V_{DL} + V_{n\nu}) \quad (13)$$

in the second approximation. Equation (12) represents the *current condition for the relativistic multicomponent double layer implying that all the positive particles carry a current that is equal to the current carried by all the negative particles*.

4. The potential drop

It is of some interest to study how the potential drop across the multicomponent double layer depends on various parameters such as thickness, electric current, and composition of the layer. Combining (7) and (8) we obtain an equation for the square of the electric field

$$\left(\frac{dV}{dx}\right)^2 = \frac{2}{\epsilon_0 c} \left\{ \sum_{\mu} i_{p\mu} (V_1^2 + 2V_{p\mu}V_1)^{1/2} + \sum_{\nu} i_{n\nu} [(V^2 + 2V_{n\nu}V)^{1/2} - (V_{DL}^2 + 2V_{n\nu}V_{DL})^{1/2}] \right\} \quad (14)$$

Non-relativistic case: In the non-relativistic limit (14) can be approximated by

$$\left(\frac{dV}{dx}\right)^2 = \frac{2^{3/2}}{\epsilon_0 c} \left[V_1^{1/2} \sum_{\mu} i_{p\mu} V_{p\mu}^{1/2} + (V^{1/2} - V_{DL}^{1/2}) \sum_{\nu} i_{n\nu} V_{n\nu}^{1/2} \right] \quad (15)$$

For a two-component double layer with $\mu=1$ and $\nu=1$ this expression is reduced to

$$\left(\frac{dV}{dx}\right)^2 = \frac{2^{3/2}}{\epsilon_0 c} [i_{p1} V_{p1}^{1/2} V_1^{1/2} + i_{n1} V_{n1}^{1/2} (V^{1/2} - V_{DL}^{1/2})] \quad (16)$$

With $m_{p1} = m_i$, $m_{n1} = m_e$, and $q_{p1} = -q_{n1} = e$ we find that this is the same expression as the one which *Langmuir* [1929] derived for the two-component layer consisting of electrons and singly charged positive ions. For this kind of layer Langmuir obtained the following expression for the electron current density

$$i_e = \frac{2^{5/2} \epsilon_0 C_2}{9} \left(\frac{e}{m_e}\right)^{1/2} \frac{V_{DL}^{3/2}}{d^2} \quad (17)$$

Here C_2 is a constant having the numerically calculated value of 1.865 [*Raadu*, 1982]).

Taking (10) into consideration we find that (15) is of the same form as (16). Consequently we can obtain a solution of (15) by replacing i_e by

$$\frac{1}{V_e^{1/2}} \sum_{\nu} i_{n\nu} V_{n\nu}^{1/2} = \left(\frac{e}{m_e}\right)^{1/2} \sum_{\nu} \frac{m_{n\nu}^{1/2} i_{n\nu}}{(-q_{n\nu})^{1/2}} \quad (18)$$

in (17). Hence, we get the general expression for the potential drop across the non-relativistic multi-

component double layer

$$V_{DL} = \left[\frac{9d^2}{2^{5/2} \epsilon_0 C_2} \sum_v \frac{m_{nv}^{1/2} i_{nv}}{(-q_{nv})^{1/2}} \right]^{2/3} \quad (19)$$

By means of the current condition (10) and the volt-equivalents of the rest masses the potential drop may also be expressed as

$$V_{DL} = \left\{ \frac{9d^2}{2^{5/2} \epsilon_0 C_2} \left[\alpha \sum_{\mu} i_{p\mu} V_{p\mu}^{1/2} + (1-\alpha) \sum_v i_{nv} V_{nv}^{1/2} \right] \right\}^{2/3} \quad (20)$$

where α is an arbitrary number. Putting $\alpha = 1$ we realize that *the potential drop across the layer is completely determined as soon as the current densities of the positive particles $i_{p\mu}$ are given*, independently of the specific values of the current densities of the negative particles. Similarly it is clear from (20) with $\alpha = 0$, as well as for symmetry reasons, that the potential drop is also fully determined by the negative current components i_{nv} only.

Relativistic case: In the relativistic limit we may approximate (14) by

$$\left(\frac{dV}{dx} \right)^2 = \frac{2}{\epsilon_0 c} \left[(V_{DL} - V) \left(\sum_{\mu} i_{p\mu} - \sum_v i_{nv} \right) + \sum_{\mu} i_{p\mu} V_{p\mu} \right] \quad (21)$$

This equation is valid in the whole of the layer but for two thin regions next to the boundaries A and C (see Section 5). When calculating the potential drop across the layer in the relativistic limit we can neglect the influence of these two regions. From the square root of (21), where we choose the minus sign, we find

$$\left(\frac{2}{\epsilon_0 c} \right)^{1/2} \int_0^d dx = - \int_{V_{DL}}^0 \left[\sum_{\mu} i_{p\mu} V_{p\mu} + V_{DL} \left(\sum_{\mu} i_{p\mu} - \sum_v i_{nv} \right) - V \left(\sum_{\mu} i_{p\mu} - \sum_v i_{nv} \right) \right]^{1/2} dV \quad (22)$$

Using the current condition (13) and integrating we obtain the potential drop across the relativistic multicomponent layer

$$V_{DL} = \frac{d}{(2\epsilon_0 c)^{1/2}} \left[\left(\sum_{\mu} i_{p\mu} V_{p\mu} \right)^{1/2} + \left(\sum_v i_{nv} V_{nv} \right)^{1/2} \right] \quad (23)$$

It is found that the potential drop depends on the current densities and masses of the various charged particles. Furthermore, it should be noticed that *the potential drop is directly proportional to the thickness of the layer*. In this respect the relativistic double layer appears to resemble a plane plate condenser. A further discussion on this matter is given in Section 5.

5. The structure of the layer

Non-relativistic case: As pointed out above (15) is of the same form as (16). This means that the distributions of the electric field and of the potential in the non-relativistic multicomponent layer both are of the same shape as the corresponding distributions in Langmuir's two-component layer. From this fact and from (1) and (3) we realize that the density distribution of each kind of the positive particles in the multicomponent layer is of the same shape as the density distribution of the ions in Langmuir's layer. For the same reasons the density distributions of the negative particles in the multicomponent layer are of the same shape as the density distributions of the electrons in Langmuir's layer. In Figure 2a the distributions of the electric field and of the densities of the positive and negative particles in a non-relativistic four-component layer are illustrated. The thick solid and dashed curves show the distributions of the total positive and negative particle densities, respectively. It is to be noticed that these latter two curves are symmetric to each other with respect to the centre of the layer. This is consistent with the fact that charge neutrality is prevailing in the layer as a whole.

Relativistic case: In the relativistic limit the potential distribution of the layer may be found from an equation similar to (22) but now with the limits of integration equal to $x = 0$, $V = V_{DL}$ and $x = x$, $V = V$. The result obtained is

$$V = \frac{\sum_{\mu} i_{p\mu} V_{p\mu}}{\sum_{\mu} i_{p\mu} - \sum_{\nu} i_{n\nu}} + V_{DL} - \left[\left(\frac{\sum_{\mu} i_{p\mu} V_{p\mu}}{\sum_{\mu} i_{p\mu} - \sum_{\nu} i_{n\nu}} \right)^{1/2} + \left(\frac{\sum_{\mu} i_{p\mu} - \sum_{\nu} i_{n\nu}}{2\epsilon_0 c} \right)^{1/2} x \right]^2 \quad (24)$$

Differentiating (24) we find the electric field

$$E = \left(\frac{2}{\epsilon_0 c} \right)^{1/2} \left[\left(\sum_{\mu} i_{p\mu} V_{p\mu} \right)^{1/2} + \frac{\sum_{\mu} i_{p\mu} - \sum_{\nu} i_{n\nu}}{(2\epsilon_0 c)^{1/2}} x \right] \quad (25)$$

This field is composed of a constant field component E_1 and a field component E_2 which depends linearly on x . The E_1 -component may be interpreted as being generated by a positive surface charge density σ_1 at the anode boundary A of the layer and an equally large, but negative, surface charge density $-\sigma_1$ at the cathode boundary C where

$$\sigma_1 = \left(\frac{2\epsilon_0}{c} \right)^{1/2} \left(\sum_{\mu} i_{p\mu} V_{p\mu} \right)^{1/2} \quad (26)$$

It should be noticed, however, that the surface charge density $-\sigma_1$ does not necessarily represent the complete surface charge density at C.

The second field component E_2 in (25) is consistent with a constant space charge density ρ_2 within the layer and a surface charge density $-\sigma_2$ at C. From Poisson's equation $d^2V/dx^2 = -dE_2/dx = -\rho_2/\epsilon_0$ and the E_2 -component as described by (25) we obtain

$$\rho_2 = \frac{1}{c} \left(\sum_{\mu} i_{p\mu} - \sum_{\nu} i_{n\nu} \right) \quad (27)$$

Using the second approximation of the current condition (13) and (23) we find from (27) the space charge density

$$\rho_2 = \left(\frac{2\epsilon_0}{cd^2} \right)^{1/2} \left[\left(\sum_{\nu} i_{n\nu} V_{n\nu} \right)^{1/2} - \left(\sum_{\mu} i_{p\mu} V_{p\mu} \right)^{1/2} \right] \quad (28)$$

It is clear that ρ_2 may be positive, negative, or zero. The charge due to ρ_2 in a column of unit area crossing the double layer is

$$\sigma_2 = \left(\frac{2\epsilon_0}{c} \right)^{1/2} \left[\left(\sum_{\nu} i_{n\nu} V_{n\nu} \right)^{1/2} - \left(\sum_{\mu} i_{p\mu} V_{p\mu} \right)^{1/2} \right] \quad (29)$$

We denote the true surface charge density at C by σ_3 . Charge neutrality of the double layer as a whole requires that $\sigma_1 + \sigma_2 + \sigma_3 = 0$. From this expression and (26) and (29) we obtain

$$\sigma_3 = - \left(\frac{2\epsilon_0}{c} \right)^{1/2} \left(\sum_{\nu} i_{n\nu} V_{n\nu} \right)^{1/2} \quad (30)$$

In the case

$$\sum_{\mu} i_{p\mu} V_{p\mu} < \sum_{\nu} i_{n\nu} V_{n\nu} \quad (31)$$

the space charge density ρ_2 is positive as well as E_2 . In a situation opposite to condition (31) both ρ_2 and E_2 are negative. If instead

$$\sum_{\mu} i_{p\mu} V_{p\mu} = \sum_{\nu} i_{n\nu} V_{n\nu} \quad (32)$$

ρ_2 and σ_2 are equal to zero and hence also E_2 . The electric field $E = E_1$ is then constant in the main part of the layer. In this case the distributions of charge and electric field strongly resemble those of a plane plate condenser.

Above we have considered σ_1 and σ_3 as surface charge densities at A and C, respectively. This description of course constitutes a simplification of reality since no pure surface charge can exist in

free plasmas or space charges. Instead the charges σ_1 and σ_3 are found in layers of finite thickness just outside A and C. After passing A the positive particles are quickly accelerated to velocities close to c . As a result of this acceleration the densities of the various species of positive particles first rapidly decrease just outside A whereupon they asymptotically approach the constant levels $i_{p\mu}/cq_{p\mu}$ in the main part of the layer (Figure 2b). The distributions of the velocities and densities of the negative species of particles have a corresponding but reversed appearance. Hence, there is a negative spike of space charge next to C just as there is a positive spike next to A. These spikes may be identified with the surface charges mentioned above.

We can estimate the thickness of the layer just outside A as follows. Within the layer the electric field grows from zero to E_I . A conservative estimate of the mean field in the layer yields $\bar{E} \approx E_I/2$. Hence, the distance required to accelerate the positive particles having maximum $V_{p\mu}$ to a potential $V_I = V_{p\mu \max}$, corresponding to moderately relativistic velocities of these particles, is

$$x_1 \approx (2\epsilon_0 c)^{1/2} \frac{V_{p\mu \max}}{\left(\sum_{\mu} i_{p\mu} V_{p\mu} \right)^{1/2}} \quad (33)$$

which constitutes a measure of the thickness of the anode layer. Using (23) we obtain the relative thickness of the anode layer

$$\frac{x_1}{d} \approx \frac{\left(\sum_{\mu} i_{p\mu} V_{p\mu} \right)^{1/2} + \left(\sum_{\nu} i_{n\nu} V_{n\nu} \right)^{1/2}}{\left(\sum_{\mu} i_{p\mu} V_{p\mu} \right)^{1/2}} \frac{V_{p\mu \max}}{V_{DL}} \quad (34)$$

From (34) we see that for strongly relativistic double layers the relative thickness of the anode layer tends to be very small. We could also interpret this as a consequence of the fact that if we increase the potential drop of the double layer while keeping the current constant, x_1 will remain constant whereas d must increase in proportion to V_{DL} as described by (23).

The relative thickness of the cathode layer may be found in a similar way as (34). For symmetry reasons we can, however, find an expression of this thickness directly from (34) by interchanging $p\mu$ and $n\nu$.

In most of the double layer volume, where all the charged particles move with velocities close to c , the total space charge density of the positive particles is generally very nearly equal to the absolute value of the total space charge density of the negative particles. When the inequality (31) or its reverse is valid there is a small difference between the densities resulting in the constant space charge density ρ_2 . In the special case where the equality (32) holds the densities are exactly equal so that ρ_2 is zero and the electric field is constant.

6. Selective Acceleration and Signature of Multicomponent Double Layers

When a multicomponent double layer appears in a plasma many different kinds of particles are accelerated to energies that correspond to the potential drop of the layer. An important question to answer in this connection is what the abundance of the accelerated particles will be. It is, for instance, by no means certain that the abundance will be the same as the abundance of the ambient current carrying plasma. In the case the double layer occurs in a partly ionized plasma it is obvious that the accelerated particles will consist of only those particles in the plasma that are ionized. Here the two abundances will clearly differ from one another. But also when we are dealing with double layers occurring in fully ionized plasmas the abundances may differ. If the abundance of the accelerated particles is to be exactly the same as the abundance of the ambient plasma it is required that all the different kinds of particles in the plasma must drift towards the double layer with the same systematic speed. This is a very unlikely situation. Hence, the multicomponent double layer must in general give rise to a *selective acceleration* of charged particles.

Since the abundance of the accelerated particles differs from the abundance of the ambient plasma it is in a way unique. The abundance of the accelerated particles may therefore be considered a *signature* of the double layer. Such a signature may be employed for detecting double layers provided beam-plasma interaction is not too strong outside the layer so that it inhibits the release of accelerated particles from the double layer region. In the following we shall discuss a few possible methods for detecting double layers along these lines of thought.

It might seem that the most straightforward method to judge whether an observed population of energetic particles has been accelerated by a double layer or not would be to calculate the abundance of particles accelerated through the double layer and then compare the result with the abundance of the observed population. Basic input parameters here are the abundance of the ambient plasma and the potential drop of the double layer. Unfortunately, it is not an easy task to perform the calculations. In the first place the abundance of the ambient plasma may be difficult to determine unless it cannot be measured directly. The reason for this is that it may differ considerably from the abundance of the plasma far away from the layer owing to drift motions combined with chemical separation [Marklund, 1979]. Secondly, the presence of even a weak beam-plasma interaction outside the layer may make the flows of the various species of charged particles through the layer difficult to predict.

A more simple method for judging whether an observed set of energetic particles has been accelerated by a double layer or not is offered by the current conditions derived in Section 3. We may here distinguish between two different situations:

- 1) The region of acceleration is so conveniently situated that measurements of the abundance of the accelerated particles can be simultaneously performed on both sides of it. An example of such a site may be the magnetosphere of the Earth. The flux densities of the various kinds of particles can here be measured directly and checked against the relevant current condition, (10) or (12). If the measured flux densities satisfies (10) or (12) this may be considered a strong evidence for a double layer having accelerated the particles.

- 2) The region of acceleration is situated far away from the place where the energetic particles are detected. In this case the energetic particles must have been scattered after acceleration and transported to the site of measurements. This situation may preferably prevail if the acceleration takes

place in free space and at a large distance from absorbing objects. It is here a matter of measuring the abundance of a small sample of the total amount of the accelerated particles rather than the abundance of the fluxes. In order to illustrate this method we first consider a non-relativistic double layer which exists for a time τ_l and which is furthermore steady during this time. Multiplying the current condition (10) by τ_l and by the effective area of cross-section of the double layer A_e we obtain in the non-relativistic limit

$$\sum_{\mu} Q_{p\mu} V_{p\mu}^{1/2} = \sum_{\nu} Q_{n\nu} V_{n\nu}^{1/2} \quad (35)$$

where $Q_{p\mu} = \tau_l A_e i_{p\mu}$ and $Q_{n\nu} = \tau_l A_e i_{n\nu}$ denote the amounts of charges of the kinds $p\mu$ and $n\nu$, respectively, which have been accelerated by the layer during its lifetime. Equation (35) represents an *abundance condition* for the particles accelerated by the double layer. When appropriate, losses and other transport effects of course have to be taken into account.

In the relativistic limit we correspondingly obtain from (12) the abundance condition

$$\sum_{\mu} Q_{p\mu} = \sum_{\nu} Q_{n\nu} \quad (36)$$

Equal amounts of positive and negative charges are here accelerated by the double layer.

The conditions (35) and (36) are derived for steady double layers. Such layers may occur in the laboratory if a constant voltage source is applied to the plasma tube where the double layer is studied [e.g. *Sato et al.*, 1981] or in space if mass motions drive a dynamo region coupled to the double layer [*Raadu*, 1993]. In laboratory plasmas as well as in cosmic plasmas potential drops are also often maintained in an inductive way [*Torvén et al.*, 1985; *Carpenter and Torvén*, 1987; *Carlqvist*, 1968; *Alfvén*, 1981]. The presence of an inductive voltage source implies that the electric current must necessarily vary with time. Hence, a strictly steady double layer cannot exist in this case. However, if the transit times of the charged particles through the layer are all much shorter than the time constant for change of the layer, the layer may in many respects be considered quasi-steady. Among other things the equations describing the current condition, the potential drop, and the structure of the layer are then still valid with good precision. In contrast to this the abundance conditions (35) and (36) have to be reconsidered since the current may vary considerably during the lifetime of the layer. We shall here consider the abundance conditions in two simple cases assuming that the current through the layer varies with the time-constant τ_c and that the layer is quasi-steady.

In the first case where $\tau_c \gg \tau_l$ it is immediately clear that the abundance conditions (35) and (36) are still valid.

In the second case where $\tau_c \lesssim \tau_l$ the situation is more complex and we have to investigate the conditions more carefully. Both the current densities and the effective area of cross-section may here vary with time. We first approximate the time-development of the components of the current

densities by step functions having the levels $i_{p\mu\xi}$ and $i_{nv\xi}$. Similarly the effective area of cross-section is approximated by the levels $A_{e\xi}$. The relative change of $i_{p\mu\xi}$, $i_{nv\xi}$, and $A_{e\xi}$ from one level to the next is supposed to be small. The duration of each level is τ_ξ so that the sum of all τ_ξ equals τ_l . During the lifetime of the double layer the relative magnitudes of the current density components may vary, but at each level any of the current conditions (10) and (12) must be valid in the non-relativistic and relativistic limits, respectively. We now multiply the conditions (10) and (12) by $\tau_\xi A_{e\xi}$ and sum up over ξ . In the limit of infinitesimally small steps we find that *the abundance conditions (35) and (36) are valid also in the time-dependent case but now with the amounts of accelerated charges given by*

$$Q_{p\mu} = \int_0^{\tau_l} A_e i_{p\mu} dt \quad (37)$$

and

$$Q_{nv} = \int_0^{\tau_l} A_e i_{nv} dt \quad (38)$$

7. Conclusions

Since long it has been known that cosmic plasmas are composed of a great number of different chemical elements. Double layers appearing in such plasmas should therefore be of the multicomponent type. In this paper we have studied a simple model of the multicomponent layer in both the non-relativistic and relativistic limits. In particular we have investigated properties of the layer such as the structure, the potential drop, and the composition of the current densities. Moreover, the abundance of the particles accelerated by double layers has been considered.

1. The structure of the non-relativistic double layer is ruled by the interaction between charged particles and electric field. In the model studied the densities of all the positive particles decrease monotonically towards the cathode boundary while the densities of all the negative particles vary correspondingly in the opposite direction. The density distribution of each kind of the positive and negative particles is in fact of the same shape as the density distributions of the ions and electrons, respectively, in Langmuir's two-component layer. Also the distribution of the electric field is of the same shape as that in the two-component layer.

In the relativistic double layer there are charges in two thin layers — one layer with positive charges just outside the anode boundary and one layer with negative charges just outside the cathode boundary. In addition to this there is usually also a small but constant space charge

distributed all over the layer. The electric field consistent with these charges may be divided into two components. First there exists a field component the absolute value of which is proportional to the x -coordinate. Secondly there is a constant field component. The magnitudes of the space charges and electric field components depend on the current density and how it is distributed among the various kinds of particles in the layer.

2. The potential drop across the non-relativistic double layer is proportional to the thickness of the layer raised to $4/3$. It depends on the current densities carried by the various kinds of particles in the layer. Particles with a great mass to charge ratio contribute more to the potential drop than particles with a small mass to charge ratio counting per unit current density. Hence, for instance, charged dust particles in dusty plasmas may influence the potential drop substantially although the density of the particles may be relatively low. It has been shown that, given the thickness of the layer, the potential drop of the layer can unambiguously be determined by means of only the current densities of the positive particles or only the current densities of the negative particles.

In the relativistic double layer the potential drop is directly proportional to the thickness. To settle the potential drop here we need, in contrast to the non-relativistic case, information about the current densities of all the different species of particles in the layer.

3. The composition of the current densities is influenced by the momentum balance of the double layer. The momentum balance requires the currents carried by the various kinds of particles to fulfill a certain condition. Such a current condition has been derived for as well the non-relativistic as the relativistic multicomponent layer. In the non-relativistic limit the current condition involves both the current densities and the different masses of the particles present in the layer.

In the relativistic limit the current condition is much simpler and only demands the total current carried by the positive particles to be equal to the total current carried by the negative particles.

It is of some interest to see how the double layer can be adapted to the current condition. If we, for a moment, suppose that the multicomponent double layer is an isolated unit which is not surrounded by plasma we may prescribe any combination of the current densities provided it satisfies the current condition. We then only have to manage it so that the prescribed flows of particles are injected at the boundaries of the layer. However, if the layer is surrounded by plasma, as is generally the case, the situation is more complex. The total current density in the layer must then be adapted to the total current density in the plasma. As before the current condition must also be fulfilled in the layer. For the current condition to be satisfied the boundaries of the layer have to move relative to the plasma in a suitable way [Carlqvist, 1984]. In the case the boundaries move with the same velocity the thickness of the layer remains unchanged. If instead the boundaries move with different velocities the thickness must vary with time.

4. By analogy with the current condition an abundance condition has also been formulated which describes how the amounts of various kinds of charges accelerated by a double layer during a certain time must be related to one another.

5. A prominent property of the double layer is its ability to accelerate charged particles. However, in order for the layer to exist, and hence for the acceleration to take place, the potential drop across the layer has to be maintained by outer means. In the laboratory this may be managed by connecting either a sufficiently large voltage source or an inductance to the plasma tube where the double layer is to be studied [Torvén, 1982; Torvén et al., 1985]. In cosmic plasmas the potential drop is

supposed mainly to be due to induced electric fields and dynamo action of moving plasma [Alfvén, 1981].

6. The abundance of elements among the particles accelerated in double layers is in general expected to be different from the abundance of elements in the ambient plasma. Hence, it is clear that the acceleration of charged particles in double layers represents a selective process. The abundance of the accelerated particles is therefore in a way unique and may be considered a signature of the double layer.

7. It is well-known that double layers are notoriously difficult to detect, especially in space plasmas. It is therefore an important task to find good and reliable methods for detecting double layers. We suggest that the specific abundance of the accelerated particles might be used as a means to detect double layers provided beam-plasma interaction is not too strong.

Two methods for detecting double layers in this way have been proposed. One of these consist in determining whether the abundance of elements in an observed population of energetic particles satisfies the abundance condition for double layers or not. The other method is founded on a detailed comparison of observed abundance and calculated abundance. This method should lead to a more unambiguous identification as compared to the former one but is, no doubt, also more difficult to accomplish. More theoretical work and laboratory experiments are needed to refine these methods.

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Figure Captions

- Fig. 1 Grafic representation of the current condition for a non-relativistic double layer containing a) three components and b) four components. The three-component double layer is composed of e^- , H^+ , and O^+ while the four-component layer is composed of e^- , O_2^- , NO^+ , and O_2^+ . The current densities are normalized to the total current density. In a) the electron current density is found in the interval $0.9772 - 0.9942$ while the total ion current density is found in the interval $5.822 \cdot 10^{-3} - 2.280 \cdot 10^{-2}$. In b) the permitted sets of current densities are represented by the flat and truncated (outermost right-hand corner) triangular surface.
- Fig. 2 Examples of distributions of the particle densities and electric field in a four-component double layer that is a) non-relativistic and b) relativistic. The layer is composed of two species of singly charged positive particles and two species of singly charged negative particles the densities of which are shown by the full and dashed thin curves, respectively. The full thick curve shows the total density of the positive particles while the dashed thick curve shows the total density of the negative particles. The electric field is illustrated by the dashed-dotted curve. All the scales of the particle densities and electric field are linear and arbitrary.

a.

b.

Fig. 1