Measurement of the mass difference between neutral charm-meson eigenstates

LHCb collaboration†

Abstract

We report a measurement of the mass difference between neutral charm-meson eigenstates using a novel approach that enhances sensitivity to this parameter. We use $2.3 \times 10^6 \, D^0 \to K^{0}_{S} \pi^+ \pi^-$ decays reconstructed in proton-proton collisions collected by the LHCb experiment in 2011 and 2012. Allowing for $CP$ violation in mixing and in the interference between mixing and decay, we measure the $CP$-averaged normalized mass difference $x_{CP} = [2.7 \pm 1.6 \, \text{(stat)} \pm 0.4 \, \text{(syst)}] \times 10^{-3}$ and the $CP$-violating parameter $\Delta x = [-0.53 \pm 0.70 \, \text{(stat)} \pm 0.22 \, \text{(syst)}] \times 10^{-3}$. The results are consistent with $CP$ symmetry. These determinations are the most precise from a single experiment and, combined with current world-average results, yield the first evidence that the masses of the neutral charm-meson eigenstates differ.


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Flavor oscillations are transitions between neutral flavored mesons and their corresponding antimesons that follow an oscillating pattern as a function of decay time. In the standard model, these transitions are mediated by weak-interaction amplitudes involving exchanges of virtual $W^\pm$ bosons and heavy quarks. Unknown particles of arbitrarily high mass can contribute as virtual particles in the amplitude, possibly enhancing the average oscillation rate or the difference between the rates of mesons and antimesons. This makes flavor oscillations sensitive to non-standard-model dynamics at large energy scales [1].

Oscillations occur because the mass eigenstates of neutral flavored mesons are linear combinations of the flavor eigenstates. In particular, for charm mesons one writes $|D_{1,2}\rangle \equiv p|D^0\rangle \pm q|\bar{D}^0\rangle$, where $p$ and $q$ are complex parameters. In the limit of charge-parity (CP) symmetry, and by defining $D_{1,2}$ as the CP-even (odd) state, the oscillation rate depends only on the dimensionless mixing parameters $x \equiv (m_1 - m_2)\epsilon^2 / \Gamma$ and $y \equiv (\Gamma_1 - \Gamma_2) / (2\Gamma)$, where $m_{1,2}$ and $\Gamma_{1,2}$ are the mass and decay width of the $D_{1,2}$ state, respectively, and $\Gamma$ equals $(\Gamma_1 + \Gamma_2) / 2$ [2]. If CP symmetry is violated, the oscillation rates for mesons produced as $D^0$ and $\bar{D}^0$ differ. The difference is generated in the mixing amplitude if $|q/p| \neq 1$ or in the interference between mixing and decay if $\phi_f \equiv \arg(A_f / \bar{A}_f) \neq 0$. The amplitude $A_f (\bar{A}_f)$ refers to the decay $D^0 \to f (\bar{D}^0 \to f)$, where $f$ is a common final state. If CP is conserved in the decay amplitude ($|A_f|^2 = |\bar{A}_f|^2$), the CP-violating phase is independent of the final state, $\phi_f \approx \phi = \arg(q/p)$ [3,4].

Current global averages of charm-mixing parameters have large uncertainties and are consistent with CP symmetry, yielding $x = (3.6^{+2.1}_{-1.6}) \times 10^{-3}$, $y = (6.7^{+0.6}_{-1.3}) \times 10^{-3}$, $|q/p| = 0.94^{+0.17}_{-0.07}$, and $\phi = -0.13^{+0.29}_{-0.17}$ [5]. Improving the knowledge of $x$, which has not been shown to differ significantly from zero, is especially critical since the sensitivity to the small phase $\phi$ relies predominantly on observables proportional to $x \sin \phi$.

Direct experimental access to charm-mixing parameters is offered by self-conjugate multibody decays, such as $D^0 \to K^0_S\pi^+\pi^-$. Inclusion of charge-conjugate processes is implied unless stated otherwise. A joint fit of the Dalitz-plot and decay-time distributions of these decays allows the identification of a $D^0$ component that increases as a function of decay time in a sample of candidates produced as $\bar{D}^0$ mesons, and vice versa. This approach is challenging as it requires analyzing the decay-time evolution of signal decays across the Dalitz plot with a detailed amplitude model, while accounting for efficiencies, resolutions, and background [6-8]. Model-independent approaches that obviate the need for an amplitude analysis exist [9-11], but rely on an accurate description of the efficiencies.

This Letter reports on a measurement of charm oscillations in $D^0 \to K^0_S\pi^+\pi^-$ decays based on a novel model-independent approach, the bin-flip method, optimized for the measurement of the parameter $x$ [12]. The method relies on ratios between charm decays reconstructed in similar kinematic and decay-time conditions, thus avoiding the need for an accurate modeling of the efficiency variation across phase space and decay time. We express the $D^0 \to K^0_S\pi^+\pi^-$ dynamics with two invariant masses following the Dalitz formalism [13,14] where $m^2_{1,2}$ is the squared invariant mass $m^2(K^0_S\pi^\pm)$ for $D^0 \to K^0_S\pi^+\pi^-$ decays and $m^2(K^0_S\pi^\mp)$ for $\bar{D}^0 \to K^0_S\pi^+\pi^-$ decays. We partition the Dalitz plot (see Supplemental material) into disjoint regions (“bins”) proposed by the CLEO collaboration to preserve nearly constant strong-phase differences $\Delta \delta(m^2_1, m^2_2)$ between the $D^0$ and $\bar{D}^0$ amplitudes within each bin [15]. Two sets of eight bins are formed, organized symmetrically about the principal bisector $m^2_+ = m^2_-$. Bins are labeled with the indices $\pm b$, where $b = 1, ..., 8$. Positive indices refer to the (lower) $m^2_+ > m^2_-$ region, where unmixed Cabibbo-favored $D^0 \to K^+(892)^-\pi^+$ decays dominate; negative indices refer to
the symmetric (upper) \(m_+^2 < m_-^2\) region, which receives a larger contribution from decays following oscillation. The data are further split into bins of decay time, indexed with \(j\). For each, we measure the ratio \(R_{bj}^+(R_{bj}^-)\) between initially-produced \(D^0 (\bar{D}^0)\) mesons in Dalitz bin \(-b\) and Dalitz bin \(b\). For small mixing parameters and \(CP\)-conserving decay amplitudes, which is a good approximation here, the ratios are \[12\]

\[
R_{bj}^+ \approx r_b + \frac{1}{4} r_b \langle t_j^2 \rangle \text{Re}(z_{CP}^2 - \Delta z^2) + \frac{1}{4} \langle t_j^2 \rangle |z_{CP} \pm \Delta z|^2 + \sqrt{r_b} \langle t_j \rangle \text{Re}[X_b(z_{CP} \pm \Delta z)].
\]

Here \(\langle t_j \rangle (\langle t_j^2 \rangle)\) is the average (squared) decay time of unmixed decays in bin \(j\), in units of the \(D^0\) lifetime \(\tau = \hbar/\Gamma_{D^0}[2]\). The parameter \(r_b\) is the ratio of signal yields in symmetric Dalitz-plot bins \(\pm b\) at \(t = 0\), and \(X_b\) quantifies the average strong-phase difference in these bins \[12\]. The \(z_{CP}\) and \(\Delta z\) parameters, defined by \(z_{CP} \pm \Delta z = -(q/p)^{\pm 1} (y + ix)\), are obtained, along with \(r_b\), from a joint fit of the observed \(R_{bj}\) ratios in which external information on \(c_b \equiv \text{Re}(X_b)\) and \(s_b \equiv -\text{Im}(X_b)\) \[15\] is used as a constraint. The results are expressed in terms of the \(CP\)-averaged mixing parameters \(x_{CP} \equiv -\text{Im}(z_{CP})\) and \(y_{CP} \equiv -\text{Re}(z_{CP})\), and of the \(CP\)-violating differences \(\Delta x \equiv -\text{Im}(\Delta z)\) and \(\Delta y \equiv -\text{Re}(\Delta z)\). Conservation of \(CP\) symmetry in mixing, or in the interference between mixing and decay, implies \(x_{CP} = x\), \(y_{CP} = y\), and \(\Delta x = \Delta y = 0\).

Samples of \(D^0 \rightarrow K_S^0\pi^+\pi^-\) decays are reconstructed from proton-proton collisions collected by the LHCb experiment in 2011 and 2012, corresponding to an integrated luminosity of 1 fb\(^{-1}\) and 2 fb\(^{-1}\) respectively. In the 2012 data, both the strong-interaction decay \(D^*+ \rightarrow D^0\pi^+\) and the semileptonic \(b\)-hadron decay \(\bar{B} \rightarrow D^0\mu^-X\), where \(X\) generically indicates unreconstructed particles, are used to determine whether a \(D^0\) or \(\bar{D}^0\) is produced. In the 2011 data, only the \(\bar{B} \rightarrow D^0\mu^-X\) decays are used, as the online-selection efficiency for \(D^*+ \rightarrow D^0\pi^\pm\) decays was low. Throughout this Letter, \(D^*+\) indicates the \(D^{(*)}(2010)\) meson and soft pion indicates the pion from its decay.

The LHCb detector is a single-arm forward spectrometer covering the pseudorapidity range \(2 < \eta < 5\), equipped with charged-hadron identification detectors, calorimeters, and muon detectors, and designed for the study of particles containing \(b\) or \(c\) quarks \[16,17\].

The online selection of \(D^*+ \rightarrow D^0(\rightarrow K_S^0\pi^+\pi^-)\pi^+\) decays (prompt sample) uses criteria on momenta and final-state charged-particle displacements from any proton-proton primary interaction. Offline, we apply criteria consistent with the decay topology on momenta, vertex and track displacements, particle-identification information, and invariant masses of the \(D^*+\) decay products. Specifically, the mass of the \(D^0\) candidate is required to meet \(1.84 < m(K_S^0\pi^+\pi^-) < 1.89\) GeV/c\(^2\) and the difference between the \(D^*+\) and \(D^0\) candidate masses is required to satisfy \(\Delta m < 151.1\) MeV/c\(^2\). The \(D^0\) and soft pion candidates are required to point back to one of the proton-proton interactions (the primary vertex) to suppress signal candidates originating from decays of \(b\) hadrons (secondary decays). A kinematic fit constrains the tracks according to the decay topology and the \(D^*+\) candidate to originate from the primary vertex \[18\]. In the reconstruction of the Dalitz-plot coordinates, we additionally constrain the \(K_S^0\) and \(D^0\) meson masses to the known values \[2\] to ensure that all candidates populate the kinematically allowed phase space.

The online selection of \(\bar{B} \rightarrow D^0(\rightarrow K_S^0\pi^+\pi^-)\mu^-X\) decays (semileptonic sample) requires at least one displaced, high-transverse-momentum muon and a vertex con-
sistent with the decay of a $b$ hadron. Offline, we apply criteria consistent with the decay topology on momenta, vertex and track displacements, particle identifications, and invariant masses of the $D^0$ decay products. In addition, candidate $D^0\mu^-$ pairs are formed by requiring $2.5 < m(D^0\mu^-) < 6.0$ GeV/$c^2$ and the corrected mass $\sqrt{m^2(D^0\mu^-) + p_T^2(D^0\mu^-) + p_\perp(D^0\mu^-)}$, where the momentum component $p_\perp(D^0\mu^-)$ of the $D^0\mu^-$ system transverse to the $B$ flight direction partially compensates for the momentum of unreconstructed decay products, to be smaller than 5.8 GeV/$c^2$. The $B$ flight direction is inferred from the measured positions of the primary and $D^0\mu^-$ vertices. A kinematic fit constrains the $D^0$ and $K_S^0$ masses to their known values.

In both samples, two categories of signal candidates are used, those with $K_S^0 \to \pi^+\pi^-$ candidates reconstructed in the vertex detector (long $K_S^0$) and those with $K_S^0$ candidates reconstructed after the vertex detector (downstream $K_S^0$).

About 2% (3%) of the selected $D^{*-}$ ($\bar{B}$) candidates belong to events in which multiple candidates are reconstructed by pairing the same $D^0$ candidate with different soft pions (muons). For these events, we randomly choose a single candidate. We consider the prompt and semileptonic samples independent as their overlap amounts to less than 0.1% of the semileptonic sample size.

Figure 1 shows the $\Delta m$ and $m(K_S^0\pi^+\pi^-)$ distributions of the prompt and semileptonic samples, respectively. The prompt sample contains $1.3 \times 10^6$ signal decays, 45% with downstream $K_S^0$ candidates, and a small background dominated by genuine $D^0 \to K_S^0\pi^+\pi^-$ decays associated to random soft pions. Secondary $D^{*-}$ decays contribute approximately 3% to the signal yield, as determined using $D^0$ candidates not pointing to the primary vertex. The semileptonic sample contains $1.0 \times 10^6$ signal decays, 66% with downstream $K_S^0$ candidates, and a sizable background dominated by unrelated $K_S^0\pi^+\pi^-$ combinations. Genuine $D^0$ decays associated with random muons contribute less than 1% to the $D^0$ yield, as determined from the yield of false $\bar{B}$ candidates formed by associating $D^{*-} \to D^0\pi^+$ with same-sign $\mu^+$ candidates. Contributions from backgrounds due to misreconstructed $D^0$ decays, such as $D^0 \to K_S^0\pi^+\pi^-\pi^0$ and $D^0 \to K_S^0h^+h^0^-$, where $h^+h^0^-$ indicates a pair of light hadrons other than $\pi^+\pi^-$, are negligible.

Simulated [19, 20] prompt decays show that the online requirements on displacement and momenta of the $D^0$ decay products introduce efficiency variations that are correlated...
between the mass of the two final-state pions, \(m(\pi^+\pi^-)\), and the \(D^0\) decay time. Because \((m(\pi^+\pi^-), t)\) correlations can bias the results, we correct for them using data. The smallness of the mixing parameters \([5]\), along with the known \(D^0 \rightarrow K^0_S\pi^+\pi^-\) decay amplitudes \([6–8]\), rule out any measurable \((m(\pi^+\pi^-), t)\) correlation introduced by \(D^0 - \bar{D}^0\) mixing with current sample sizes. Hence, we ascribe any observed dependence between \(m(\pi^+\pi^-)\) and \(t\) to instrumental effects. We use the background-subtracted \((m(\pi^+\pi^-), t)\) distribution to determine the decay-time efficiency, normalized to the average decay-time distribution, as a function of \(m(\pi^+\pi^-)\). This two-dimensional map is smoothed and used to assign per-candidate weights proportional to the inverse of the relative efficiency at each candidate’s \((m(\pi^+\pi^-), t)\) coordinates, effectively removing the correlated nonuniformities. The corrections are determined separately for long and downstream \(K^0_S\) candidates, as they feature different correlations. Figure 2 shows the smoothed \((m(\pi^+\pi^-), t)\) map for the sample with downstream \(K^0_S\) candidates, where the correlations are more prominent. The 6% of candidates reconstructed with \(t < 0.9\tau\) are discarded because the corresponding weights cannot be determined precisely. No \((m(\pi^+\pi^-), t)\) correlations are observed in \(B \rightarrow D^0(\rightarrow K^0_S\pi^+\pi^-)\) \(\mu^- X\) decays.

We divide prompt and semileptonic samples according to \(K^0_S\) category, \(D^0\) meson flavor, Dalitz-plot position, and decay time. In each subsample we determine the signal yield, and — for each decay-time bin — the average decay time and average squared decay time of the signal candidates. Finally, we fit the decay-time dependence of the ratio of signal yields symmetric with respect to the Dalitz-plot bisector.

We determine the signal yields by fitting the \(\Delta m\) distribution, weighted to correct for the \((m(\pi^+\pi^-), t)\) correlations, for the \(D^{*+} \rightarrow D^0(\rightarrow K^0_S\pi^+\pi^-)\) candidates and the \(m(K^0_S\pi^+\pi^-)\) distribution for the \(\bar{B} \rightarrow D^0(\rightarrow K^0_S\pi^+\pi^-)\) \(\mu^- X\) candidates. All components are modeled empirically. The \(\Delta m\) model combines a \(D^{*+}\) signal with a smooth phase-space-like background. The \(m(K^0_S\pi^+\pi^-)\) model combines a \(D^0\) signal with a linear background. Signal and background shape parameters are determined independently for long and downstream \(K^0_S\) candidates, for \(D^0\) and \(\bar{D}^0\) mesons, and in each decay-time and Dalitz-plot bin. The signal model assumes the same parameters for each pair of positive and negative Dalitz-plot bins.
We fit simultaneously the prompt (pr) and semileptonic (sl) samples, separated between the decay-time bin $j$ for prompt and semileptonic samples, and for long and downstream $K^0_S$ candidates. Background is subtracted using weights derived from the mass fits [21] of candidates restricted to the lower half ($m_0^2 < m^2_\pi$) of the Dalitz plot, which is enriched in $D^0$ mesons that did not undergo oscillations. We neglect the decay-time resolutions, typically $0.1\tau$ and $0.25\tau$ for the $D^{*+} \to D^0(\to K^0_S\pi^+\pi^-)\pi^+$ and $B \to D^0(\to K^0_S\pi^+\pi^-)\mu^-X$ samples, respectively, and account for this approximation in the systematic uncertainties.

The mixing parameters are determined by minimizing a least-squares function that compares the decay-time evolution of signal yields ($N$) observed in the Dalitz bins $-b$ and $+b$, along with their uncertainties ($\sigma$), with the expected values reported in Eq. (1),

$$
\chi^2 \equiv \sum_{pr,sl} \sum_{l,d} \sum_{b,j} \sum_{\pm,-} \frac{(N_{-bj}^\pm - N_{+bj}^\pm R_{+bj}^\pm)^2}{(\sigma_{-bj}^\pm)^2 + (\sigma_{+bj}^\pm R_{+bj}^\pm)^2} + \sum_{b,b'} (X_b^{\text{CLEO}} - X_b) (V_{\text{CLEO}}^{-1})_{bb'} (X_{b'}^{\text{CLEO}} - X_{b'}). \quad (2)
$$

We fit simultaneously the prompt (pr) and semileptonic (sl) samples, separated between long (l) and downstream (d) $K^0_S$ candidates, and between $D^0$ (+) and $\bar{D}^0$ (−) flavors, across all decay-time bins $j$ and Dalitz-plot bins $b$. We constrain the parameters $X_b$ to the values $X_b^{\text{CLEO}}$ measured by the CLEO collaboration through a Gaussian penalty term that uses the sum $V_{\text{CLEO}}$ of the statistical and systematic covariance matrices [15]. In the fit, the parameters $r_b$ are determined independently for each subsample (pr, sl, l, d) because they are affected by the sample-specific variation of the efficiency over the Dalitz plot [12]. The values of $x_{CP}$, $\Delta x$, and $\Delta y$ were kept blind until the analysis was finalized.

Figure 3 shows the yield ratios with fit projections overlaid for prompt and semileptonic data. The offsets between semileptonic and prompt data are due to sample-specific efficiency variations across the Dalitz plot; their slopes, due to charm oscillations, are consistent across samples. Table 1 lists the results. The data are consistent with $CP$ symmetry ($\Delta x = \Delta y = 0$). The precision is dominated by the statistical contribution, which incorporates a subleading component due to the precision of the CLEO measurements.

The dominant systematic uncertainties on $x_{CP}$ are associated with the 3% contamination from secondary $D^{*+}$ decays in the prompt sample ($0.24 \times 10^{-3}$) and from the 1% contamination of genuine $D^0$ mesons associated with random muons in the semileptonic sample ($0.34 \times 10^{-3}$). Biases due to the neglected decay-time and $m_0^2$ resolutions, and efficiency variations across decay time and Dalitz plot, constitute the dominant systematic uncertainty on $y_{CP}$ ($0.94 \times 10^{-3}$). Possible asymmetric nonuniformities with respect to the

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value [10^{-3}]</th>
<th>Stat. correlations</th>
<th>Syst. correlations</th>
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</thead>
<tbody>
<tr>
<td>$x_{CP}$</td>
<td>2.7 ± 1.6 ± 0.4</td>
<td>0.17 0.04 0.02 0.15 0.01 0.02</td>
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</tr>
<tr>
<td>$y_{CP}$</td>
<td>7.4 ± 3.6 ± 1.1</td>
<td>0.03 0.01 0.05 0.03</td>
<td></td>
</tr>
<tr>
<td>$\Delta x$</td>
<td>0.53 ± 0.70 ± 0.22</td>
<td>-0.13</td>
<td>0.14</td>
</tr>
<tr>
<td>$\Delta y$</td>
<td>0.6 ± 1.6 ± 0.3</td>
<td></td>
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Figure 3: (Top) $CP$-averaged yield ratios and (bottom) differences of $D^0$ and $\bar{D}^0$ yield ratios as functions of $t/\tau$ for (closed points) prompt and (open points) semileptonic data, and for the various Dalitz bins. The fit projections over the (solid line) prompt and (dashed line) semileptonic data are overlaid.

bisector in the Dalitz plot induced by reconstruction inefficiencies dominate the systematic uncertainty on $\Delta x$ ($0.22 \times 10^{-3}$) and $\Delta y$ ($0.25 \times 10^{-3}$). Other minor effects, such as mismodeling in the signal-yield fits or in the determination of the bin-averaged decay times, are also considered. The consistency between results on the prompt and semileptonic
Table 2: Point estimates and 95.5% confidence-level (CL) intervals for the derived parameters. The uncertainties include statistical and systematic contributions.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>95.5% CL interval</th>
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<tbody>
<tr>
<td>$x$ $[10^{-2}]$</td>
<td>$0.27^{+0.17}_{-0.15}$</td>
<td>$[-0.05, 0.60]$</td>
</tr>
<tr>
<td>$y$ $[10^{-2}]$</td>
<td>$0.74^{+0.37}_{-0.00}$</td>
<td>$[0.00, 1.50]$</td>
</tr>
<tr>
<td>$</td>
<td>q/p</td>
<td>$</td>
</tr>
<tr>
<td>$\phi$</td>
<td>$-0.09^{+0.11}_{-0.16}$</td>
<td>$[-0.73, 0.29]$</td>
</tr>
</tbody>
</table>

sample (see Suppemental material), and on various partitions of the data, supports the robustness of the analysis, including the correction of the $(m(\pi^+\pi^-), t)$ correlations.

In summary, we report a measurement of the normalized mass difference between neutral charm-meson eigenstates using the recently proposed bin-flip method. Allowing for CP violation in charm mixing, or in the interference between mixing and decay, we measure the CP-averaged normalized mass difference $x_{CP} = [2.7 \pm 1.6 \text{ (stat)} \pm 0.4 \text{ (syst)}] \times 10^{-3}$, and the CP-violating parameter $\Delta x = [-0.53 \pm 0.70 \text{ (stat)} \pm 0.22 \text{ (syst)}] \times 10^{-3}$. In addition, we report the CP-averaged normalized width difference $y_{CP} = [7.4 \pm 3.6 \text{ (stat)} \pm 1.1 \text{ (syst)}] \times 10^{-3}$, along with the corresponding CP-violating parameter $\Delta y = [0.6 \pm 1.6 \text{ (stat)} \pm 0.3 \text{ (syst)}] \times 10^{-3}$. We use the results to form a likelihood function of $x$, $y$, $|q/p|$, and $\phi$ and derive confidence intervals (Table 2) using a likelihood-ratio ordering that assumes the observed correlations to be independent of the true parameter values [22]. The resulting determination of the mass difference is the most precise from a single experiment. While our result is consistent with $x = 0$ within two standard deviations, combined with the current global knowledge it offers the first evidence of a nonzero (positive) mass difference between neutral charm mesons (see Supplemental material).

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and Yandex LLC (Russia); GVA, XuntaGal and GENCAT (Spain); the Royal Society and the Leverhulme Trust (United Kingdom); Laboratory Directed Research and Development program of LANL (USA).

References


[15] CLEO collaboration, J. Libby et al., *Model-independent determination of the strong-phase difference between $D^0$ and $\bar{D}^0 \rightarrow K^{0}_{\pm,\mp} h^+ h^-$ ($h = \pi, K$) and its impact on the measurement of the CKM angle $\gamma/\phi_3$*, Phys. Rev. D82 (2010) 112006, arXiv:1010.2817.


Supplemental material

Figure 4 shows the decay-time-integrated Dalitz-plot distribution of the background-subtracted $D^0 \to K_S^0 \pi^+ \pi^-$ candidates used in the analysis, together with the Dalitz-plot binning scheme. No efficiency corrections are applied. All samples are combined.

Figure 4: (Left) Dalitz-plot distribution of background-subtracted $D^0 \to K_S^0 \pi^+ \pi^-$ candidates. (Right) Iso-\(\Delta\delta\) binning of the $D^0 \to K_S^0 \pi^+ \pi^-$ Dalitz plot, reproduced from Phys. Rev. D82 (2010) 112006. The bins are symmetric with respect to the \(m^2_+ = m^2_-\) bisector; positive indices refer to bins in the (lower) \(m^2_+ > m^2_-\) region; negative indices refer to those in the (upper) \(m^2_+ < m^2_-\) region. Colors indicate the absolute value of the bin index \(b\).

Prompt- and semileptonic-only fit results

Tables 3 and 4 report results obtained by fitting independently the prompt and semileptonic data samples, respectively.

Table 3: Results of the fit to the prompt data sample. The first contribution to the uncertainty is statistical, the second systematic.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value ([10^{-3}])</th>
<th>Stat. correlations</th>
<th>Syst. correlations</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x_{CP})</td>
<td>3.0 ± 1.9 ± 0.5</td>
<td>0.14</td>
<td>0.05</td>
</tr>
<tr>
<td>(y_{CP})</td>
<td>6.5 ± 4.3 ± 1.5</td>
<td>-0.03</td>
<td>0.02</td>
</tr>
<tr>
<td>(\Delta x)</td>
<td>-0.41 ± 0.78 ± 0.24</td>
<td>-0.12</td>
<td>0.17</td>
</tr>
<tr>
<td>(\Delta y)</td>
<td>0.3 ± 1.8 ± 0.3</td>
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Updated world average of charm-mixing parameters

We combine the results presented in this Letter with current knowledge of charm-mixing parameters to assess their impact on the world average. The combination procedure follows...
Table 4: Results of the fit to the semileptonic data sample. The first contribution to the uncertainty is statistical, the second systematic.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Stat. correlations</th>
<th>Syst. correlations</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_{CP} )</td>
<td>2.3 ± 2.8 ± 1.0</td>
<td>-0.08 0.03</td>
<td>0.14 0.00 -0.03</td>
</tr>
<tr>
<td>( y_{CP} )</td>
<td>8.5 ± 6.3 ± 0.7</td>
<td>0.01 0.02</td>
<td>-0.06 -0.03</td>
</tr>
<tr>
<td>( \Delta x )</td>
<td>-0.9 ± 1.7 ± 0.3</td>
<td>-0.07</td>
<td>0.13</td>
</tr>
<tr>
<td>( \Delta y )</td>
<td>2.2 ± 4.0 ± 0.2</td>
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</table>

closely the methods of the Heavy Flavor Averaging Group. In addition to the results presented in this Letter, the following measurements are included in the combination:

- LHCb collaboration, R. Aaij et al., Updated determination of \( D^0 \rightarrow \bar{D}^0 \) mixing and CP violation parameters with \( D^0 \rightarrow K^+\pi^- \) decays, Phys. Rev. D97 (2018) 031101, arXiv:1712.03220;


- CDF collaboration, T. Aaltonen et al., Observation of \( D^0 \rightarrow \bar{D}^0 \) mixing using the CDF II detector, Phys. Rev. Lett. 111 (2013) 231802, arXiv:1309.4078;

- BaBar collaboration, B. Aubert et al., Evidence for \( D^0 \rightarrow \bar{D}^0 \) mixing, Phys. Rev. Lett. 98 (2007) 211802, arXiv:hep-ex/0703020;

- CLEO collaboration, D. M. Asner et al., Updated measurement of the strong phase in \( D^0 \rightarrow K^+\pi^- \) decay using quantum correlations in e^+e^- \( \rightarrow D^0\bar{D}^0 \) at CLEO, Phys. Rev. D86 (2012) 112001, arXiv:1210.0939;


- LHCb collaboration, R. Aaij et al., Measurement of the CP violation parameter \( A_f \) in \( D^0 \rightarrow K^+K^- \) and \( D^0 \rightarrow \pi^+\pi^- \) decays, Phys. Rev. Lett. 118 (2017) 261803, arXiv:1702.06490;

- Belle collaboration, M. Starić et al., Measurement of \( D^0 \rightarrow \bar{D}^0 \) mixing and search for CP violation in \( D^0 \rightarrow K^+K^- \), \( \pi^+\pi^- \) decays with the full Belle data set, Phys. Lett B753 (2016) 412, arXiv:1509.08266;

- LHCb collaboration, R. Aaij et al., Measurement of indirect CP asymmetries in \( D^0 \rightarrow K^-K^+ \) and \( D^0 \rightarrow \pi^-\pi^+ \) decays using semileptonic B decays, JHEP 04 (2015) 043, arXiv:1501.06777;

Table 5: Updated global combinations of charm-mixing measurements.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Allowed interval</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>68.3% CL</td>
</tr>
<tr>
<td>$x \times 10^{-2}$</td>
<td>0.38 ± 0.12</td>
<td>[0.26, 0.50]</td>
</tr>
<tr>
<td>$y \times 10^{-2}$</td>
<td>0.655 ± 0.062</td>
<td>[0.588, 0.717]</td>
</tr>
<tr>
<td>$</td>
<td>q/p</td>
<td>$</td>
</tr>
<tr>
<td>$\phi$</td>
<td>-0.070 ± 0.079</td>
<td>[-0.151, 0.009]</td>
</tr>
</tbody>
</table>

- LHCb collaboration, R. Aaij et al., *Model-independent measurement of mixing parameters in $D^0 \to K_S^0 \pi^+\pi^-$ decays*, JHEP 04 (2016) 033 [arXiv:1510.01664]
- Belle collaboration, T. Peng et al., *Measurement of $D^0-\bar{D}^0$ mixing and search for indirect CP violation using $D^0 \to K_S^0 \pi^+\pi^-$ decays*, Phys. Rev. D89 (2014) 091103 [arXiv:1404.2412]
- BaBar collaboration, P. del Amo Sanchez et al., *Measurement of $D^0-\bar{D}^0$ mixing parameters using $D^0 \to K_S^0 \pi^+\pi^-$ and $D^0 \to K_S^0 K^+K^-$ decays*, Phys. Rev. Lett. 105 (2010) 081803 [arXiv:1004.5053]
- LHCb collaboration, R. Aaij et al., *First observation of $D^0-\bar{D}^0$ oscillations in $D^0 \to K^+\pi^-\pi^+\pi^-$ decays and measurement of the associated coherence parameters*, Phys. Rev. Lett. 116 (2016) 241801 [arXiv:1602.07224]
- BaBar collaboration, J. P. Lees et al., *Measurement of the neutral $D$ meson mixing parameters in a time-dependent amplitude analysis of the $D^0 \to \pi^+\pi^-\pi^0$ decay*, Phys Rev. D93 (2016) 112014 [arXiv:1604.00857]
- BaBar collaboration, B. Aubert et al., *Measurement of $D^0-\bar{D}^0$ mixing from a time-dependent amplitude analysis of $D^0 \to K^+\pi^-\pi^0$ decays*, Phys. Rev. Lett. 103 (2009) 211801 [arXiv:0807.4544]

The results are reported in Table 5 and Figure 5.
Figure 5: Impact of the results reported in this Letter on current global averages of charm-mixing parameters. The hatched and shaded areas in the bottom panels indicate the 68% and 95% confidence regions, respectively.
<table>
<thead>
<tr>
<th>Institution Name</th>
<th>City</th>
<th>Country</th>
<th>Associated to</th>
</tr>
</thead>
<tbody>
<tr>
<td>Massachusetts Institute of Technology</td>
<td>Cambridge, MA</td>
<td>United States</td>
<td></td>
</tr>
<tr>
<td>University of Cincinnati</td>
<td>Cincinnati, OH</td>
<td>United States</td>
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<tr>
<td>University of Maryland</td>
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<tr>
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<tr>
<td>Laboratory of Mathematical and Subatomic Physics</td>
<td>Constantine, Algeria</td>
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<tr>
<td>Pontificia Universidade Católica do Rio de Janeiro (PUC-Rio)</td>
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<tr>
<td>South China Normal University</td>
<td>Guangzhou, China</td>
<td>Associated to 3</td>
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<tr>
<td>School of Physics and Technology, Wuhan University</td>
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<td>Associated to 3</td>
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<tr>
<td>Institute of Particle Physics, Central China Normal University</td>
<td>Wuhan, Hubei, China</td>
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<tr>
<td>Departamento de Física, Universidad Nacional de Colombia</td>
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<td>Institut für Physik</td>
<td>Universität Rostock, Rostock, Germany</td>
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<td>Van Swinderen Institute</td>
<td>University of Groningen, Groningen, Netherlands</td>
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<td>National Research Tomsk Polytechnic University</td>
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<td>Associated to 35</td>
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<tr>
<td>Instituto de Física Corpuscular, Centro Mixto Universidad de Valencia - CSIC</td>
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<td>Associated to 42</td>
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<tr>
<td>University of Michigan</td>
<td>Ann Arbor, United States</td>
<td>Associated to 63</td>
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<tr>
<td>Los Alamos National Laboratory (LANL)</td>
<td>Los Alamos, United States</td>
<td>Associated to 63</td>
<td></td>
</tr>
</tbody>
</table>

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- Laboratoire Leprince-Ringuet, Palaiseau, France
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- Università di Milano Bicocca, Milano, Italy
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† Deceased