CLASSICAL DISTRIBUTIONS FOR INTERACTING PARTICLES¹

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Abstract We propose a set of non-Maxwellian statistical distributions, stationary solutions to the Fokker-Planck equation, in which the diffusion and drift coefficients are different from the Brownian case (non interacting particles). These distributions, slightly different from the Maxwellian, describe classical interacting particles and, at equal temperature, have depleted tails compared to the Maxwellian one. This feature could be of great interest in many astrophysical problems such as the solar neutrino production.

Statistical distributions are very important in astrophysics and particularly in the calculation of the astrophysical thermonuclear rates [1]. Usually Maxwellian distribution is assumed in such calculation. We show a particular set of stationary solutions to the Fokker-Planck equation [2], slightly different from the Maxwellian, which could be relevant in this context.

Let us consider the three dimensional Fokker-Planck equation [3]:

$$\frac{\partial n(t, \mathbf{v})}{\partial t} = \nabla \{ [\mathbf{J}(t, \mathbf{v}) + \nabla D(t, \mathbf{v})] n(t, \mathbf{v}) + D(t, \mathbf{v}) \nabla n(t, \mathbf{v}) \} \quad (1)$$

In this equation $n(t, \mathbf{v})$ is the occupational number of the particles in the velocity space, $\mathbf{J}(t, \mathbf{v})$ is the drift coefficient and $D(t, \mathbf{v})$ is the diffusion coefficient. We define also the distribution function $f(t, v)$ by means of

$$f(t, v) = 4\pi v^2 n(t, v) \quad , \quad (2)$$

and recall that $f(t, v)dv$ represents the number of particles per unit volume with a speed $v = |\mathbf{v}|$ in the range $[v, v + dv]$. We define also the distribution function in the energy space by means of $p(t, E)dE = f(t, v)dv$; the quantity $p(t, E)dE$ represents the number of particles per unit volume with an energy in the range $[E, E + dE]$. Eq.(1) can be written as a continuity equation in the energy space:

$$\frac{\partial p(t, E)}{\partial t} + \frac{\partial \Phi(t, E)}{\partial E} = 0 \quad . \quad (3)$$

The quantity $\Phi(t, E)$ represents the particle current in the energy space and gives the flux of particles outcoming from a sphere of radius $v = \sqrt{2E/m}$ in the velocity space and has the following expression:

$$\Phi(t, E) = - \left(\hat{J}(t, E) + \frac{\partial \hat{D}(t, E)}{\partial E} \right) p(t, E) - \hat{D}(t, E) \frac{\partial p(t, E)}{\partial E} \quad , \quad (4)$$

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$$\widehat{D}(t, E) = 2mED(t, E) \quad , \quad (5)$$

$$\widehat{J}(t, E) = 2EJ_s(t, E) - 3mD(t, E) \quad . \quad (6)$$

The scalar drift coefficient $J_s(t, E)$ is defined through:

$$\mathbf{J}(t, \mathbf{v}) = \mathbf{v}J_s(t, E) \quad . \quad (7)$$

The stationary distributions, solutions of Eq.(3), can be obtained by setting the flux of Eq.(4) equal to zero. We obtain:

$$p(E) = A_1 \frac{\sqrt{E}}{D(E)} \exp \left[- \int \frac{J_s(E)}{mD(E)} dE \right] \quad . \quad (8)$$

We have also

$$f(E) = \sqrt{2mE}p(E) \quad (9)$$

and taking into account Eq.(8) we find:

$$f(E) = A_2 \frac{E}{D(E)} \exp \left[- \int \frac{J_s(E)}{mD(E)} dE \right] \quad . \quad (10)$$

A_1 and A_2 are normalization constants obtained by imposing the condition

$$\int_0^\infty f(t, v) dv = \int_0^\infty p(t, E) dE = \left(\frac{h}{m} \right)^3 \frac{N}{V} \quad , \quad (11)$$

where h is Planck's constant and N is the number of particles contained in the volume V . In the following we consider interacting particles. Their interaction can be simulated by means of additional terms in the drift and diffusion coefficients in comparison to the Brownian expressions. We pose

$$J_s(E) = c_0 + c_1 \alpha \epsilon \quad , \quad (12)$$

$$D(E) = \frac{c_0}{\beta m} + \frac{c_2}{\beta m} \alpha \epsilon \quad , \quad (13)$$

$$\epsilon = \beta E \quad , \quad \beta = 1/kT \quad . \quad (14)$$

In the case $c_1 = c_2 = 0$, Eqs. (12) and (13) define Brownian motions. We introduce the parameters $\chi_0 = c_0/c_2$ and $\chi_1 = c_1/c_2$, the expression of the distribution $p(E)$ and $f(E)$ becomes

$$p(E) = B_1 \sqrt{\alpha \epsilon} \exp(-\chi_1 \alpha \epsilon) \left(1 + \frac{\alpha \epsilon}{\chi_0} \right)^{\chi_1 \chi_0 - \chi_0 - 1} \quad , \quad (15)$$

$$f(E) = B_2 \alpha \epsilon \exp(-\chi_1 \alpha \epsilon) \left(1 + \frac{\alpha \epsilon}{\chi_0} \right)^{\chi_1 \chi_0 - \chi_0 - 1} \quad , \quad (16)$$

B_1 and B_2 are normalization constants that can be determined from Eq. (11). Eqs. (15) and (16) represent the main result of our work.

Fig.1 shows the behavior of the function $f(E)$, given by Eq.(16), when the parameters are chosen in such a way to have $\chi_0 = 1, \alpha = 1$, for different values of $\chi_1(0.5, 1, 3, 10)$. The temperatures of the systems described by the distributions b, c, d, e are less than the temperature of the Maxwellian distribution (curve a). We note a clear depletion in the region of the tails respect to the Maxwellian. As the depletion of the tail increases the temperature of the system decreases.

Fig.1 The distribution function $f(E)$ with the parameters $\chi_0 = 1, \alpha = 1$ and different values of χ_1 : a (Maxwellian), b ($\chi_1 = 10$), c ($\chi_1 = 3$), d ($\chi_1 = 1$), e ($\chi_1 = 0.5$).

In Fig.2 we report few (b, c, d, e) distributions $f(E)$ together with the Maxwellian (a) having chosen particular values of α (0.620, 0.760, 0.885, 0.995) in order to obtain the same temperature for the different distributions. The different curves correspond to different values of χ . We take $\chi_0 = \chi_1 = \chi = 1.6, 2.5, 5, 100$. We note that the curve $b(\alpha = 0.620, \chi = 1.6)$ is identical to the Maxwellian distribution. We obtain that the same system can be described by distributions corresponding to the same temperature but with different tails.

Fig.2 The distribution function $f(E)$ with $\chi_0 = \chi_1 = \chi$ and different values of χ and α . The curve a is the Maxwellian and coincides with the curve b ($\chi = 1.6, \alpha = 0.620$). All the curves a, b, c ($\chi = 2.5, \alpha = 0.760$), d ($\chi = 5, \alpha = 0.885$), e ($\chi = 100, \alpha = 0.995$) correspond to the same temperature.

At present, we are still working on the problem and we want to find the expression of the internal interactions and the external force field corresponding to the coefficients J and D as given by Eqs. (12) and (13). Of course, these interactions must represent both the collective effects of the plasma and the many-particle correlations. The values the coefficients α , χ_0 and χ_1 of Eqs. (15), (16) should assume in specific problems could be fixed ad hoc by inserting the statistical distributions in specific models.

Even with these limitations, we conclude the following. It is well known that the largest contribution to the reaction rates comes from the high-velocity tail of the distribution. The possibility of depleting the tail, without altering the temperature and the pressure could be relevant in the solution of the solar neutrino problem and in other astrophysical processes [4].

References

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