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EFFECT OF RADIATIVE RECOMBINATION IN THERMONUCLEAR FUSION RATES¹

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Abstract We show that radiative recombination in dense astrophysical plasma, like the solar interior, can give rise to Coulomb screening effects which affect the thermonuclear reaction rates, and depend on the nuclear electron screening factor, temperature and electron density in the plasma.

Thermonuclear reactions in stars and of course in the sun are affected by electronic screening [1]. This effect, studied also in laboratory nuclear reaction experiments, is described by different authors using various conceptual approaches but a standard treatment, particularly in the case of dense plasma, has not yet been established [2-4]. In the field of electron screening, we want to call the attention on a particular shielding mechanism, which so far has been neglected, which is due to radiative recombination processes [5] in the astrophysical plasma.

We now give an example to explain our previous statement. Let us suppose we have a one-component neutral plasma of electrons and protons in the stellar interior. These protons and electrons recombine radiatively, forming unstable neutral hydrogen atoms of lifetime τ , and this instability is due to the fact that the plasma temperature is higher than the binding energy of such atoms. Before one of these atoms dissociates, a fusion process between the atom and a proton or another such atom can occur, and in this case, the reaction rate is enhanced compared to the case of two bare protons.

We address the question of the importance of this enhancement and the physical conditions that have influence on it. Let us now consider the more general case of a neutral plasma composed by two species of nuclei, a and b , and electrons e^- . We define the fusion rate F_1 of bare nuclei as

$$F_1 = \frac{1}{1 + \delta_{a,b}} n_a n_b < \sigma_{ab} v > , \quad (1)$$

where n_a and n_b are the densities of the nuclei a and b , σ_{ab} is the fusion cross section and v is the relative velocity and $\delta_{a,b}$ is Kronecker's delta. If one takes into consideration the radiative recombination of the nuclei with the electrons in the plasma

$$a + e^- \rightarrow (a - e^-)_{atom} + \gamma , \quad (2)$$

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then one obtains a fusion rate F_2 given by

$$F_2 = \frac{1}{1 + \delta_{a,b}} \left[(n_a - \epsilon_a)(n_b - \epsilon_b) < \sigma_{ab} v > + \epsilon_a \epsilon_b < \sigma_{(ae^-)(be^-)} v > \right. \\ \left. + (n_a - \epsilon_a) \epsilon_b < \sigma_{a(be^-)} v > + \epsilon_a (n_b \epsilon_b) < \sigma_{(ae^-)b} v > \right] , \quad (3)$$

where ϵ_a and ϵ_b are the densities of the recombined atomic systems $(a - e^-)$ and $(b - e^-)$ respectively, $\sigma_{a(be^-)}$ is the cross sections for the fusion of the nucleus a with the atomic system $(b - e^-)$, $\sigma_{(ae^-)b}$ is the fusion cross sections for $(a - e^-)$ and b and $\sigma_{(ae^-)(be^-)}$ is the fusion cross sections for $(a - e^-)$ and $(b - e^-)$. Let us suppose that also in a dense plasma the relation between fusion cross section of bare nuclei and fusion cross section of a bare nucleus with a neutral atom is the following:

$$\sigma_{(ae^-)b} = \sigma_{a(be^-)} = f \sigma_{ab} , \quad (4)$$

where f is the enhancement factor due to the Coulomb screening [6]. If we assume that the screening factor f is independent on the relative velocity, we find that the ratio between F_2 and F_1 is

$$\frac{F_2}{F_1} = 1 + \frac{\epsilon_a}{n_a}(f - 1) + \frac{\epsilon_b}{n_b}(f - 1) + \frac{\epsilon_a}{n_a} \frac{\epsilon_b}{n_b} (f - 1)^2 . \quad (5)$$

We may write the densities ϵ_a and ϵ_b (for temperatures T much higher than the binding energies) in the form

$$\epsilon_{a,b} = n_{a,b} n_{e^-}^{eff(a,b)} < \sigma_R^{a,b} v > \tau_{(a,b)e^-} , \quad (6)$$

where σ_R^a and σ_R^b are the radiative recombination cross sections for the nuclei a and b , $< \sigma_R^a v >$ and $< \sigma_R^b v >$ are the recombination rates (averaged over a Maxwellian distribution or over stationary non-Maxwellian distributions [7]), τ_{ae^-} and τ_{be^-} are the mean lifetimes of the atomic systems $(a - e^-)$ and $(b - e^-)$ respectively, and $n_{e^-}^{eff(a)}$ and $n_{e^-}^{eff(b)}$ are the "effective" densities of electrons available for the nuclei a and b and are given by $n_{e^-}^{eff(a,b)} = Z_{a,b} n_{a,b}$, where $-Z_a e$ is the charge of the nucleus a and $-Z_b e$ is the charge of the nucleus b . An expression for $\epsilon_{a,b}$ in a neutral plasma that is correct also at low temperatures can be derived considering that $\epsilon_{a,b}$ must satisfy the following differential equation:

$$\frac{d\epsilon_{a,b}}{d\tau} = \frac{\epsilon_{a,b}(\infty) - \epsilon_{a,b}}{\tau_0} , \quad (7)$$

where $\epsilon_{a,b}(\infty) = n_{a,b}$ and τ_0 has the expression

$$\tau_0 = \frac{1}{n_{e^-}^{eff(a,b)} < \sigma_R^{a,b} v >} . \quad (8)$$

Solving Eq.(7), we obtain

$$\epsilon_{a,b} = n_{a,b} \left[1 - \exp \left(-n_e^{eff(a,b)} < \sigma_R^{a,b} v > \tau \right) \right] . \quad (9)$$

The lifetime τ of an atomic system at temperature T is given by the well-known formula [6]

$$\tau = \left(\frac{2\pi\hbar^2}{\mu kT} \right)^{3/2} \frac{2J_3 + 1}{(2J_1 + 1)(2J_2 + 1)} \frac{\exp(-Q/kT)}{< \sigma_R v >} , \quad (10)$$

where $\mu \simeq m_e$ is the reduced mass of the atomic system, Q is the ionization energy of the system and is always negative, and J_1, J_2 and J_3 are the spins of the nucleus, the electron and the atomic system respectively. The ionization energy Q , for a system $(a - e^-)$ is given by $Q = -Z_a^2 \alpha^2 m_e c^2 / 2$, where α is the fine-structure constant. Using Eqs. (6),(10) we obtain

$$\frac{\epsilon_{a,b}}{n_{a,b}} = Z_{a,b} n_{a,b} \left(\frac{2\pi\hbar^2}{m_e kT} \right)^{3/2} \left[\frac{2J_3 + 1}{(2J_1 + 1)(2J_2 + 1)} \right]_{a,b} \exp \left(\frac{Z_{a,b}^2 \alpha^2 m_e c^2}{2kT} \right) \quad (11)$$

where the spin part is labeled a or b if it belongs to the atomic system $(a - e^-)$ or $(b - e^-)$. From this last equation it is easily seen that the ratio F_2/F_1 depends only on the temperature T , the screening factor f , the densities n_a and n_b and the charges Z_a and Z_b of the nuclei a and b .

We now want to elucidate our arguments with a few numerical results. Let us consider fusion between two hydrogen atoms or between an hydrogen atom and a proton. If we use for the screening factor the well known expression $\exp(\Delta/kT)$, we find that in the solar core, taking $\Delta \simeq 30$ eV (see ref. [1]), the pure screening enhancement f is about 2%. The effective electron density in the solar core is of the order of 10^{26} cm^{-3} and therefore the evaluation of the ratio F_2/F_1 shows an enhancement, due to the atomic recombination, of the order of few percent. This enhancement decreases very rapidly with increasing temperature and decreasing electron density.

The same approach can be used to evaluate the recombination effect in the fusion of helium isotopes. In this case one has to consider the recombination with one and with two electrons and the lifetime of the neutral or singly ionized He atoms. We have found that the enhancement is slightly larger than in the hydrogen isotopes case. We have used the expression $\exp(\Delta/kT)$ for the screening factor, and this may be criticized [3]. In ref.[3] it is argued that a proton that gives rise to fusion with a H atom from a distance approximately equal to the Bohr radius and not from infinity, and therefore one should expect $f = 1$ in this case. However, we can show that the neutral systems present in the solar core plasma must have dimensions smaller than the free atoms, for instance, the neutral system $(\alpha + 2p + 4e)$ occupies a spherical volume of

radius about one half the Bohr radius. Therefore we argue that one can still use the expression $\exp(\Delta/kT)$ for the screening factor.

In conclusion we would like to remark that, in a high temperature dense plasma, the radiative recombination effect is quite important among the various screening mechanisms that take place and that probably the effect could be even bigger if one was able to consider the collective effects that bring together electrons and ions to form neutral systems [8] (the collective effects could be included by using stationary non-Maxwellian distributions, as discussed in [9]).

It is well known that in the center of the sun the mass density of H is only about 36%, while the mass density of He is about 64%. As a consequence if we fix our attention to the solar core we should consider also the formation of neutral or singly ionized He atoms. In this case one or two electrons can be radiatively captured by the He atoms and we should also consider the fusion of two neutral He atoms or the fusion of singly ionized He with similar systems or with H and protons. In addition we must say that the dimensions of neutral systems in a dense plasma are probably smaller than in normal matter condition, therefore their binding energy is stronger than in the free case and their lifetime are longer than the values we can obtain from Eq.(7). Nevertheless, in this work, we are not interested in a full and complete description of these various processes in the sun interior, rather in calling the attention to the importance of the screening mechanism due to recombination. In this context the preliminary results on the ratio F_2/F_1 we report in this work are probably underestimated and their importance might even be greater than indicated in this contribution.

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