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June 1994

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ABSTRACT

We calculate the leading weak corrections at $O(\frac{m_t^2}{s})$ to the QCD production of heavy top quark pairs via $q\bar{q} \rightarrow t\bar{t}$ at hadron colliders and compare them with the complete one-loop weak corrections. We find that these corrections dominate the threshold region for a heavy top quark if the Higgs boson is light. For a heavy Higgs boson, these corrections are generally small. At large $M(t\bar{t})$, the final state weak corrections of $O(g^2)$ become almost as large as the $O(\frac{m_t^2}{s})$ corrections (or even slightly larger if the Higgs boson is heavy). The parity violation appearing in $q\bar{q} \rightarrow t\bar{t}$ from the SM weak corrections is a few percent for all $M(t\bar{t})$, so any observation of large parity violation in this process would indicate new physics. The polarization of the $t\bar{t}$ pairs is also discussed, including the effect that this has on proposed techniques for measuring the top quark mass.

1. Introduction

Studies on radiative corrections concluded that the mass of a Standard Model (SM) top quark (m_t) has to be less than 200 GeV.[1] From the direct search at the Tevatron, assuming a SM top quark, m_t has to be larger than 131 GeV.[2] Recently, evidence was presented by the CDF group at the FNAL to support the existence of a heavy top quark with a mass of $m_t = 174 \pm 15$ GeV.[3] Nonetheless, there are no compelling reasons to believe that the top quark couplings to other particles should be of the SM nature. Because the top quark is heavy relative to other light fundamental particles, one expects that the underlying theory at a very high energy scale may easily reveal itself at low energy through the effective behavior of the top quark and its couplings to the light fields. Furthermore, because the top quark mass is of the order of the Fermi scale $v = (\sqrt{2}G_F)^{-1/2} = 246$ GeV, which characterizes the energy scale for electroweak symmetry breaking, the top quark may be used to probe the symmetry breaking sector. Therefore, it is important to study the couplings of the top quark to the light fields in the theory. In this paper we examine the effective couplings of the top quark to the gluon field through the production process $q\bar{q} \rightarrow t\bar{t}$ in hadron collisions.

At the (upgraded) Tevatron, the dominant production mechanism for $t\bar{t}$ pairs is via $q\bar{q}$ annihilation. Although at the LHC (Large Hadron Collider) the top quark pairs are mainly produced from the gluon fusion process in the SM, it is possible that some large, new, physics effects might appear in the production of $t\bar{t}$ pairs via $q\bar{q}$ annihilation. Some of these possibilities were discussed in Refs. 4 and 5. It is therefore important to know the production rate for top quarks as precisely as possible.

At the upgraded Tevatron and the LHC, top quarks have a production rate large enough to make them a potentially serious background, particularly in the pursuit of new physics such as Higgs boson searches. Since the decay products of the top quark can mimic the signature of signal events of interest, it is also useful to know the polarization of the top quark which in turn governs the kinematics of the detected particles created by the top quark decays.

The leading SM weak corrections of $O(\frac{m_t^2}{v^2})$ to the production of heavy $t\bar{t}$ pairs from the QCD subprocess $q\bar{q} \rightarrow t\bar{t}$ were calculated by Stange and Willenbrock[6] and independently by us.[7] These results are in agreement. We find that the corrections to the event rate are small with a few percent increase near the threshold region for a light (around 100 GeV) Higgs boson and about the same percent decrease at high subprocess energies. These corrections can be taken most seriously for higher top quark masses, where not only is the $O(\frac{m_t^2}{v^2})$ approximation most valid, but also the nonperturbative effects[8] which can modify the threshold behavior

become less significant since the faster decay of the top quark prevents bound states from forming. [9]

The complete one-loop weak corrections to the production rate of $t\bar{t}$ pairs from the QCD process $q\bar{q} \rightarrow g \rightarrow t\bar{t}$ was recently presented by Beenakker, *et al.* [10] This result confirms the conclusion in their previous publication [11] that in the high energy region (large $M(t\bar{t})$) the corrections of $O(\frac{m_t^2}{s})$ do not dominate the production rate for $t\bar{t}$ pairs.

We compare our results with the results of Refs. 6 and 10. After separately calculating the weak corrections to the initial state and the final state for the scattering process $q\bar{q} \rightarrow t\bar{t}$, we find that the large difference in the high energy limit between the corrections at $O(\frac{m_t^2}{s})$ and the complete one-loop corrections in the SM does not come from the initial state radiative corrections. It is the inclusion of the complete one-loop weak corrections in the final state that makes the suppression of the event rate roughly a factor of two different from that predicted by including the $O(\frac{m_t^2}{s})$ corrections alone.

Continuing to examine the effective couplings of the $g-t-\bar{t}$ vertex due to weak corrections, we find that for heavy top quarks produced near the threshold region ($M(t\bar{t})$ near $2m_t$) that the $O(\frac{m_t^2}{s})$ corrections dominate if the Higgs boson is light, but not so if the Higgs boson is heavy.

Though the QCD corrections to the $q\bar{q} \rightarrow t\bar{t}$ production rate [12] are larger than the weak corrections presented here, the parity violation manifest in the electroweak interactions produces effects unobtainable by theories like QCD that maintain parity (P) and charge conjugation (C) symmetries separately. These effects are realized, for example, in the difference between the K -factors describing the higher order corrections for final state polarizations related by parity transformations, *i.e.*, the difference between the event rates for the production of $t_R\bar{t}_L$ and $t_L\bar{t}_R$ (where the subscripts R, L respectively indicate right-handed and left-handed helicity). Such differences are often largest as the invariant mass of the $t\bar{t}$ pair gets large, making the polarization effects most relevant when considering the backgrounds to signal events like those discussed for probing the electroweak symmetry breaking mechanism in the TeV region. Using helicity amplitude techniques in the computation of K -factors for both polarized and unpolarized final states, we find that the parity violation effects derived from the SM weak corrections to the $g-t-\bar{t}$ interaction are a few percent, so any observation of large parity violation in this process would indicate new physics.

The effect that polarization has on the observed kinematics of the $q\bar{q} \rightarrow t\bar{t}$ events is reflected in the decay products of the top quarks. [13] Consequently, there may be a change in the efficiency of experimental cuts used to remove the top quark

background or to observe the top quark signal depending upon the spin asymmetries in top quark production. For example, the charged lepton produced from the decay of a top quark with a right-handed helicity via $t \rightarrow b l^+ \nu$ preferentially receives a greater boost along the direction of motion of the top quark than the charged lepton does from a top quark with a left-handed helicity. An enhancement of top quarks with a right-handed (left-handed) polarization thereby produces charged leptons with more (less) energy in the laboratory. Realizing this, it is plausible that a fixed cut on lepton energies determined from the leading order QCD production of top quarks may automatically produce a different efficiency in removing backgrounds or collecting signals than would be obtained from the production rate that contained electroweak corrections. Perhaps even more pertinent to present interests is the effect that the polarization asymmetry has on the distribution of $M(eb)$, the mass of the charged lepton and bottom quark system in $t \rightarrow e^+ b \nu$, since it is through a related mass distribution (the invariant mass of the e^+ and μ^- , $M(e\mu)$, where the muon comes from fragmentation of the bottom quark) that the best techniques for determining the top quark mass are derived. [14, 15]

The remainder of this paper is organized as follows. In Sec. 2 we present the analytical results of our calculations in terms of form factors. Using these form factors, we give the numerical results describing the production rate and the degree of polarization for top quarks in Sec. 3. In Sec. 4 we examine the polarization effects on the $M(eb)$ distribution and discuss how this relates to measurements of the top quark mass. Sec. 5 contains our conclusion.

2. The Loop Corrections

We consider two calculations in this section. First, we give the leading weak corrections of $O(g_s^2 m_t^2/v^2)$ to top quark production using an effective theory to describe the production of heavy top quark pairs via $q\bar{q} \rightarrow t\bar{t}$. Second, we consider the complete SM weak corrections to this process at the one-loop level.

2.1 CORRECTIONS FROM AN EFFECTIVE THEORY

The effective theory considered in this section is obtained by taking the $g \rightarrow 0$ limit of the electroweak coupling after replacing the mass of the W^+ boson (M_W) with $gv/2$ in the SM Lagrangian. It is an effective theory that reproduces the Standard Model results in the limit where the gauge coupling g vanishes. The neutral and charged Goldstone bosons (G^0 and $G^\pm$) are massless. (This is the case for the SM in the Landau gauge.) The parameter $v \approx 246$ GeV characterizes the scale of the electroweak symmetry breaking and corresponds to the vacuum expectation value (VEV) of the Higgs field in the SM.

We show that the form factors describing the g - t - $\bar{t}$ interaction are infrared-safe and that it is not necessary to include the real emission diagrams with an additional G^0 or $G^\pm$ associated with the $t\bar{t}$ production in our calculations. In the loop diagrams, the ultraviolet divergences are regularized by dimensional regularization with the regulator $\Delta \equiv 2/(4-N) - \gamma_E + \ln(4\pi)$, where N is the spacetime dimension and γ_E is the Euler constant.

Wave Function Renormalization

In this effective theory there are three diagrams from H , G^0 , and $G^\pm$ contributing to the self energy of the top quark and its wave function renormalization. The wave function renormalization constant Z_t can be written as

$$Z_t = 1 + \frac{1}{16\pi^2}[\delta Z_V - \delta Z_A \gamma_5]. \quad (2.1)$$

Hereafter, we use m and m_t interchangeably. Employing the on-shell renormalization scheme, we obtain [16]

$$\begin{aligned} \delta Z_V &= \frac{m^2}{v^2} \left\{ -\frac{1}{2} \left[3\Delta - 3\ln\left(\frac{m^2}{\mu^2}\right) + 1 - 2I(r) + 2J(r) + 4L(r) + i\pi \right] \right\}, \\ \delta Z_A &= \frac{m^2}{v^2} \left\{ \frac{1}{2} \left[\Delta - \ln\left(\frac{m^2}{\mu^2}\right) + 2 + i\pi \right] \right\}, \end{aligned} \quad (2.2)$$

where μ is the 't Hooft mass parameter, $r = m_H^2/m^2$ and m_H is the mass of the Higgs boson. The integrals $I(r)$, $J(r)$ and $L(r)$ are defined as

$$\begin{aligned} I(r) &= \int_0^1 \ln[x^2 + r(1-x) - i\epsilon] dx, \\ J(r) &= \int_0^1 x \ln[x^2 + r(1-x) - i\epsilon] dx, \\ L(r) &= \int_0^1 \frac{x(1-x^2)}{(1-x)^2 + rx - i\epsilon} dx. \end{aligned} \quad (2.3)$$

Vertex Corrections

The g - t - $\bar{t}$ vertex can be expressed as

$$ig_s \bar{u}(p) T^a \Gamma_\mu v(q), \quad (2.4)$$

where g_s is the strong coupling and the T^a are the $SU(3)$ matrices with $T\text{r}(T^a T^b) = \frac{1}{2} \delta^{ab}$.

The $u(p)$ and $v(q)$ are the Dirac spinors of the t and $\bar{t}$ with momenta p and q , respectively. Both momenta p and q are defined as flowing out of the vertex.

The tree level vertex function is $\Gamma_\mu^{tree} = \gamma_\mu$. At the one-loop level, the vertex function can be written as

$$\Gamma_\mu^{loop} = \frac{1}{16\pi^2} \frac{m^2}{v^2} \bar{u}(p) \Lambda_\mu v(q), \quad (2.5)$$

$$\Lambda_\mu = \gamma_\mu (A' - B' \gamma_5) + \frac{1}{2} (p_\mu - q_\mu) (C' - D' \gamma_5) + \frac{1}{2} (p_\mu + q_\mu) (E' - F' \gamma_5),$$

where

$$\begin{aligned} A' &= \frac{3}{2} [\Delta - \ln(\frac{m^2}{\mu^2}) + 1] + \frac{1}{4} (\frac{1}{\beta^2} - 3) \ln(\frac{s}{m^2}) - \beta \ln(\frac{1+\beta}{1-\beta}) \\ &\quad + \frac{m_H^2}{s\beta^2} [-I(r) - 2 + \beta \ln(\frac{1+\beta}{1-\beta})] \\ &\quad - (\frac{m_H^4}{s\beta^2} + 4m^2) C_0^H - \frac{s}{16} (\frac{1}{\beta^2} + 2 - 3\beta^2) C_0^{G^+} \\ &\quad + i\pi (-\frac{m_H^2}{s\beta} + \frac{1}{2} + \beta), \\ B' &= -\frac{1}{2} [\Delta - \ln(\frac{m^2}{\mu^2}) + 1] + \frac{1}{4} (\frac{3}{\beta^2} - 1) \ln(\frac{s}{m^2}) \\ &\quad + \frac{s}{16} (-\frac{3}{\beta^2} + 2 + \beta^2) C_0^{G^+} - i(\frac{\pi}{2}), \\ C' &= \frac{m}{s\beta^2} \{ (\frac{3}{\beta^2} - 1) \ln(\frac{s}{m^2}) - 18 - 8I(r) + 4\beta \ln(\frac{1+\beta}{1-\beta}) \\ &\quad + (\frac{12m_H^2}{s\beta^2}) [-I(r) - 2 + \beta \ln(\frac{1+\beta}{1-\beta})] + 2r[I(r) - \ln(r) + 1] \\ &\quad - 12m_H^2 (\frac{m_H^2}{s\beta^2} + 1) C_0^H - \frac{s}{4} (\frac{3}{\beta^2} - 2 - \beta^2) C_0^{G^+} - 4i\pi (\frac{3m_H^2}{s\beta} + \beta) \}, \\ F' &= \frac{m}{s} [(1 - \frac{3}{\beta^2}) \ln(\frac{s}{m^2}) - 2 + \frac{s}{4} (\frac{3}{\beta^2} - 2 - \beta^2) C_0^{G^+}], \\ D' &= E' = 0, \end{aligned} \quad (2.6)$$

and

$$\begin{aligned} C_0^H &= C_0(m^2, m^2, s, m^2, m_H^2, m^2), \\ C_0^{G^+} &= C_0(m^2, m^2, s, 0, 0, 0), \end{aligned} \quad (2.7)$$

given that $s = (p + q)^2$ is the square of the $t\bar{t}$ center-of-mass energy and $\beta = \sqrt{1 - 4m^2/s}$.

The 3-point function C_0 is defined as

$$C_0(p_1^2, p_2^2, p_5^2, m_1^2, m_2^2, m_3^2) \equiv \frac{1}{i\pi^2} \int d^N q \frac{1}{[q^2 - m_1^2][(q + p_1)^2 - m_2^2][(q + p_1 + p_2)^2 - m_3^2]}, \quad (2.8)$$

where $p_5 = p_1 + p_2$. The loop integrals in the form factors have been evaluated with the code LOOP.[17,18] For simplicity, the mass of the bottom quark (m_b) is taken to be zero.[†]

The renormalized vertex function can be expressed as

$$\begin{aligned} \Gamma_\mu^R &= \gamma_\mu \\ &+ \frac{1}{16\pi^2} \{ \gamma_\mu (A - B\gamma_5) \\ &+ \frac{1}{2} (p_\mu - q_\mu) (C - D\gamma_5) + \frac{1}{2} (p_\mu + q_\mu) (E - F\gamma_5) \}. \end{aligned} \quad (2.9)$$

In this effective theory, $A = \frac{m^2}{v^2} A' + \delta Z_V$, $B = \frac{m^2}{v^2} B' + \delta Z_A$, $C = \frac{m^2}{v^2} C'$, $F = \frac{m^2}{v^2} F'$ and $D = E = 0$. We note that vector current conservation demands that $E = 0$ and $B = -sF/4m$ and that the form factor D is zero in theories that conserve CP.

It can be seen explicitly that the terms with the regulator Δ and the mass parameter μ cancel exactly among themselves; therefore, the renormalized vertex function is free of ultraviolet divergence and independent of the μ parameter, as expected. In addition, it has been checked that Γ_μ^R is free of infrared divergence.

When the Higgs boson mass is very large, i.e., $m_H^2 \gg m^2$, we have

$$\begin{aligned} rI(r) &\rightarrow -\frac{1}{2} + r \ln(r) + \ln(r) - r, \\ J(r) &\rightarrow -\frac{3}{4} + \frac{1}{2} \ln(r), \\ L(r) &\rightarrow 0, \\ C_0^H &\rightarrow -\frac{1}{m_H^2} [\ln(r) + 1 - \beta \ln(\frac{1+\beta}{1-\beta}) + i\pi\beta] + \frac{1}{m_H^4} \ln(r) (\frac{s}{2} - 3m^2). \end{aligned} \quad (2.10)$$

It is straightforward to check that the dependence of the renormalized vertex function Γ_μ^R on the Higgs boson mass vanishes as $m_H \rightarrow \infty$. (Recall that $r = m_H^2/m^2$.) Therefore, m_H decouples for this case in the limit of a large Higgs boson mass. This is in contrast to the usual one-loop SM electroweak corrections which grow like $\ln(m_H^2)$ in the large Higgs boson mass limit.[19]

[†] We checked that the difference between using $m_b = 5$ GeV and zero is less than 0.4% in the numerical results of the form factors.

2.2 SM WEAK CORRECTIONS

In this subsection, we consider the one-loop weak corrections to $q\bar{q} \rightarrow t\bar{t}$ in the SM. The diagrams are shown in Fig. 1. As done in Ref. 10, the pure electromagnetic corrections are not included in this study. Since we are mainly interested in the $g-t-\bar{t}$ couplings, we do not present the analytical form of the initial state corrections, whose numerical contributions are usually small. The ISRCs are discussed in the next section. Following the same notation used in the previous subsection, we give our analytical results for the additional weak corrections to the $g-t-\bar{t}$ vertex in terms of the standard loop integrals defined by 't Hooft and Veltman in Ref. 18.

In the SM the one-loop weak radiative corrections to the $g-t-\bar{t}$ vertex have contributions from the Higgs boson (H), the neutral and charged Nambu-Goldstone bosons ($G^0, G^\pm$), the Z boson (Z) and the W bosons ($W^\pm$). We have employed the 't Hooft-Feynman gauge for the loop calculations. In this gauge, $M_{G^0} = M_Z$ and $M_{G^\pm} = M_W$. In this work, we use $M_W = 80 \text{ GeV}$, $M_Z = 91.17 \text{ GeV}$, and we do not include the quark mixing from the CKM matrix elements; thus, CP is conserved.

Wave Function Renormalization

The wave function renormalization constant for the top quark is given by

$$\begin{aligned}\delta Z_V &= \delta Z_V^H + \delta Z_V^0 + \delta Z_V^+ + \delta Z_V^Z + \delta Z_V^W, \\ \delta Z_A &= \delta Z_A^+ + \delta Z_A^Z + \delta Z_A^W,\end{aligned}$$

where

$$\begin{aligned}\delta Z_V^H(m^2, m^2, m_H^2) &= (m^2/v^2)[B_1 + 2m^2(+\bar{B}_0 - \bar{B}_1)], \\ \delta Z_V^0(m^2, m^2, M_Z^2) &= (m^2/v^2)[B_1 + 2m^2(-\bar{B}_0 - \bar{B}_1)], \\ \delta Z_V^+(m^2, m_b^2, M_W^2) &= (m^2/v^2)[B_1 + 2m^2(-\bar{B}_1)], \\ \delta Z_A^+(m^2, m_b^2, M_W^2) &= (m^2/v^2)[-B_1], \\ \delta Z_V^Z(m^2, m^2, M_Z^2) &= (g_v^2 + g_a^2)(2B_1 + 1 - 2\bar{B}_1) + (g_v^2 - g_a^2)(2m^2)(-4\bar{B}_0), \\ \delta Z_A^Z(m^2, m^2, M_Z^2) &= 2g_v g_a(2B_1 + 1), \\ \delta Z_V^W(m^2, m_b^2, M_W^2) &= \frac{g^2}{4}[(2B_1 + 1) + 2m^2(-2\bar{B}_1)], \\ \delta Z_A^W(m^2, m_b^2, M_W^2) &= \frac{g^2}{4}(2B_1 + 1).\end{aligned}\tag{2.11}$$

The superscripts $H, 0, +, Z, W$ stand for contributions from the diagrams with H, G^0, G^+, Z, W^+ , while $g_v = \frac{g}{4 \cos \theta_W}(1 - 8 \sin \theta_W/3)$, $g_a = \frac{g}{4 \cos \theta_W}$ and θ_W is the

weak-mixing angle. The derivative of the two-point function is defined as

$$\bar{B}_{0,1} = \frac{\partial}{\partial p^2} B_{0,1}(p^2, m_1^2, m_2^2)|_{p^2=m^2}. \quad (2.12)$$

The Vertex Corrections

The one-loop vertex corrections from the spin-0 bosons are

$$\begin{aligned} A^H(m^2, m^2, s, m^2, m_H^2, m^2) &= (m^2/v^2)[-1/2 + 2C_{24} + m^2(3C_0 - 2C_{11} - C_{21} - C_{22}) \\ &\quad + (s - 2m^2)(-C_{12} - C_{23})] + \delta Z_V^H, \\ C^H(m^2, m^2, s, m^2, m_H^2, m^2) &= (m^2/v^2)[m(-C_0 + C_{21} + C_{22} - 2C_{23})], \\ A^0(m^2, m^2, s, m^2, M_Z^2, m^2) &= (m^2/v^2)[-1/2 + 2C_{24} + m^2(-C_0 - 2C_{11} - C_{21} - C_{22}) \\ &\quad + (s - 2m^2)(-C_{12} - C_{23})] + \delta Z_V^0, \\ C^0(m^2, m^2, s, m^2, M_Z^2, m^2) &= (m^2/v^2)[m(C_0 + 2C_{11} - 2C_{12} + C_{21} + C_{22} - 2C_{23})], \\ A^+(m^2, m^2, s, m_b^2, M_W^2, m_b^2) &= (m^2/v^2)[-1/2 + 2C_{24} + m_b^2 C_0 + m^2(-2C_{11} - C_{21} - C_{22}) \\ &\quad + (s - 2m^2)(-C_{12} - C_{23})] + \delta Z_V^+, \\ C^+(m^2, m^2, s, m_b^2, M_W^2, m_b^2) &= (m^2/v^2)[m(C_{11} - C_{12} + C_{21} + C_{22} - 2C_{23})], \\ B^+(m^2, m^2, s, m_b^2, M_W^2, m_b^2) &= -(m^2/v^2)[-1/2 + 2C_{24} + m_b^2 C_0 + m^2(-2C_{12} - C_{21} - C_{22}) \\ &\quad + (s - 2m^2)(-C_{12} - C_{23})] + \delta Z_A^+, \\ F^+(m^2, m^2, s, m_b^2, M_W^2, m_b^2) &= -(m^2/v^2)[m(C_{11} + 3C_{12} + C_{21} + C_{22} + 2C_{23})]. \end{aligned} \quad (2.13)$$

The corrections from the gauge bosons are

$$\begin{aligned}
A^Z(m^2, m^2, s, m^2, M_Z^2, m^2) &= 2(g_v^2 + g_a^2)[-1 + 2C_{24} + m^2(C_0 - 2C_{12} - C_{21} - C_{22}) \\
&\quad + (s - 2m^2)(-C_{11} - C_{23})] + \delta Z_V^Z, \\
C^Z(m^2, m^2, s, m^2, M_Z^2, m^2) &= 2(g_v^2 + g_a^2)[m(-C_{11} + C_{12} + C_{21} + 2C_{22} - 2C_{23})] \\
&\quad + 2(g_v^2 - g_a^2)[m(+2C_{11} - 2C_{12})], \\
B^Z(m^2, m^2, s, m^2, M_Z^2, m^2) &= 4g_v g_a[-1 + 2C_{24} + m^2(C_0 - 2C_{11} - C_{21} - C_{22}) \\
&\quad + (s - 2m^2)(-C_{11} - C_{23}) + \delta Z_A^Z], \\
F^Z(m^2, m^2, s, m^2, M_Z^2, m^2) &= 4g_v g_a[m(3C_{11} + C_{12} + C_{21} + C_{22} + 2C_{23})], \\
A^W(m^2, m^2, s, m_b^2, M_W^2, m_b^2) &= \frac{g^2}{2}[-1 + 2C_{24} + m_b^2 C_0 + m^2(-2C_{12} - C_{21} - C_{22}) \\
&\quad + (s - 2m^2)(-C_{11} - C_{23})] + \delta Z_V^W, \\
C^W(m^2, m^2, s, m_b^2, M_W^2, m_b^2) &= \frac{g^2}{2}[m(-C_{11} + C_{12} + C_{21} + C_{22} - 2C_{23})], \\
B^W(m^2, m^2, s, m_b^2, M_W^2, m_b^2) &= \frac{g^2}{2}[-1 + 2C_{24} + m_b^2 C_0 + m^2(-2C_{11} - C_{21} - C_{22}) \\
&\quad + (s - 2m^2)(-C_{11} - C_{23})] + \delta Z_A^W, \\
F^W(m^2, m^2, s, m_b^2, M_W^2, m_b^2) &= \frac{g^2}{2}[m(3C_{11} + C_{12} + C_{21} + C_{22} + 2C_{23})].
\end{aligned} \tag{2.14}$$

The form factors of the g - t - $\bar{t}$ vertex with full SM weak radiative corrections can be expressed as

$$\begin{aligned}
A &= A^H + A^0 + A^+ + A^Z + A^W, \\
B &= B^+ + B^Z + B^W, \\
C &= C^H + C^0 + C^+ + C^Z + C^W, \\
F &= F^+ + F^Z + F^W, \\
D &= E = 0.
\end{aligned} \tag{2.15}$$

Again, note that $D = 0$ in theories that conserve CP and that vector current conservation demands that $E = 0$ and $B = -sF/4m$.

3. Numerical Results

3.1 GENERAL

It has been shown that the modifications of the g - t - $\bar{t}$ vertex due to the weak corrections given by Eqs. (2.9) and (2.15) appear as finite modifications to the form factors. In particular, the values of E and F are irrelevant for the $q\bar{q} \rightarrow t\bar{t}$ process considered, since these terms vanish when the momentum factor in front of them couples to the annihilation vertex for the massless quarks in the initial state. With the corrections written in the manner of Eq. (2.9), it becomes feasible to use helicity amplitude techniques for computing the loop effects in $t\bar{t}$ production. [13] In particular, this allows us to preserve the polarization information. Since the form factors are energy dependent, we give in Fig. 2 (for $m_H = 100, 1000$ GeV) the various form factors of the g - t - $\bar{t}$ vertex predicted from the SM as a function of $M(t\bar{t})$ for a 180 GeV top quark.

In Fig. 2 we also show the corrections due to the effective theory, hereafter called the Yukawa contributions. We find that the Yukawa contributions to the form factors B and F , which violate parity, are larger (in absolute magnitude) than those from the full SM weak corrections (called TOTAL). This is due to the cancellation between the pure $O(g^2)$ and $O(\frac{m_t^2}{v^2})$ contributions in TOTAL. In the 't Hooft–Feynman gauge, this cancellation occurs because contributions from the diagrams with virtual G^+ and W^+ interfere destructively.

To compare our results with those in the literature, we show in Figs. 3 (for the effective theory) and 4 (for the SM weak corrections) the fractional change in the constituent cross section $\hat{\sigma}(q\bar{q} \rightarrow t\bar{t})$ as a function of $M(t\bar{t})$ for $m_t = 180$ GeV and $m_H = 100$ or 1000 GeV. In Fig. 3, the individual contributions from the Higgs boson H and the unphysical Goldstone bosons G^+ and G^0 are also shown separately. Our results agree with those in Ref. 6 for the same m_t and m_H . In addition to the Yukawa contributions, we show in Fig. 4 the contributions labeled as the ISRC (initial state radiative weak correction), the FSRC (final state radiative weak correction, including the Yukawa contributions), and the TOTAL (complete SM weak correction). The results of our TOTAL agree with those in Ref. 10. From Fig. 4 we learn that at large $M(t\bar{t})$, the FSRC dominates the radiative weak corrections, *i.e.*, the Yukawa contributions of $O(\frac{m_t^2}{v^2})$ do not dominate the pure weak corrections of $O(g^2)$ in the high energy region. This is because FSRCs contain large logarithms of $M^2(t\bar{t})/M_V^2$, [6] where M_V is the mass of the W or Z bosons. At small $M(t\bar{t})$, the Yukawa contributions dominate for a light Higgs boson, but become less important if the SM Higgs boson is heavy. This can be understood from the results in Figs. 3 and 4.

Since the contributions from the ISRCs are generally small, they may be ignored. Henceforth, we describe our results in terms of the form factors of the g - t - $\bar{t}$ vertex and only include the contributions from the FSRCs in our numerical results.

In Figs. 5 and 6 we present the variation of the hadron level distribution $d\sigma/dM(t\bar{t})dy$ with the subprocess center-of-mass energy at a fixed rapidity of $y = 0$ for the Fermilab Tevatron and the LHC for $m_t = 180$ GeV and $m_t = 150$ GeV, respectively. We use the parton distribution functions of Morfin and Tung[20] (set SL) using a scale of $Q = \sqrt{s}$. This same scale, $\sqrt{s} = M(t\bar{t})$, which is the center-of-mass energy of the $t\bar{t}$ pair, is also used in evaluating the strong coupling constant. We note that at the LHC the dominant production mechanism for the $t\bar{t}$ pairs is $gg \rightarrow t\bar{t}$, even for $t\bar{t}$ masses as high as 10 TeV. Nevertheless, in the invariant mass region about or above 1 TeV, $q\bar{q}$ annihilation has to be considered as a background to the study of electroweak symmetry breaking.[21]

Definition of K -Factor

Denoting the higher order cross section at the parton level as $\hat{\sigma}^{H.O.}$, we define the K -factor,

$$\hat{K}(s, m_t, m_H) \equiv \frac{d\hat{\sigma}^{H.O.}}{ds} / \frac{d\hat{\sigma}^{Born}}{ds}, \quad (3.1)$$

which quantifies the weak corrections to the parton cross section at the Born level, $\hat{\sigma}^{Born}$. When only considering FSRCs, like those for our s -channel process $q\bar{q} \rightarrow t\bar{t}$, we find that this K -factor is valid also at the hadron level for all system rapidities,

$$y = \frac{1}{2} \ln \frac{x_a}{x_b}, \quad (3.2)$$

given our choice of scale, where x_a, x_b are the fractions of momenta that the q and $\bar{q}$ take from their parent hadrons.

The hadron level K -factor is defined as

$$K(s, m_t, m_H) \equiv \frac{d\sigma^{H.O.}}{ds} / \frac{d\sigma^{Born}}{ds}. \quad (3.3)$$

Eq. (3.3) differs from Eq. (3.1) in that the ratio is with regard to the hadronic differential cross sections, which are simply the parton differential cross sections convoluted with the parton distribution functions,

$$\sigma = \sum_{a,b} \int dx_a dx_b f_{a/A}(x_a, Q) f_{b/B}(x_b, Q) \hat{\sigma}(a + b \rightarrow t + \bar{t}), \quad (3.4)$$

where $f_{a/A}(x_a, Q)$ provides the density for partons of flavor a carrying momentum fraction x_a of the total momentum of hadron A . As with the strong coupling, the

scale Q in the parton densities has been set to $\sqrt{s}$. After converting the $dx_a dx_b$ integrals over the parton momentum fractions to $dy ds/S$ integrals in Eq. (3.4), where $S = s/x_a x_b$ is the center-of-mass energy of the hadron-hadron system, the parton cross section factors outside the dy integration because the parton level cross section depends only on s and masses. (It is implied that the integration over top quark polar angle has been performed and recall we are only considering the FSRCs for these K -factors.) Computing the integral over dy yields a cancellation of the contribution from the parton distribution functions between the numerator and the denominator in Eq. (3.3), such that the parton level K -factor of Eq. (3.1) is the same as the analogous hadron level K -factor of Eq. (3.3).

This is independent of whether the initial hadrons are protons or antiprotons. So, the parton level K -factors presented may be considered as the hadron level K -factors at the Fermilab Tevatron,

$$\hat{K}(s, m_t, m_H) = K_{FNAL}(s, m_t, m_H) \equiv \frac{d\sigma_{FNAL}^{H.O.}}{ds} / \frac{d\sigma_{FNAL}^{Born}}{ds}, \quad (3.5)$$

and at the LHC,

$$\hat{K}(s, m_t, m_H) = K_{LHC}(s, m_t, m_H) \equiv \frac{d\sigma_{LHC}^{H.O.}}{ds} / \frac{d\sigma_{LHC}^{Born}}{ds}, \quad (3.6)$$

where $\sigma^{H.O.}$ and σ^{Born} respectively represent the hadron level cross sections for the production of $t\bar{t}$ pairs through quark-antiquark annihilation at the one-loop level and at the tree level. Note that Eq. (3.5) and Eq. (3.6) are true, provided that no kinematic cuts are applied to the t or $\bar{t}$.

One of the advantages of computing the higher order corrections through a modification of the g - t - $\bar{t}$ form factors is that we can conveniently implement the corrections at the amplitude level and examine the consequent changes in the production of polarized top quarks. The higher order effects vary somewhat when we compare the production of unpolarized $t\bar{t}$ pairs to the production of polarized final states. The results for polarized top quarks are given in Figs. 7–9.

Magnitude of Results

In general the higher order weak effects in $q\bar{q} \rightarrow t\bar{t}$ yield only a small correction of a few percent to the cross section. In Figs. 7 and 8 the K -factors that describe these corrections are greater than unity near the threshold region for a light Higgs boson, reaching magnitudes around 1.08 (1.04) for $m_t = 180$ (150) GeV and $m_H = 100$ GeV; a drop in value occurs as we go to subprocess energies of 3 TeV. For the production of $t\bar{t}$ pairs in the threshold region given a heavy Higgs boson (see Fig. 9), we find a small decrease in rate yielding K -factors just under unity. Despite the

large size of the K -factor for the lighter Higgs boson when $M(t\bar{t})$ is extremely close to the mass threshold of producing the $t\bar{t}$ pair, the effect on the total cross section is small because of the suppression of the phase space indicated in Figs. 5 and 6. At subprocess energies far from threshold, the event rate for top quark production is smaller (by about a ten percent at $M(t\bar{t}) \approx 1$ TeV for unpolarized $t\bar{t}$ pairs); nevertheless, it is useful to know that the weak corrections cause a decrease in the event rate for large invariant masses, $M(t\bar{t})$, of the $t\bar{t}$ pairs because it is in the high invariant mass region at the LHC that the top quark is a background to signals needed to study the electroweak symmetry breaking sector (given that no light Higgs boson is found). [21]

The K -factor has a dependence on both the mass of the top quark and the mass of the Higgs boson. The general outcome of an increase in top quark mass from 150 GeV to 180 GeV is that $|1 - K|$ is slightly larger for the heavier quark either near threshold or at high $M(t\bar{t})$. If we fix m_H at either 100 GeV or 1 TeV while changing the top quark mass from 150 GeV to 180 GeV, we find the K -factor deviation from unity behaves similarly for the two m_t values. This is more explicitly demonstrated in Fig. 10 over the mass range $150 < m_t < 250$ GeV for $m_H = 100$ GeV. From the perspective of fixed m_t , we also see the variation in the K -factor between $m_H = 100$ GeV and $m_H = 1$ TeV is steeper at low $M(t\bar{t})$ for the lighter Higgs boson mass. As mentioned previously, in the limit that the mass of the Higgs boson is taken to infinity, m_H decouples from the renormalized vertex function Γ_μ^R in Eqs. (2.9) and (2.15).

3.2 NEW EFFECTS THAT DID NOT APPEAR AT THE TREE LEVEL

Though the effects of the weak corrections are small when compared to the total cross section, there are conditions where the form factor modifications can yield a relatively significant change. In particular, because the form factors become complex, there are polarization asymmetries which develop nonzero values in contradistinction to their Born level counterparts. Such is the case when considering inclusive cross sections for top quark production where the polarization of the observed top quark is transverse to the scatter plane.

As was discussed in Ref. 13, the Born level amplitudes for $q\bar{q} \rightarrow t\bar{t}$ are all real. For this reason, a single particle asymmetry is zero when considering the transverse polarization perpendicular to the scatter plane. [22] In computing the higher order corrections, however, an imaginary portion is generated in the form factors that takes the polarization perpendicular to the scatter plane to nonzero values. These nonzero values can in principle be used to test for CP violation effects. [13]

The radiative corrections computed here make the helicity amplitudes complex in value. This produces a nonzero value for the polarization of single top quark

spins directed perpendicular to the scatter plane, $P_{\perp}$. Let's define

$$P_{\perp} \equiv \frac{d\hat{\sigma}^{\uparrow}/d\cos\theta - d\hat{\sigma}^{\downarrow}/d\cos\theta}{d\hat{\sigma}^{\uparrow}/d\cos\theta + d\hat{\sigma}^{\downarrow}/d\cos\theta}, \quad (3.7)$$

where $\hat{\sigma}^{\uparrow}$ ($\hat{\sigma}^{\downarrow}$) describes the parton level cross section when the transverse spin of the top quark is pointing “up” (“down”) with respect to the scatter plane. (Think of “up” as the $+Y$ direction given that the $q\bar{q} \rightarrow t\bar{t}$ hard scattering occurs in the $X-Z$ plane with the initial quark moving in the $+Z$ direction and the top quark carrying a positive value for its X component of momentum.) Though $P_{\perp}$ carries nonzero values, numerical results indicate that the polarization for single top quark spins directed perpendicular to the scatter plane due to the weak corrections considered here are small, yielding $P_{\perp}$ values not much larger than 10^{-3} for $m_t = 180$ GeV. (Recall, one-loop QCD corrections provide $P_{\perp}$ values of a couple percent for $t\bar{t}$ pairs produced from gluon fusion. [13, 23]) In Fig. 11, we present curves of $P_{\perp}$ vs. $\cos\theta$, where θ is the angle the top quark subtends with the incoming quark in the center-of-mass frame of the subprocess. We found an interesting qualitative change in the plots as the Higgs boson mass increases. For the lower Higgs boson masses (around 100 GeV) the polarization is positive for $\cos\theta < 0$ while for the higher Higgs boson masses (around 1 TeV) the polarization for $\cos\theta < 0$ is negative. Such quantities are mainly relevant for proton-antiproton collisions, where a convenient means is available for determining “up” and “down” directions with respect to the scatter plane by defining the scatter plane with the proton and antiproton beams as opposed to the annihilating quark and antiquark. Although the $P_{\perp}$ plots exhibit this interesting feature, it might be extremely difficult to measure this polarization because of its small value in the SM.

Another new effect which is absent at the tree level is the double polarization asymmetry $P_{\perp}(in, out)$ which is also sensitive to higher order corrections. [24] Specifically, $P_{\perp}(in, out)$ refers to the asymmetry produced when the top quark spin is perpendicular to the scatter plane while the transverse spin of the top antiquark is in the scatter plane. This quantity, analogous to Eq. (3.7), is zero at the Born level and only achieves its nonzero value because of the imaginary correction to the form factors. Analogous to the single spin asymmetry presented for $P_{\perp}$, $P_{\perp}(in, out) \sim 10^{-3}$ for $m_t = 180$ GeV.

So, though we can say we have an effect with $K = \infty$, the statistics are too poor for any reasonable study in the SM.

3.3 K -FACTORS FOR SPIN EFFECTS PRESENT AT THE BORN LEVEL

Because QCD is C and P invariant, the single particle polarization of the top quark has to vanish at the tree level for the process $q\bar{q} \rightarrow t\bar{t}$. Nevertheless, the top quark can have a single particle polarization if weak effects are present in their production. The effects of top quark polarization for the Born level electroweak reaction $q\bar{q} \rightarrow (\gamma, Z) \rightarrow t\bar{t}$ were discussed in Ref. 26. The contribution of this process to the total cross section for the production of $t\bar{t}$ pairs is small (at the percent level for the Tevatron), so any spin effects present in $q\bar{q} \rightarrow (\gamma, Z) \rightarrow t\bar{t}$ are diluted by the QCD production of $t\bar{t}$ pairs. Considering the larger rate for the QCD production of $t\bar{t}$ pairs, similar spin effects that appear when considering the degree of polarization due to the weak corrections to $q\bar{q} \rightarrow g \rightarrow t\bar{t}$ at the loop level can be more significant. The degree of polarization for a single top quark due to weak corrections may be obtained from the curves (a) in Figs. 7–9. Typically, this effect is of the order of a few percent for large $M(t\bar{t})$.

Besides examining single particle polarizations, there are also double particle asymmetries in the spin dependence which can be investigated. In the following, we consider the longitudinal and transverse spins of the top quark and top antiquark.

Longitudinal Spins

When considering the longitudinal polarizations for the top quarks, we classify the states as carrying either right-handed (R) or left-handed (L) helicity. In the figures and the text the correlated spin states for the $t\bar{t}$ pairs are labelled either RR, RL, LR, LL, where the first letter is the top quark helicity and the second letter is the top antiquark helicity. Since the interactions described by Eqs. (2.9) and (2.15) conserve CP, the higher order corrections make no distinction between the RR and LL states because they are CP transforms of each other. For all cases of m_H and m_t considered, the $|1 - K|$ value for the LR state is larger than that for the RL state when the K -factors for both of these helicity combinations are below unity. The reverse is true when these K -factors are above unity. When m_H becomes larger, the $|1 - K|$ values of the RR, LL spin states become smaller, as shown in Figs. 7 and 9.

The difference in the production rates for LR and RL derives from the parity violation in the weak interactions. The effect of this parity violation can be quantified using the variable

$$\mathcal{A} = \frac{\hat{\sigma}(RL) - \hat{\sigma}(LR)}{\hat{\sigma}(RL) + \hat{\sigma}(LR)},$$

which is zero for parity conserving interactions like those of QCD. For the one-loop, SM, weak corrections in $q\bar{q} \rightarrow g \rightarrow t\bar{t}$, $\mathcal{A}$ is approximately 1 – 2% at $M(t\bar{t}) \approx$

1 TeV and continues to grow slowly with increasing $M(t\bar{t})$ for the values of m_t and m_H considered. Therefore, any large parity violation observed would indicate new physics.

As discussed in Sec. 3.1, the parity violating form factors B and F , as defined in Eq. (2.9), are smaller after including the weak corrections of $O(g^2)$ to the pure Yukawa contributions of $O(\frac{m_t^2}{\Lambda^2})$. This is due to the cancellations between these two contributions. Hence, it is important to include $O(g^2)$ weak corrections when discussing the parity violation effects in the $q\bar{q} \rightarrow t\bar{t}$ process.

Transverse Spins

Though we do not present plots of the K -factors when both the t and $\bar{t}$ quarks are polarized transversely to their direction of motion, we discuss some of the results here. We consider transverse spins for the top quark and antiquark to be either perpendicular to the scatter plane or within it. Since both quarks are considered simultaneously, the t and $\bar{t}$ spins are further classified as being either aligned or antialigned.

As guided by the unpolarized results, for $m_t = 180$ GeV and $m_H = 100$ GeV the K -factor is about 1.08 near the threshold for the production of top quarks for any top quark polarization. For the case where both $t, \bar{t}$ spins are perpendicular to the scatter plane, the K -factor decreases to about $K = 0.93$ as we move to $M(t\bar{t}) \approx 1$ TeV. For the lighter top quark ($m_t = 150$ GeV), the K -factor lies only slightly closer to unity. Considering the case where the transverse spins for the two top quarks live in the scatter plane, the K factor again drops to around $K = 0.93$ for $m_t = 180$ GeV at $M(t\bar{t}) \approx 1$ TeV. No matter whether $m_t = 150$ GeV or $m_t = 180$ GeV, the configuration where the transverse spins are aligned receives more suppression from the high order corrections than the configuration where the two top quark spins are antialigned for $M(t\bar{t}) > 2$ TeV.

4. Polarization and Top Quark Mass Measurements

In Ref. 14 the most effective method considered for measuring the mass of the top quark concentrated on the analysis of the invariant mass distribution, $M(e\mu)$, which is determined from the combined momentum of the charged e^+ lepton from the decay $t \rightarrow b e^+ \nu$ and the muon from the fragmentation of the bottom quark. An error of about 1.6% was estimated in the determination of the top quark mass for $m_t = 150, 250$ GeV using a series of kinematic cuts on the unpolarized production of top quarks. It is known, however, that if a polarization asymmetry were to develop in the top quark production that the kinematics of the observed particles would change. If there were no kinematic cuts, this would be of no consequence since

integrating out the angular dependence washes out the polarization effects on this measurement; however, with kinematic cuts, as required in reality, a polarization asymmetry can affect the $M(e\mu)$ mass spectrum.[27] With this observation it becomes necessary to investigate the effects such an asymmetry may produce and whether it interferes with the precision of the mass measurement.

We proceed by examining an analogous quantity, namely, the invariant mass of the bottom quark and charged lepton from the top quark decay. Respectively denoting the momenta of the e^+ , ν , b quark, W -boson, and t quark as p_e , p_ν , p_b , p_W , p_t , the amplitude squared for the three-body decay $t \rightarrow bW^+ \rightarrow be^+\nu$ is given by [25]

$$|\mathcal{M}|^2 = \frac{64G_F^2 M_W^4}{(p_W^2 - M_W^2)^2 + M_W^2 \Gamma_W^2} (p_\nu \cdot p_b) [(p_e \cdot p_t) - m_t(p_e \cdot \zeta)], \quad (4.1)$$

where G_F is the Fermi coupling constant and ζ describes the polarization of the top quark.[26] The spin averaged amplitude is obtained by setting $\zeta = 0$ in Eq. (4.1). The neutrino and positron have been taken as massless and the masses of the top quark, W boson and bottom quark are respectively given by m_t , M_W , m_b . With the conventions chosen, the top quark decay rate is given by

$$d\Gamma_t = \frac{1}{2m_t} |\mathcal{M}|^2 d\Phi_3, \quad (4.2)$$

where the three-body phase space is

$$d\Phi_3 = \frac{1}{32m_t^2(2\pi)^5} dM(eb)^2 dM_W^2 d\Omega_b d\phi_\nu^* \Theta[-G(M(eb)^2, p_W^2, m_t^2, 0, m_b^2, 0)] \quad (4.3)$$

with

$$G(M(eb)^2, p_W^2, m_t^2, 0, m_b^2, 0) = 2p_W^2 [m_b^2 M(eb)^2 - M(eb)^4 - M(eb)^2 p_W^2 - m_b^2 m_t^2 + M(eb)^2 m_t^2]. \quad (4.4)$$

We choose to perform the calculation in the rest frame of the top quark. The angular dependence in the phase space factor of Eq. (4.3) is comprised of the differential for the solid angle of the bottom quark, $d\Omega_b = d\cos\theta_b d\phi_b$, and $d\phi_\nu^*$, which we choose to be the azimuthal angle of the neutrino measured from the coordinate system that is rotated such that the bottom quark momentum defines the z -axis.

For the decay of unpolarized top quarks, Eq. (4.1) indicates that there is no angular dependence to the $M(eb)$ distribution. The phase space integration may

be easily performed in the narrow width approximation yielding the unpolarized decay distribution,

$$\frac{d\Gamma_t}{dM(eb)^2} = \frac{G_F^2}{16\pi^2\Gamma_W} \left(\frac{M_W}{m_t}\right)^3 [m_t^2 - M_W^2 - M(eb)^2](M_W^2 - m_b^2 + M(eb)^2). \quad (4.5)$$

It is also clear from Eq. (4.1) that for polarized top quark decay there is a spin component that contributes to the behavior of the $M(eb)$ distribution, and this term does have an angular dependence. To demonstrate the effect of the spin-dependent term in Eq. (4.1), we plot the $M(eb)$ distribution for $q\bar{q} \rightarrow t\bar{t} \rightarrow be^+\nu\bar{b}q_1\bar{q}_2$ at the LHC in Fig. 12, separating the contributions for left-handed and right-handed helicities of the top quark. These curves were created for top quarks of mass 180 GeV by restricting the rapidities of the e^+ and b quark to within 2.5 in magnitude and insisting that the transverse momentum for each of these two particles be greater than 40 GeV for top quark production via $q\bar{q} \rightarrow g \rightarrow t\bar{t}$ at the LHC. (In our result we also impose the same rapidity and transverse momentum cuts for the $\bar{b}$, q_1 , $\bar{q}_2$.) A difference in the two curves for pure helicity states, created solely by the kinematic constraints, is realized mainly in the low $M(eb)$ region. To understand how this affects the $M(e\mu)$ mass distribution (where μ comes from the fragmentation of the bottom quark) and the measurement of the top quark mass, one has to convolute our results with the hadronization of the bottom quark to produce the muon. This is beyond the scope of this paper but has been performed for hadron collision energies of 40 TeV in Ref. 27.

5. Conclusion

We have computed the leading weak correction of $O(\frac{m_t^2}{s})$ for $q\bar{q} \rightarrow t\bar{t}$ and compared it with the results that include the complete one-loop weak correction. We learned that at large $M(t\bar{t})$, the Yukawa contributions of $O(\frac{m_t^2}{s})$ did not dominate the pure weak corrections of $O(g^2)$ in the high energy region. At small $M(t\bar{t})$, the Yukawa contributions dominated for a light Higgs boson, but became less important for a heavy SM Higgs boson.

We found the complete weak corrections provided only a very slight change in the total cross section of no more than a few percent, which is smaller than the typical uncertainty in the prediction of the top quark event rate in the usual QCD processes.

A decrease appeared in $d\hat{\sigma}/dM(t\bar{t})$ of about ten percent compared to the lowest order result for subprocess center-of-mass energies around 1 TeV. The perturbative results indicate an increase in the K -factor just under 10% near the threshold region

for the $m_t = 180$ GeV and $m_H = 100$ GeV values considered here. These results do not include any relevant nonperturbative physics, [8] but the effect on the total cross section is small because of the suppression of the phase space at threshold. With $m_H = 1$ TeV all helicity combinations for the final state were suppressed, though the K -factor was close to unity near threshold. At large $M(t\bar{t})$ the K -factor is rather insensitive to m_H .

Small transverse polarizations were obtained from the imaginary contributions to the form factors (generated by the loop corrections), as we found that the polarization when considering solely the spin of the top quark perpendicular to the scatter plane was about 10^{-3} .

The parity violation due to the electroweak couplings appears in the K -factors for the production of polarized $t\bar{t}$ pairs, where for $m_H = 100$ GeV the RL states generally received a larger K -factor enhancement near threshold than that for the production of LR states, while the K -factor suppression the LR states received in the TeV region was greater than the suppression for the production of RL states. The effects of parity violation appear in the ratio

$$\mathcal{A} = \frac{\hat{\sigma}(RL) - \hat{\sigma}(LR)}{\hat{\sigma}(RL) + \hat{\sigma}(LR)},$$

which is zero for parity conserving interactions like those of QCD and QED, but for the corrections we have computed, $\mathcal{A}$ is approximately 1 – 2% at $M(t\bar{t}) \approx 1$ TeV. Any large parity violation observed would therefore indicate new physics.

We also showed that the invariant mass spectrum of $M(eb)$ (and hence $M(e\mu)$ from the decay $t \rightarrow bW^+ \rightarrow be^+\nu$) depends on the polarization of the top quark when kinematic constraints are invoked. (μ comes from the fragmentation of the bottom quark b .) To ensure that $M(e\mu)$ is a good variable for measuring the mass of the top quark, one has to understand the effect of the top quark polarization when performing the analysis.

In conclusion, we find in $q\bar{q} \rightarrow t\bar{t}$ that the Yukawa contributions of $O(\frac{m_t^2}{v^2})$ are inadequate by themselves for an accurate description of either the cross section at large $M(t\bar{t})$ with a light m_H or the effects of parity violation over all invariant mass; the pure weak corrections of $O(g^2)$ must be included. This situation where the Yukawa contributions do not dominate the weak contributions of $O(g^2)$ for a heavy Higgs boson may also occur in other processes, so when using such an effective model (taking the gauge coupling $g \rightarrow 0$ limit), one must be cautious about interpreting such results as representative of the full theory.

Acknowledgements

We would like to thank Ansgar Denner, Michael Peskin, Xerxes Tata, Mark Wise and Scott Willenbrock for beneficial discussions and comments. C.K. was supported in part by the U. S. Department of Energy under contract #DE-FG05-87ER40319. The work of G.A.L. and C.P.Y. was supported in part by TNRLC grant #RGFY9240 and NSF grant #PHY-9309902.

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FIGURE CAPTIONS

1. Diagrams contributing to (a) the top quark self energy and wave function renormalization and (b) the weak corrections to the g - t - $\bar{t}$ vertex. Both momenta p and q are defined as flowing out of the vertex.
2. Real part of the form factors (a) A and B, (b) C and F, which are defined in Eqs. (2.9) and (2.15) as a function of $M(t\bar{t})$ for $m_t = 180$ GeV with $m_H = 100$ GeV and $m_H = 1000$ GeV; (c) separate contributions to the real part of the form factor B. Note that the contribution from G^+ is largely cancelled by that from W^+ in deriving TOTAL.
3. Yukawa corrections to the cross section of $q\bar{q} \rightarrow g \rightarrow t\bar{t}$, $(\Delta\hat{\sigma}/\hat{\sigma})$ vs. $M(t\bar{t})$, for $m_t = 180$ GeV with (a) $m_H = 100$ GeV or (b) $m_H = 1000$ GeV. The contributions from the Higgs bosons (dashed), the neutral Goldstone boson (dot-dash), the charged Goldstone boson (dot-dot-dash) and the total Yukawa corrections (solid) are presented separately.
4. Weak corrections to the cross section of $u\bar{u} \rightarrow g \rightarrow t\bar{t}$, $(\Delta\hat{\sigma}/\hat{\sigma})$ vs. $M(t\bar{t})$, for $m_t = 180$ GeV with (a) $m_H = 100$ GeV and (b) $m_H = 1000$ GeV. The contributions from the Yukawa corrections (dashed), the initial state radiative corrections (dot-dash), the final state radiative corrections (dot-dot-dash) and the total weak corrections (solid) are presented separately.
5. Taking $m_t = 180$ GeV, we plot the distribution $[d\sigma/dM(t\bar{t})dy]_{y=0}$ vs. $M(t\bar{t})$ from the Born level QCD subprocess $q\bar{q} \rightarrow t\bar{t}$ in (a) proton-antiproton collisions at the Tevatron with an energy of 2.0 TeV and (b) proton-proton collisions for the LHC with an energy of 16 TeV.
6. These figures (a) and (b) are the same as Fig. 5, except that $m_t = 150$ GeV.
7. Taking $m_t = 180$ GeV and $m_H = 100$ GeV, we plot the K -factor of Eq. (3.5) and Eq. (3.6) against the subprocess center-of-mass energy $M(t\bar{t})$ considering (a) t helicity states with an unpolarized $\bar{t}$ (R indicates right-handed t , L indicates a left-handed t); (b) t and $\bar{t}$ helicity states (RL indicates right-handed t , left-handed $\bar{t}$, etc.) .
8. These figures (a)–(b) are the same as Fig. 7, except that $m_t = 150$ GeV.
9. These figures (a)–(b) are the same as Fig. 7, except that $m_H = 1$ TeV.
10. This plot shows the variation of the K factor with the mass of the $t\bar{t}$ pair ($M(t\bar{t})$) using $m_t = 150, 200, 250$ GeV and $m_H = 100$ GeV in the unpolarized production rates.
11. Taking $m_t = 180$ GeV and $\sqrt{s} = 500$ GeV, we plot the single particle asymmetry $P_\perp$ as described by Eq. (3.7) when the top quark spin is perpendicular

to the scatter plane against $\cos \theta$. The two curves represent the asymmetry for two different values of the Higgs boson mass.

12. These two curves represent the distribution $d\sigma/dM(eb)$ vs. $M(eb)$ at the LHC for right-handed and left-handed top quark helicities using $m_t = 180$ GeV and $m_b = 0$. Kinematic constraints in the lab frame on the rapidity ($|\eta| < 2.5$) and transverse momentum ($p_T > 40$ GeV) were imposed for the quarks and visible leptons from the t and $\bar{t}$ decays.

(a)

(b)

Figure 1

Figure 2 (a)

Figure 2(b)

Figure 2(c)

Figure 3(a)

Figure 3(b)

Figure 4(a)

Figure 4(b)

Figure 5(a)

Figure 5(b)

Figure 6(a)

Figure 6(b)

Figure 7(a)

Figure 7(b)

Figure 8(a)

Figure 8(b)

Figure 9(a)

Figure 9(b)

Figure 10

Figure 11

Figure 12